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STUDY OF THE DYNAMIC BEHAVIOR OF PLATES IMMERSED IN A FLUID

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Abstract: *This work is about the study of the dynamic behavior of cantilever plates, partially or totally immersed in water. To evaluate the contribution of the liquid on the body, an extensive experimental campaign, dedicated to the study of the resonance frequencies, was performed. On the basis of the results obtained, a model was developed that can determine the influence of water as a function of the width of the plate, the immersion's depth and the vibration mode. This was then implemented into a FEM code and the results were compared with the experimental tests.*

Key words: *plates, vibration, added mass, fluid, FEM.*

1. Introduction

The study of the dynamic fluid-structure interaction is of fundamental importance in all fields of engineering where structures come into contact with a fluid. Examples vary from the vibration analysis of structures immersed in water (vessels, offshore platform, submarine), to liquid retaining structures (dams, storage tank) that are subjected to earthquakes. This study is about the influence of fluids on the natural frequencies and mode shapes of vertical cantilever plates, partially or totally immersed in water.

In the literature, there are several articles that focus on the dynamic behavior of plates immersed in water, however in this work only the most integral studies will be mentioned.

Lindholm et al [1] experimentally investigated the first six-resonance frequencies of steel cantilever plates of differing thickness and width at changing immersion depths. The results were compared to a theoretical prediction based on the beam and thin plate theory in combination with the hydrodynamic strip theory. In order to agree with the experiments, the models where adjusted by using an empirical correction factor that depended on the vibration mode and the geometry of the plate.

Linag et al. [5] adopted an empirical added mass formulation to determine the resonance frequencies of vertical cantilevered plates completely submerged. To account for the aspect ratio of the plates (length/width) a correction factor was used, proposed by Pabst (1930). A commercial FEM code was also used, together with the added mass considered above, to calculate the resonance frequency. The results were then compared with Lindholm's experiments and other finite element methods.

Y. Kerboua et al. [8] studied the dynamic behavior of simply supported plates and cantilevered plates. They adopted Bernoulli's equation and the velocity potential together with the transverse displacement function of the plate, to express the fluid pressure acting on the structure. Then, the added mass was evaluated integrating the fluid pressure across the plate surface and solving it with the finite element method.

In this article we present the results of experiments and an empirical study of vertical cantilever plates partially or totally immersed in water. The effects of the aspect ratio (length/width), immersion depth, damping ratio and mode shapes on the first three resonance frequencies were experimentally investigated. Based on analytical models from literature (which are limited to the study of rigid and two-dimensional bodies) and experimental observations, an empirical method was developed that was able to determine the distribution of the added mass as a function of the width of the plate, immersion depth and the vibration mode.

2. Experiments

2.1. Experimental setup

An experimental setup was designed to investigate dynamic behavior of vertical cantilevered plates at varying immersion depths (D). Fig. 1 shows a draft of the setup.

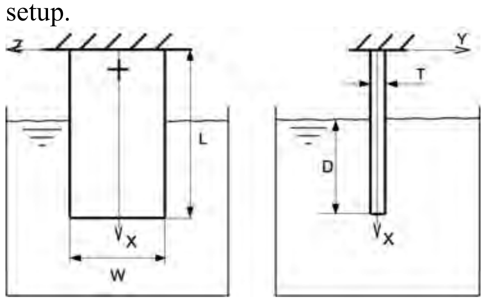

Fig. 1. *Draft of the experiment setup.*

To measure the resonance frequencies a strain gauge was selected that was sealed with silicon for waterproofing. In order for the sensor to register, it was positioned at 1/10 of plate's length from the joint, where the longitudinal deformations of the first three bending modes didn't have nodes. The sample was excited with an impulsive load to stimulate every vibrational mode. The signal recorded by the strain gauge was converted into digital and then analyzed with a code written by the authors. The program is based on an algorithm called Short-Time Fourier Transform that allows for the determination of the frequency and the amplitudes of the signal as it change over time. Briefly described, this technique consists of cutting the original signal into shorter segments, where each one corresponds to a precise point over time. Then the Fast Fourier Transform algorithm is applied to each segment and the generated results are assembled together in a spectrogram, which is a graph as a function of time, frequency and amplitude. Furthermore, the "Hanning window" together with the zero padding technique was applied at each segment in order to reduce the leakage of the spectrum and increase the frequency resolution. As will be demonstrated below, this particular technique allows us to estimate the resonance frequencies in addition to the damping ratio, and determine whether the dynamic contribution of the fluid on the body is constant or not.

