IMEKO TC15 - Experimental Mechanics - Proceedings of the
11th Youth Symposium on Experimental Solid Mechanics
Brasov, ROMANIA, 2012, May 30-June 2

Simulation of the Tensile Modulus and the Tensile Strength of Short Fibre Reinforced Polymers (SFRP)

P. Stelzer[1], M. Reiter[1] and Z. Major[1]

Abstract: *This paper presents the simulation of the tensile modulus and the tensile strength of short glass fibre reinforced Polypropylene. Various formulae of the shear-lag theory were implemented in an analytical calculation program to evaluate the tensile strength values. Based on the homogenization approach the modulus values and the strength values, applying an existing failure criterion, were calculated with a material modelling software. It is shown the microstructure dependence, especially of the fibre weight fraction and fibre orientation, of those mechanical properties.*

Key words: *short fibre reinforced composites, strength, shear-lag model, homogenization, fibre angle, and weight fraction.*

[1]Institute of Polymer Product Engineering, Johannes Kepler University Linz, Austria.

1. Introduction and Objectives

Short fiber reinforced polymer matrix composites are increasingly used for lightweight structural components in demanding automotive applications. With increasing amount of short glass fibers, temperature and polymer matrix dependent tensile strength values in the range of 60 to 160 MPa can be achieved [1] simultaneously keeping the production costs of the parts on a low level. Moreover, the stiffness and the strength of these materials also highly depend on the fiber length and on the fiber orientation. In order to tap the full potential of these materials, accurate and reliable simulations considering all above effects are needed for describing the deformation and damage behavior.
During the processing the short fibers are oriented in the flow direction in the cavity. Based on various physical models and assumptions [2, 3], the fiber orientation is frequently predicted in injection molding simulation but this prediction can be deviated from the reality. Furthermore, computer tomography is an efficient tool for determining the real fiber orientation and length [4], but it is time consuming and expensive. The combination of both methods results in a realistic material structure model for the entire component volume.
Various models of the well-known shear-lag theory were implemented in an analytical calculation program (Mathcad of PTC, Needham, MA, US) and are partly described in this paper [5]. The fibre content and the fibre orientation dependence of the critical tensile strength were calculated using the experimental data generated previously [6].

There are some basic assumptions in the shear-lag theory:

- It doesn´t consider the non-linear material behaviour of the polymer matrix → linear elastic models
- Calculation with a constant interfacial shear strength.
- It doesn´t regard the interactions of the shear and tensile deformation.

The tensile strength of the composite is given by the rule-of-mixture as follows [7]:

$$\sigma_{uc} = \nu\sigma_F + (1-\nu)\sigma_M \qquad (1)$$

where σ_F, σ_M are the mean stresses of fibre and matrix at composite failure and ν is the fibre volume fraction.
Also important is the critical fibre length which is the minimal length when the axial fibre stress reaches the value of the fibre tensile strength, σ_{uF}:

$$l_c = \frac{r_f \sigma_{uF}}{\tau} \qquad (2)$$

with r_f the fibre radius and τ the inter-facial shear stress.
For an unidirectionally aligned SFRP with an uniform fibre length, l we get the mean fibre stress to:

$$\sigma_F = \sigma_{uF}\frac{l}{2l_c} \qquad \text{for } l < l_c \qquad (3)$$

$$\sigma_F = \sigma_{uF}\left(2 - (3l_c)/2l\right) \qquad \text{for } l \geq l_c \qquad (4)$$

Now we can rewrite equation (1) to

$$\sigma_{uc} = \chi\sigma_{uF}\nu + (1-\nu)\sigma_M \qquad (5)$$

with χ as fibre length factor defined as

$$\chi = \frac{l}{2l_c} \qquad \text{for } l < l_c \qquad (6)$$

$$\chi = \left(2 - \frac{5l_c}{2l}\right) \quad \text{for } l \geq l_c \qquad (7)$$

A more detailed description of these calculations along with explanation is found in [3].

Furthermore, microcells were constructed and the homogenization approach in Digimat MF (eXstream engineering, Foetz, Lx) module was used to calculate the the modulus values and the strength values applying various existing composite failure models [8].
The objective of this work was the calculation of the tensile strength by means of the shear-lag theory and to compare them with the results of the homogenization based simulation. In addition, tensile modulus values for various fiber orientations, weight fractions and for various matrix Poisson's ratio values were derived.

2. Materials and Simulation Parameters

The material parameters were estimated for the simulations based on a data set available in the laboratory. Nevertheless, the accurate determination of some parameters is not easy and requires special measurement technique and considerations. A homopolymer Polypropylene (PP(H)) was defined as matrix and glass fibres as inclusions. The material parameters for the shear-lag calculations are shown in Table 1. The tensile strength was calculated for three temperatures (-30°C, 23°C and 80°C) and for the fibre weight fractions listed below in the Table. The fibre aspect ratio (AR) is the ratio of the length and the diameter of the fibre and it varies usually over the range from 10 to 50 for a typical short fibre reinforced composite. Both the composite stiffness and the strength depends on the AR value or its distribution on the composite [9].

