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COMBINING EXPERIMENTAL DATA AND NUMERICAL PROCESSING FOR FAILURE PROBABILITY EVALUATION OF COIL RETAINING RINGS

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Abstract: *Turbogenerator coil retaining rings (CRRs) are s hrunk fitted onto the rotor over the coils, in order to restrain them against the centrifugal force. They are typically subjected to low cycle fatigue (LCF), with a cycle being completed at every machine switch-on and switch-off. The object of this paper is to show a computationally efficient methodology, based on the AMV, to determine the failure probability of a CRR. Its estimation is based on the experimental evaluation of material static, cyclic and fatigue properties and on the knowledge of the entity of cyclic loads. The determined value, in the order of 10^{-11} at the last stage of the machine life, is compatible with reference values for structures under fatigue in the aeronautical field.*

Key words: *Turbogenerator Coil Retaining Rings (CRRs), Low Cycle Fatigue (LCF), Machine Safety, Probability of Failure, AMV Method.*

1. Introduction

The main components of a turbogenerator, the rotor and the coil retaining ring (CRR), are typical examples of devices experiencing low cycle fatigue (LCF). They are incredibly large sized machines: the rotor has 1.2 m diameter and 4.5 m length, while the CRRs applied at the rotor ends, have a quite short thickness, about 80 mm. The LCF load acting on the CRR arises from the constructive details of a turbogenerator, which can be briefly summarized as follows. The rotor exhibits uniformly spaced longitudinal slots. Copper conductors and insulating materials are packed into the slots and emerge at their ends, to form a coil. As the rotor spins (nominal speed of 3,000 rpm), the copper conductors are subjected to high centrifugal forces and must be restrained. Along most of the rotor length, they are constrained by metal wedges applied along the slots. At the ends, constraining is achieved by the shrink fit of CRRs onto the rotor body over the coils (Figure 1). Each CRR is finally fixed by locking keys against displacements in the axial direction. As the rotor spins, a strong centrifugal force is transmitted to the CRR by the copper masses. As a consequence, the ring swells in tension, and the stress state at its central part reaches its maximum values. Subsequently, when the machine is stopped, the centrifugal force decreases to zero and the stress is released. Consequently, a LCF cycle is completed at every machine switch on and switch off. A

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similar effect causes LCF on the rotor too, in particular, in the zone involved by the shrink-fit coupling. The expected number of cycles in the whole machine life is between 10,000 and 15,000. The consequences of an on service failure involving the rotor or the CRR may be very serious, since an explosion is likely to occur [7].

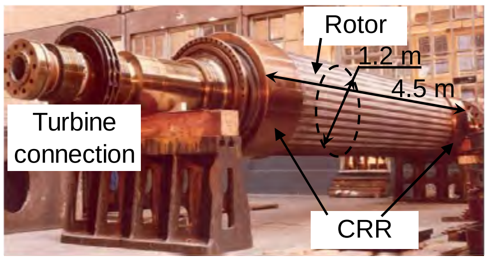

Fig. 1. *Turbogenerator layout*

The performance and the resistance of an engineered system is often affected by several uncertainties [3]. Being able to manage and estimate the effects of uncertainty on the design performance is now becoming more and more important, since a not full comprehension may lead to unexpected failures. Probabilistic analyses are often based on the experimental knowledge of the structural behaviour [3]. In the turbogenerator field a reliability quantitative analysis is also important: the conventional approach to design requires the execution of deterministic calculations, based on the typical models for LCF, with the simulation of the stress-strain hysteresis loops at most stressed locations and the estimation of the residual life by the adopted strain-life model. Managing the uncertainties affecting this result and their effects means being able to determine the probability of failure for the most critical failure modes.

The aforementioned analysis can be implemented as follows. Let $\underline{U}$ be the vector of the r input variables U_i and let Y be the output variable. Let g be the functional relationship between the effect Y and its causes $\underline{U}$: $Y = g(\underline{U})$. The goal of the probabilistic analysis consists in the determination of the probability P of $Y \leq y$, given the distributions of the random inputs and being y a threshold value. The curve relating P to y, for different threshold values can be regarded as the cumulative distribution function (CDF) of Y. This problem can be approached in different ways. A possible solution consists in the integration of the joint probability density function of the random variable vector. However, this approach is usually too complicated [3]. A possible alternative consists in the use of the Monte Carlo (MC) method, however this sampling procedure has the drawback of being computationally expensive and inefficient for high reliability devices.

Otherwise, numerical methods can be adopted. A detailed description of the evolution of these methods and of related advantages is contained in [7]. Most of them are based on the determination of a polynomial approximation of the relationship between Y and the inputs. This approximation was initially determined in the neighbourhood of a specific point (MVFOSM): an important upgrade consisted in the development of the most probable point (MPP) methods. They performed the polynomial expansion at iteratively determined points, thus improving the accuracy of results. The most efficient numerical methods, FORM, AFOSM, and AMV [3], [9] can be regarded within this category.

2. Objectives

A survey of the literature concerned with turbines and turbogenerators showed that very few practical applications of probabilistic analyses are described [1], [5]. Moreover, in none of these publications the procedure for determining random variable distributions, based on the experimentally studied structural behaviour, is exposed.

The AMV method is often cited as a suitable and powerful tool for probabilistic analyses [8], but a detailed description of the analytical steps to be followed as a general procedure is often missing.