Samples of different size and material were analyzed in order to investigate which variables really influence the grade of interaction between the fluid and the plate. Two types of material where used for the specimens: aluminum 6061-T6 (E=68900 Mpa, ρ=2700 Kg/m^3) and fiberglass epoxy (E=30300 Mpa, ρ=2000 Kg/m^3). The samples had diverse thicknesses (T=2-4 mm) and widths

(W=25-50-75-150-250 mm) while the length remained constant (L=300 mm). Each one was then tested at different immersion depths (D=20-30-40-50-75-100-125-150-200 mm) and their first three banding modes where analyzed.

The size of the chosen tank for immerging the plates was 800x450x400 mm (LxWxH), big enough to avoid the reflection of the waves against the walls and idealize it as an infinite fluid.

2.2. Experimental results

It is known that the dynamic behavior of any mechanical system, and therefore its resonance frequencies, dep ends on three parameters: inertia, damping and stiffness. However, while for a pl ate vibrating in vacuum these factors are considered constant, the same assertion is not so foreseeable when the structure is immerged in water. Th is is because i t is not considered a p riori if the dynamic contribution of the flui d on the body will be constant. For t his reason, the f ree vibrations of the plate immerged in the water were studied at different points in time. Graphs a and b in Fig. 2 respectively show the evolution of the first resonance frequency over time and the amplitude of a plate with half of its length immersed.

The two graphs sho w that the resonance frequency is independent from its oscillation amplitude; this means that the fluid-plate system can be considered linear and the d ynamic parameters constant. Therefore, under the simplified hypothesis of viscous damping proportional to the velocity (linear system), the free vibration of the plate immerged can be described by the following equation:

$$M\ddot{x} + C\dot{x} + Kx = 0 \qquad (1)$$

where M, C and K denote the generalized mass, damping and stiffness matrices. Solving the eigenvalue problem, expressed by Eq. 1 (in case of proportional damping) yields the *r*th damped natural frequency ω_{d_r} [rad/s]:

$$\omega_{d_r} = \omega_{n_r}\sqrt{1-\xi_r{}^2} \;\; with\text{: } \omega_{n_r} = \sqrt{\frac{k_r}{m_r}} \qquad (2)$$

where m_r, ξ_r and k_r are the *r*th modals mass, damping ratio and stiffness, respectively. ω_{n_r} is the *r*th natural frequency [rad/s]. The goal is t o assess which of these parameters are relevant for the natural frequencies and which are really affected by the fluid. To evaluate the order of magnitude of the modal damping ratio in water, the log arithmic decrement method was used:

$$\ln\left(\frac{X_r(t_1)}{X_r(t_1+\Delta t)}\right) = \frac{2\pi f_{d_r}\Delta t \xi_r}{\sqrt{1-\xi_r^2}} \qquad (3)$$

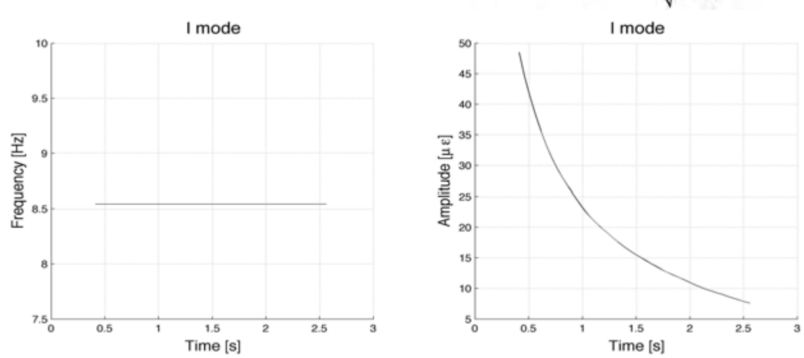

Fig. 2. *Evolution in time of the first resonance frequency and its amplitude for an aluminum plate of dimension 2x300x50 mm, immerse for 150 mm.*

where f_{d_r} is the *r*th damped natural frequency [1/s]; $X_r(t_1)$ and $X_r(t_1+\Delta t)$ the amplitudes of the *r*th vibration mode [$\mu\varepsilon$], registered by the strain gauge in two different temporal instants, Δt [s] apart. The first three modal damping ratios calculated for the plate described in Fig. 2 result: $\xi_1 = 0.018$, $\xi_2 = 0.0063$ and $\xi_3 = 0.0023$. Then overestimating the modal damping ratio to a value of $\xi_r = 0.1$, the percentage error committed neglecting the damping in evaluate the natural frequencies of the system is:

$$E\% = \frac{\omega_{n_r} - \omega_{d_r}}{\omega_{d_r}} * 100 = 0.5\ \% \qquad (4)$$

Therefore, being that the dumping is irrelevant to the resonance frequency, it will be neglected in this model.