Material parameters for the calculations based on the shear-lag theory Table 1

Glass Fibre: σ_{uF}	2400 MPa
PP: σ_M (-30°C)	40 MPa
PP: σ_M (23°C)	25 MPa
PP: σ_M (80°C)	12 MPa
Interfacial shear strength τ	20 Mpa
Fibre weight fraction	{0,05;0,1;…;0,5}
Fibre diameter:	6 µm
AR	10
const. length (from AR)	60 µm

For the Digimat MF software simulations additional parameters were defined which are shown in Table 2.

Material parameters for the Digimat MF Table 2

Materials	**PP (RT)**	**Glass fibre**
Material model	Elasto-plastic	Liner elastic
Young´s modulus; MPa	1500	72000
Tensile strength; MPa	25	2400
Poisson´s ratio	0,42	0,2
Shear strength; MPa	20	

The microstructure parameters are shown in Table 3.

Microstructure paramter for the Digimat MF simulation Table 3

Fibre weight fraction	{0,05 ;0,1 ;…;0,5}
Fibre angle	0° to 90°

AR	10

To define the tensile strength the Tsai-Hill 2D failure indicator was used [5]. The Homogenization scheme was Mori-Tanaka which is well described in [8].
The simulation was conducted for room temperature (23°C) and additionally for various matrix Poisson´s ratios over a range of {0,05; 0,1;…;0,45 ;0,49}.

3. Results and discussion

First, the results of the shear-lag calculations are presented. The fibre weight fraction dependence of calculated tensile strength values based on the shear-lag theory for a short fibre reinforced PP compound with a fibre aspect ratio of 10 (represents rather a lower bound of an AR value for SFRPs) for three temperatures (-30°C, 23°C and 80°C) is shown in Fig. 1. As expected, the tensile strength of the composites increases with rising fibre weight fraction and is in good accordance with experimental data. The experimental data were obtained in [6].

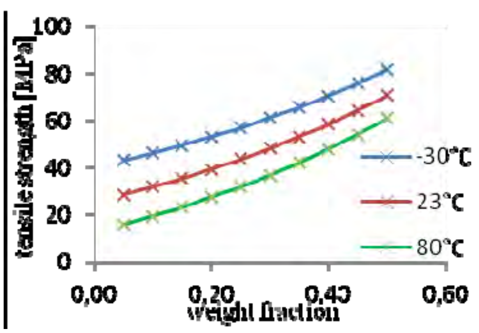

Fig. 1: *Calculated tensile strength over the weight fraction for 3 temperatures of an ud PP with glass fibres (AR of 10)*

Furthermore, the simulation results performed in Digimat MF for room temperature (23°C) are illustrated in terms of tensile modulus and tensile strength values. The fibre angle dependence of simulated tensile modulus values for a short fibre reinforced PP compund with a fibre aspect ratio of 10 for various weight fractions from 5 to 50 w% is shown in Fig. 2. One can observe that the tensile modulus strongly decreases up to about 40° and until a distinct local minimum value of fibre angle $\varphi_{Et,min}$. Then increases because of confinement due to Poisson´s ratio of the composite structure (fibres having lower angles to direction 2 (transverse) hinder the lateral contraction of the composite.

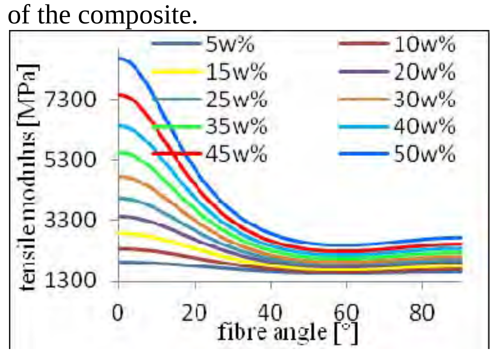

Fig. 2: *Tensile modulus over the fibre angle for various weight fractions of PP with glass fibres at 23°C (AR of 10)*

Furthermore, the fibre angle dependence of the simulated tensile modulus for a fibre weight fratcion of 10w% for various Poisson´s ratios is shown in Fig. 3.
For smaller Poisson´s ratios you get higher tensile modulus values above a fibre angle of about 60°. The higher the Poisson´s ratio the stronger is the constraint of the lateral contraction and hence the tensile modulus values are bigger after $\varphi_{Et,min}$. Furthermore with higher Poisson´s ratios the values of $\varphi_{Et,min}$ are shifted to lower angles.

