IMEKO TC15 - Experimental Mechanics - Proceedings of the
11th Youth Symposium on Experimental Solid Mechanics
Brasov, ROMANIA, 2012, May 30-June 2

STOCHASTIC MODELING THE CHANGE IN AMPLITUDE RATIO AND MODULUS FACTOR IN CASE OF FATIGUE TEST OF PP SPECIMENS

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Abstract: *To realize a life-time prediction of polymer specimen in case of fatigue load a fatigue model based on laboratory test has been introduced in this paper. This fatigue model provides an approximate description of the three life stage of polypropylene (PP) specimens. Using a stochastic module containing the probabilistic calculations of the fatigue model developed will serve for qualitative analysis.*

Key words: *fatigue test, creep, fiber bundle, polypropylene, modeling fatigue*

1. Introduction

In the recent years we have conducted fatigue experiments on various specimens in the Biomechanical Research Centre of the Budapest University of Technology and Economics (BME). After studying the damage process, and the stiffness retardation process of the material we have built a unique mechanical model based on the *fiber bundle* theory.

The fiber bundle theory is based on the probability assumption that a fibrous structure is built of close fiber assemblies called bundles, but only capable of determining the stress-strain function in static tensile or bending case. Our model material model is capable of simulating the fatigue process of various homogenous, filled and fiber reinforced materials under tensile-tensile fatigue load.

An auxiliary stochastic model was built to cope with the stochastic nature of the fiber and the molecule chain orientation. In this paper the stochastic theory behind our model is described.

2. Experimental results

In 2006 we have conducted a long series of fatigue tests on polypropylene specimens, and we have we have described the three life stages [1] of the material. In order to predict the life expectancy of the polymer specimen we have presented a new fatigue model [2] based on our laboratory experiments. Although our fatigue model provided approximate description of the three life stages of the

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polypropylene specimen, and by applying the results of the stochastic calculations we can conduct qualitative analysis.

2.1. Modelling the material response

The model used to describe the specimen's damage mechanism consists of n_k Maxwell fibers and one Kelvin-Voight fiber in the kth cycle. The Maxwell fibers are representing the long PP macro-molecules while the Kelvin-Voight element represents the tab at the end of the specimen. [3,4].

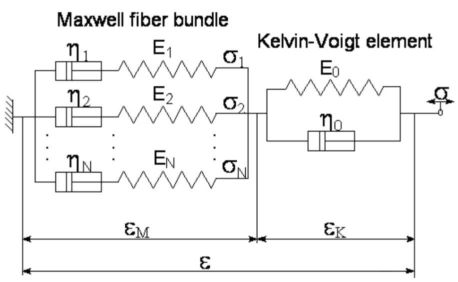

Fig. 1. *Analogous mechanical model*

This is a linear model with N Maxwell fibres at t=0. According to the linear behavior of the model the strain response can be calculated ad the sum of the strain response of the sub models for the stress excitation

$$(\sigma = \sigma_0 + \sigma_A \sin \varpi t = \sigma_K = \sigma_M).$$

By solving every component's differential equation we determined the local extremities of the strain response function in each and every cycle. Hence we can calculate the modulus factor in every cycle, which is the ratio of the tensile modulus of elasticity and the Young's modulus measured during static tensile tests.

2.2. Stochastic effects

The number of the untorn fibres $0 \leq n_k \leq N$ as a function of cycles can be described only by a stochastic probability function. In this function the number of the Maxwell fibers decreasing by one in random $0<t_{Bi}<\infty$ (i=1,…,N) moment, although these random moments are in connection with the excitation, material and model parameters. These connections are determined by the fiber breakage mechanism as shown in *Figure 2*, where the periodic part of the local extremities are separated from the stress and strain functions by high pass filters.