Now, being that the water is a fluid, by definition it is incapable of retaining elastic energy and consequently it doesn't contribute to the rigidity of the system. In conclusion, the effect of the fluid on the vibration behavior of the plate immerged in water can be express with good precision in terms of inertia. In the literature this contribution is known as added-mass.

The problem is to understand how this added mass is influenced by the plate's characteristics (and its dynamic behavior in a vacuum). Thus, Fig. 3 plots the experimental results of the first resonance frequency for plates of the same material, thickness and length, but with different widths, at varying depts. of the immersion.

As can be seen in the graph, all the plates show a reduction in the resonance frequency with an increasing immersion depth, and thus an increase in the wetted surface. This behavior can be explained by an increase of the added mass, and therefore the inertia of the system, which reduces the resonance frequency, according to equation (2). Therefore, the added mass seems to be strictly linked to the surface area of the body actually immerged.

It is also noted that at the beginning the curves do not diverge too much, while in the case of increasing depths the difference becomes more marked.

Now, considering the added mass as a volume of water that is moved by the motion of the plate: intuitively the height of this volume will be the greatest at the center of the wet surface with a tendency to decrease the closer it gets to the edges of

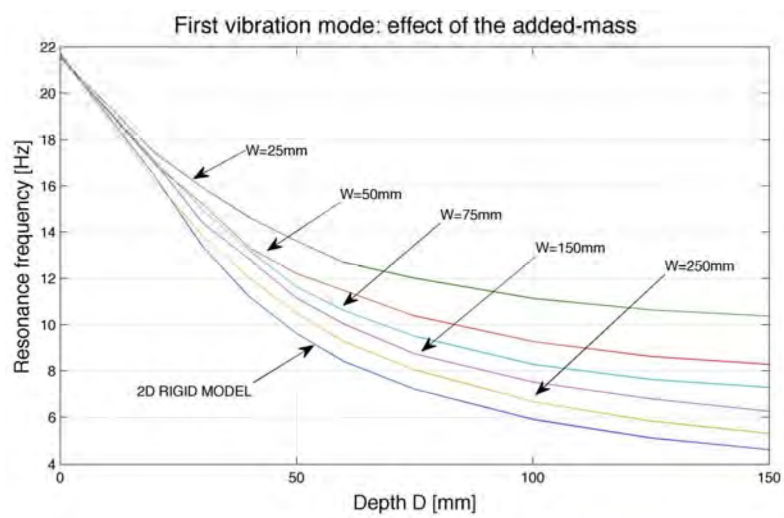

Fig. 3. *Chart summarizing the evolution of the first resonance frequency as function of its depth for aluminum plates of dimension 2x300 mm and different width (W)*

the plate. This is because the water is not restrained by any walls, and it will tend to slip away. The prior phenomenon has been defined as the border effects, and it will be more marked towards the shorter edge of the rectangular area that represents the wetted surface.

Thus, narrower plates at the growing immersion depths will be more affected by the borders effect, having a ratio perimeter/ wetted surface higher than the larger plates. Consequently, the resonance frequency for the narrow plates will decrease less then that of the large ones.

3. Model

There are many analytical models that evaluate the added mass associated with a rigid body translating in ideal fluid ($\rho = cost.$ & $\mu = 0$) with irrotational flow. In the literature the body with the most similar shape to the one studied, is a rigid plate modeled in two dimensions (therefore with a supposed infinite width). The added-mass associated to the plate with length $2a$ and width 1 (bi-dimension), translating with perpendicular direction to its surface, described by:

$$M = \rho \, \pi \, a^2$$

that corresponds to the "water area" of the circumference circumscribed to the plate.

For this model, the hypothesis of incompressible fluid ($\rho = cost.$) is realistic being that the water is a liquid; also the assumption of inviscid fluid ($\mu = 0$) is acceptable, and is confirmed by the experimental results (dumping negligible). The hypothesis to idealize the flux as irrotational is reasonable because it has been successfully applied in many problems involving a body translating in water. But there are two limits to the analytical model:

-the body is considered bi-dimensional, while, as is observed in the experiments, the width (borders effects) influences the vibrational behavior of the plate,

-the body is considered rigid, while in this study it is elastic, otherwise it wouldn't vibrate.