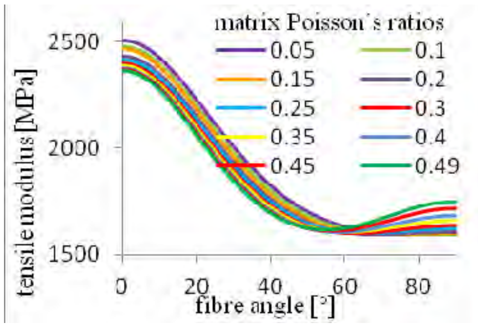

Fig. 3: *Tensile modulus over the fibre angle for various Poisson´s ratios of 10 w% of PP with glass fibres at 23°C (AR of 10)*

Moreover, the fibre angle dependence of simulated tensile strength values for a short fibre reinforced PP compound with a fibre aspect ratio of 10 for various weight fractions from 5 to 35w% is shown in Fig. 4. Similar to fibre angle dependence of tensile modulus values, the tensile strength decreases strongly until about 40° and then slightly. The simulations provided good results for weight fractions to about 35w%.

To define when the composite failure occurs we need failure indicators, which compare a given stress state with strengths of the composites [8]. Such an indicator is normalized and dimensionless whereas an indicator value smaller than one would mean a safe state of the composite and an indicator value bigger or equal one would stop the simulation and failure would occur.
For this work the Tsai-Hill 2D failure model was chosen. It considers also the shear deformation and assumes a planar stress state. This criterion was originally developed based on anisotropic yield assumptions for sheet metal forming. Hence it does not have any physical background for polymer matrix composites. On the other hand it is relatively simple and the values required for this model can easily be generated in conventional tests. The Tsai-Hill 2D failure indicator is given by:

$$f_A = \frac{\sigma^2_{11}}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma^2_{22}}{Y^2} + \frac{\sigma^2_{12}}{S^2} \quad (7)$$

With X as axial tensile or compressive strength, Y as in-plane tensile strength or compressive strength and S as transverse shear strength. A more detailed explanation is found in [8].

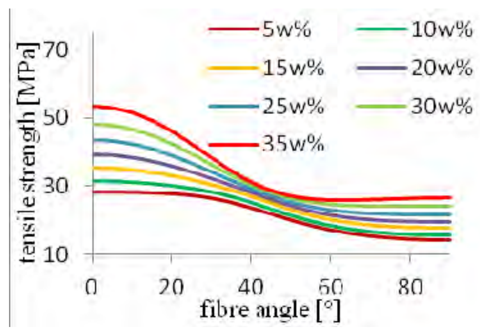

Fig. 4: *Tensile strength over the fibre angle for various weight fractions of PP with glass fibres at 23°C (AR of 10) with the Tsai-Hill 2D failure criterion*

4. Summary and Conclusion

While good agreement was found for fibre content with both approaches, only the homogenization approach provided reliable results for the fibre orientation dependence of the failure strength. These investigations will be extended with the first pseudo grain failure model (FPGFM, meso-scale model) which is implemented in the latest version of the DigiMat MF micromechanics software tool.

Acknowledgements

The research work of this paper was performed in the K_{APMT} project with contributions by Borealis Austria GmbH (Linz, A). The K_{APMT} is funded by the Austrian Government and the State Government of Upper Austria.

References

1. Kim, JK., Mai, YW.: Chapter 4, in *Engineered interface in fiber reinforced composites*, Elsevier, 1998, p. 93-170.
2. Fu, SY., Lauke B. and Mai, YW.: Chapter 2, in *Science and engineering of short fibre reinforced polymer composites*, Woodhead Publishing Ltd, 2009, p. 6-25.
3. Sigma GmbH: Sigmasoft v4.8 manual, Germany.
4. Cosmi, F., Bernasconi , A. and Sodini, N., *Phase contrast micro-tomography and morphological analysis of a short carbon fibre reinforced polyamide*, Composites Science and Technology 71 ,2011, p. 23–30.
5. Stelzer, P.: BSc. Thesis, Johannes Kepler University, Linz, 2012 in preparation.
6. Freudenthaler, P.: BSc. Thesis, Johannes Kepler University, Linz, 2012 in preparation.
7. Fu, SY., Lauke B. and Mai, YW.: Chapter 5, in *Science and engineering of short fibre reinforced polymer composites*, Woodhead Publishing Ltd, 2009, p. 80-115.
8. E-Xstream engineering: Digimat v4.2.1 documentation, January 2012.
9. Jerabek, M., Gastl, S., Herbst, H. and Major, Z.: *Modeling and Simulation of the Deformation Behavior of Particle Filled PP compounds*, 14^{TH} European Conference on Composite Materials, 7-10 June 2010, Budapest, Hungary, Paper ID: 172-ECCM14.