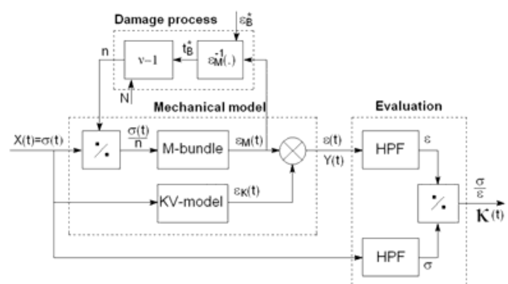

Fig. 2. *Block diagram of the signal processes and the evaluation*

In our analogous model we assume that the Kelvin-Voight element cannot be torn during cyclic loading, only the number of the Maxwell filaments is decreasing. The rupture of the ith Maxwell fiber happens at moment t_{Bi} if the strain response of the Maxwell bundle ε_M reaches the break elongation of the ith Maxwell fiber ε_{Bi}. Hence the following events are equivalent at an arbitrary t moment:

$$\{ith - fiber \quad torn \quad at \quad time \quad t\} =$$
$$\{t \geq t_{Bi}\} = \{\varepsilon_M(t) \geq \varepsilon_{Bi}\}$$

Because in the moment of the rupture of the i^{th} fiber (i=1,…,N) the following equation defines t_{Bi}

$$t_{Bi}: \ \varepsilon_M(t_{Bi}) = \varepsilon_{Bi},$$

which is also true for ordered sequences $\{t_{Bi}^*\}$ and $\{\varepsilon_{Bi}^*\}$. This is represented in figure 2 by the feedback block of the model.

It was assumed that fiber breakage occurs where the Maxwell strain function $\varepsilon_M(t)$ has its local peaks, and the break strain of the single fiber ε_{Bi} (i=1,…,N) is independent probabilistic value with same coefficient of variation and distribution function $Q_{\varepsilon B}(x)$, x>0 as the Maxwell fiber's model parameters. The $\varepsilon_M(t)$ strain function is monotonous increasing in it's local peaks hence t_{Bi} and ε_{Bi} variables have the following connection between:

$$Q_{t_B}(t) = P\left(t_{Bi} < t\right) = P\left(\varepsilon_M(t_{Bi}) < t\right) =$$
$$P\left(\varepsilon_{Bi} < \varepsilon_M(t)\right) = Q_{\varepsilon_B}\left(\varepsilon_M(t)\right)$$

It hase defined the stress load of a single Maxwell fiber as $\sigma_{Mi} = \frac{\sigma_0}{n} + \frac{\sigma_A}{n}\sin\omega t$ which means that the load of the i^{th} fiber at time moment 't' is a function of the number of the intact fibers. This makes the stress and the strain functions of the Maxwell fibers probabilistic in nature: $\sigma_i=\sigma_i(t, \nu(t))$, $\varepsilon_M=\varepsilon_M(t, \nu(t))$. The stress of the i^{th} Maxwell fiber can be calculated as follows:

$$\sigma_i = \sigma_i(t,\nu(t)) = \frac{\sigma(t)}{\nu(t)}$$

Because the Maxwell boundle's strain value where the stress function has its local maximum higher than the strain at the stress function's local minimum at the same cycle the Maxwell boundle's ε_M strain at the moment of fiber breakage:

$$\varepsilon_{M\max} = \frac{\sum_{i=1}^{n_k}\left(c_M + \frac{\sigma_{0i}\left(k+0.25\right)}{\eta_i f} + \frac{\sigma_{Ai}}{E_i}\right)}{n_k}$$

$$c_M = \frac{\sigma_0}{E_{stat}} - \frac{\sigma_0}{E_0} + \frac{\sigma_A \eta_0 \varpi}{E_0^2 + \eta_0^2 \varpi^2} + \frac{1}{N\varpi}\sum_{i=1}^{N}\frac{\sigma_{Ai}}{\eta_i} -$$
$$\left(\frac{\sigma_A \eta_0 \varpi}{E_0^2 + \eta_0^2 \varpi^2} + \frac{\eta_0}{E_0 N}\sum_{i=1}^{N}\frac{\sigma_{0i}}{\eta_i} + \frac{\eta_0 \varpi}{E_0 N}\sum_{i=1}^{N}\frac{\sigma_{Ai}}{E_i} - \frac{\sigma_A \eta_0 \varpi}{E_0 E_{stat}}\right)$$

If all Maxwell fibers are broken the strain value of the bundle is not defined, we treat it as infinite. In the beginning of the simulating algorithm when we define the fibers we generate the Maxwell model parameters and the break strain value of the fibers. After generating the fibers we arrange the fibers in an order with increasing break strain $\varepsilon_{B1}^* < \varepsilon_{B2}^* < \ldots < \varepsilon_{BN}^*$ and we can calculate the $\nu(t)$ function for the entire fatigue process. [5].