Therefore an empirical model was developed based on the analytical model. To take into account the third dimension, the area of the circumference, with a radius equal to half of the immersion depth, was multiplied by the width of the plate. Then part of the volume was subtracted using a geometrical function dependent on the aspect ratio of the wetted surface (depth/width). In order to account for the elastic behavior of the body, the previous added mass distribution was multiplied by the mode shapes of the plate. An example of the mass distribution for the first three vibration modes for a plate with ¾ of its length immersed is shown in Fig. 4.

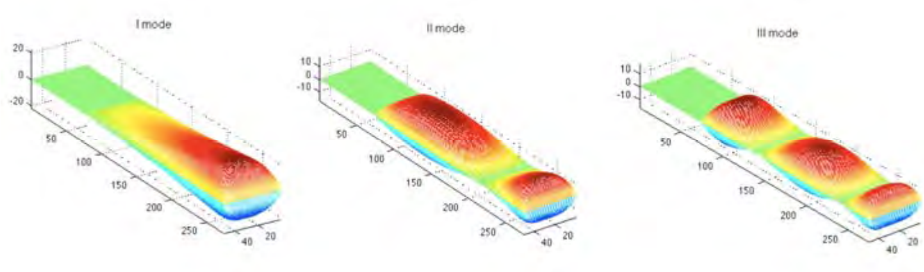

Fig. 4. *Example of the mass distribution for the first three vibration modes for a plate of dimension 2x50x275 mm, immersed 225 mm.*

4. Results and comparison

The proposed mass distribution was then implemented into a FE M code that generated the natural frequency and mode shapes of the cantilever plates. The results were then compared to the experimental data to confirm the validity of the model.

As can be observed in the ex amples in Fig. 5 shows the trend of th e first three resonance frequencies over immersion depth, the results generated by the model precisely describe the real behavior of the plate at any length/immersion depth ratio.

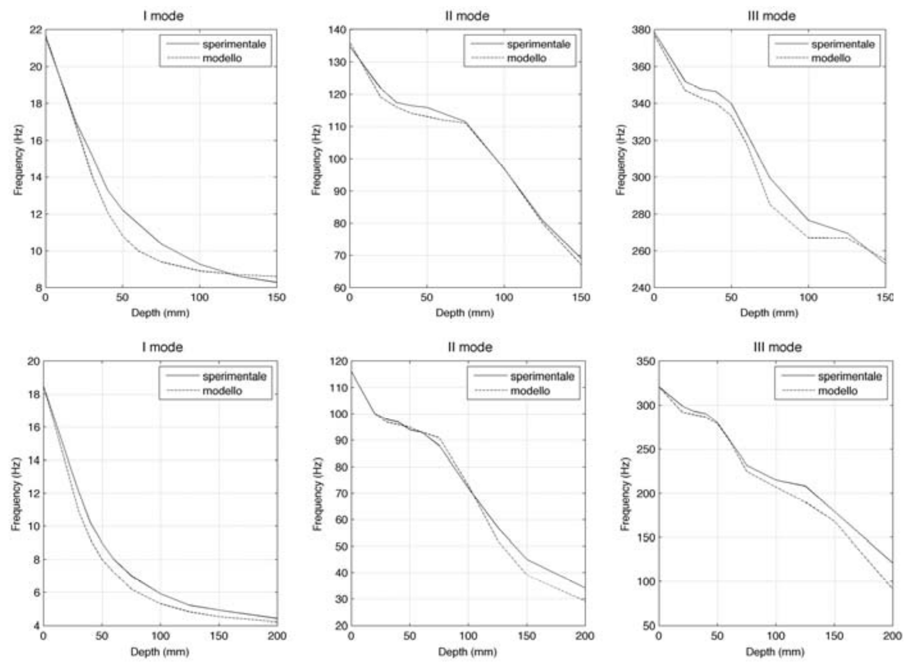

Fig. 5. *Variation of the I, II, and III r esonance frequencies as a function of immersion depth, for an aluminum plate with d imensions of 2x50x275 mm. and a fiberglass plate with dimensions of 2x150x275 mm.*

3. Conclusions

With the proposed model it is possible to estimate the dynamic behavior of elastic plates of any size that interact with a fluid, ensuring accuracy of the results independent of the material, geometry and depth of immersion.

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