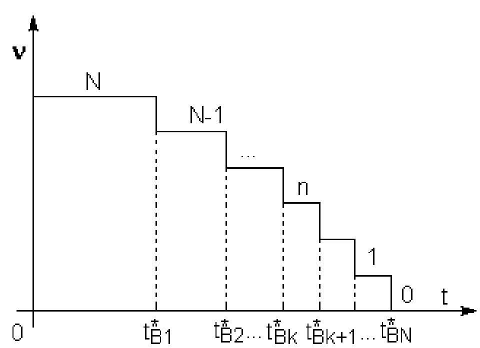

Fig. 3. *Number of intact fibers*

The probability of having n=N-k intact fibers at time moment 't' equals with the probability that time moment 't' is between the k^{th} and the $(k+1)^{th}$ fiber breakage moment. Based on the theory of ordered samples this probability can be calculated from the distribution function of the independent fiber breakage $Q_{tB}(t)$, t>0 by using the binomial distribution function [6, 7]:

$$p_n(t) = P(\nu(t) = n) = P\left(t^*_{B,N-n} < t < t^*_{B,N-n+1}\right) = \binom{N}{N-n}\left[Q_{t_B}(t)\right]^{N-n}\left[1 - Q_{t_B}(t)\right]^n$$

The prospective value and the variance of the number of the intact fibers at an arbitrary t≥0 time we can get the following relations deduced from the binomial distribution:

$$\bar{\nu}(t) = E(\nu(t)) = N\left(1 - Q_{t_B}(t)\right)$$

$$D^2(\nu(t)) = NQ_{t_B}(t)\left(1 - Q_{t_B}(t)\right)$$

It is well known that by increasing N the binomial distribution tends to a fixed $\Lambda(t)=N(1-Q_{tB}(t))$ parameter – independent from N – Poisson distribution which gives the number of torn fibers k (k=0,1,2,…) at time 't':

$$P(\nu(t) = n) = P(\kappa(t) = k) \xrightarrow[N \to \infty]{} \frac{\Lambda^k(t)}{k!} e^{-\Lambda(t)}$$

Where

$$E(\nu(t)) = D^2(\nu(t)) = \Lambda(t) = N\left(1 - Q_{t_B}(t)\right)$$

But this prospective value:

$$E(\nu(t)) = \frac{\sum_{n=1}^{N} \frac{1}{n}\binom{N}{n}\left[Q_{t_B}(t)\right]^{N-n}\left[1 - Q_{t_B}(t)\right]^n}{1 - \left[Q_{t_B}(t)\right]^N}$$

Is very difficult to calculate during simulation, so we are using the following simplifying formula which is much easier and quicker to calculate if n>0:

$$\frac{1}{n+1} < \frac{1}{n} \le \frac{1}{n+1} + \frac{3}{(n+1)(n+2)}$$

For great N we can under and overestimate the previous prospective value:

$$\frac{1}{(N+1)\left(1 - Q_{t_B}(t)\right)} < E(\nu(t)) \le \frac{1}{(N+1)\left(1 - Q_{t_B}(t)\right)} + \frac{1}{(N+1)(N+2)\left(1 - Q_{t_B}(t)\right)^2}$$

If t is fix the difference between the under and over estimates decreases with $1/N^2$ as N increasing, so we can estimate the Young's modulus at time 't' by using the following equation:

$$E(\nu(t)) \approx \frac{1}{(N+1)\left(1 - Q_{t_B}(t)\right)}$$

By using the equation above we can calculate the Young's modulus in each and every cycle during simulation, and we are able to calculate the modulus factor in every cycle.

$$\kappa(k) = \frac{E(\nu(t))}{E_{stat}}$$

3. Discussion

We have simulated the fatigue process with the same load parameters ($\sigma = 11.25 + 7.5\sin 62.83t$) as used during laboratory tests.

The model parameters were determined by curve fitting using the results of the static tensile tests:

KV element's stiffness: 3159.6 MPa

Dumping ratio of the KV element: 2194 MPas

Prospective value of the Maxwell fibers' stiffness: 5064MPa
Variance of the Maxwell fibers' stiffness: 500MPa
Prospective value of the Maxwell fibers' dumping ratio: 13140 MPas
Variance of the Maxwell fibers' dumping ratio: 1300 MPas
Prospective value of the Maxwell fibers' break strain: 0.214
Variance of the Maxwell fibers' break strain: 0.02

Every specimen was manufactured with the same injection molding technological parameters, so we assume that the model parameters have normal distribution.

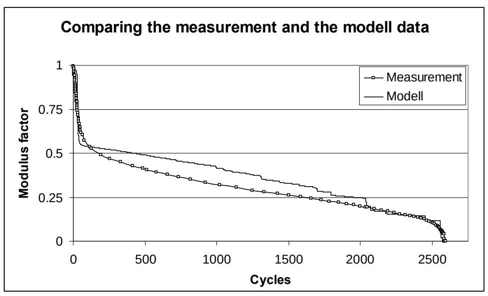

Fig. 4. *Comparing the measurement and the model data*

3. Conclusion remarks

The damage mechanism can be described quite well by this stochastic model, as the fiber breakage is the function of the cumulated damage in each fiber. This model represents exactly the actual damage process, because of the fact that fiber breakage is of probabilistic nature. Certainly, the developed auxiliary stochastic module is also capable of determining whether a Maxwell fiber is torn or just yield, as well as the tensile strain of the specimen.

We have studied the fatigue process of non-reinforced polypropylene specimens, but we have completed a series of laboratory tests on nanoparticle-reinforced polyamide [8] and carbon nanotube-reinforced epoxy specimens. Our further goal is to build up a knowledge center with data of several different materials and material parameters to use it as a design tool in the future.

Acknowledgement

This work is connected to the scientific program of the "*Development of quality-oriented and harmonized R+D+I strategy and functional model at BME*" This projects is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002). This work was also supported by the Hungarian Scientific Research Fund through grants OTKA T049069, NI 62729

References

1. Pető I.N., Borbás L. - Experimental results of the change in amplitude ratio during fatigue test of non reinforced polypropylene specimens, GEP, 2006/8
2. Pető I.N., Borbás L. – Modeling the change in modulus factor under tensile –tensile fatigue load, 7th International Multidisciplinary Conference Baia Mare, 2007
3. Bodor G., Vas L.M. - Structure and Properties of Polymers. (in Hungarian),. Műegyetemi Kiadó, Budapest, 2000
4. Aklonis, J.J., MacKnight W.J., and Shen, M. -Introduction to Polymer Viscoelasticity, Wiley-Interscience, J.Wiley&Sons, New York, 1972
5. Vas L.M., Rácz Zs. - (2004) Modeling and Testing the Fracture Process of Impregnated Carbon Fiber Roving Specimens During Bending Part I., Fiber Bundle Model. Journal of Composite Materials, Vol. 38. No.20. 1757-1785

6. Müller P.H. - Wahrscheinlichkeitsrechnung und Mathematische Statistik. Lexikon der Stochastik. 2. Auflage., Akademie-Verlag, Berlin, 1975
7. Feller W. - An Introduction to Probability Theory and its Applications. (in Hungarian), Műszaki Könyvkiadó, Budapest, 1978
8. Pető I.N., Borbás L. – Experimental results and modeling of the change in amplitude ratio during fatigue test of nano-paritcle filled polyamide specimens; 24th Danubia-Adria Symposium on Developments in Experimental Mechanics (DAS), Sibu, Romania, 2007; ISBN 978-973-739-456-9