The subject of the present paper consists in the determination of the probability of failure and of the safety index (having the meaning of a safety coefficient in the probabilistic environment) of the previously mentioned generator CRR. In particular, the present study focuses on the failure mode of the CRR cracking in tension, when it swells under centrifugal force. The calculation is performed by combining experimental data [6], related to the static, cyclic and fatigue curves of the involved material, with information regarding the local stress and strain states. From the methodological point of view, full details are provided on the main steps of the applied numerical procedure, based on the AMV method. The present paper describes in detail the efficient and general procedure to process the experimental data and to determine the distributions of the material variables.

3. Material variables and distributions

The most widely used material for CRR manufacturing is 18Mn18Cr. In the previous stage of the current research [6] it was experimentally characterized, by running static, cyclic and LCF tests on specimens machined from blanks of real CRRs. The Manson-Coffin curve was determined according to [4], by running 27 tests in the strain controlled mode, considering different strain amplitudes. The curve was determined by decomposing the total strain amplitude into its elastic and plastic parts and by running linear regressions in the logarithmic scale. The analytical procedure is summarized by the following equations, Eqs. (1-3).

$$\Delta\varepsilon/2 = \Delta\varepsilon_{el.}/2 + \Delta\varepsilon_{pl.}/2 = \\ = \left(\sigma'_f/E\right)\left(2N\right)^b + \varepsilon'_f\left(2N\right)^c \tag{1}$$

$$Lg\left(\Delta\varepsilon_{el.}/2\right) = \\ = Lg\left(\sigma'_f/E\right) + b\cdot Lg\left(2N\right) \tag{2}$$

$$Lg\left(\Delta\varepsilon_{pl.}/2\right) = \\ = Lg\left(\varepsilon'_f\right) + c\cdot Lg\left(2N\right) \tag{3}$$

The subscripts el. and pl. indicate strain amplitudes in the elastic and plastic fields, while σ'_f, ε'_f, b, c are respectively the fatigue strength and ductility coefficients with related exponents. A suitable statistical model [2] was adopted to determine the standard deviations of the slopes and of the constant terms of the straight lines in Eqs. (2-3). The important outcome was the consequent estimation of the Normal distributions for the variables $Lg(\sigma'_f)$, $Lg(\varepsilon'_f)$, b, c . The adoption of these types of distributions for the aforementioned coefficients was supported by [8], while the elastic modulus E was considered as a deterministic constant in agreement with [8]. The determination of the coefficient distributions has two important implications. First of all, it is possible to determine tolerance ranges to be applied to each material coefficient. From the graphical point of view, it is possible to determine lower (all the coefficients having their minimum value) and upper (max. values) bounds to be applied to the fatigue curve, accounting for the worst scenario of twice the standard deviation. The internal band indicates the maximum likelihood states in the strain-life model (Figure 2). The second reason of importance consists in being the first step for the development of a probabilistic calculation. The experimental campaign dealt with the determination of the static and cyclic curves too, again with a suitable number of specimens involved.

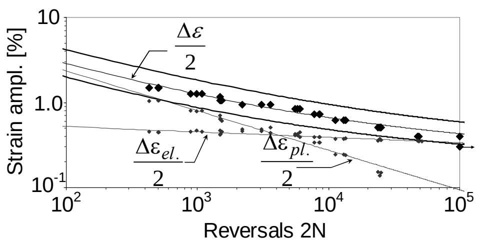

Fig. 2. *LCF curve with its bounds*

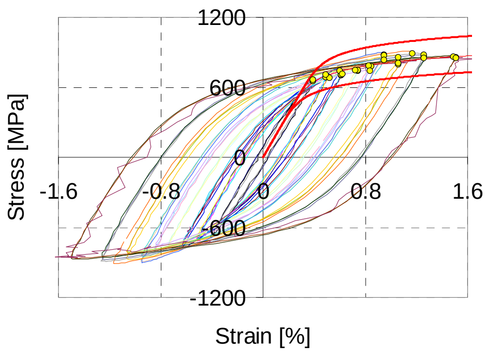

Fig. 3. *Cyclic curve as an envelope of the tips of steady-state loops with its bounds*

A similar model, based on logarithmic scale regressions and standard deviation estimations, was applied for static and cyclic curve determination (Figure 3). This further procedure led to the determination of the Normal distributions of *Lg(K)*, *Lg(K')*, *n* and *n'*, where the *K*-terms indicate Monotonic and Cyclic strength coefficients, while the *n*-terms are the related strain hardening exponents. The eight random variables, together with their Normal distribution parameters, are summarized in Table 1 (μ = mean nominal value, σ = standard deviation). The sampling of these eight coefficients made it possible to achieve a complete simulation of the local stress-strain hysteresis loops and to perform a computation of the residual life.

There is of course another factor having an influence on the structural response: the nominal state of load. In the present study it was intended as a deterministic constant, based on the stiff procedures of CRR mounting on the rotor, and of rotation at a strictly controlled speed. A detailed discussion on this topic is contained in [7]. Finally, the CRR life *N* was regarded as the output variable *Y*. According to [8], it was considered in a logarithmic scale.

The eight random variables Table 1.

Variable	μ	σ
$Lg(\sigma'_f) = U_1$	3.120	0.019
$Lg(\varepsilon'_f) = U_2$	-0.701	0.059
$b = U_3$	-0.063	0.005
$c = U_4$	-0.465	0.016
$n = U_5$	0.008	$3{\cdot}10^{-5}$
$Lg(K) = U_6$	3.112	$4{\cdot}10^{-5}$
$n' = U_7$	0.098	0.009
$Lg(K') = U_8$	3.131	0.023

4. Methodology

The AMV utilizes a Taylor expansion to achieve a suitable approximation of the function g in the neighbourhood of an expanding point $\underline{a} = [a_1, \ldots, a_r]^T$, in the multi-dimensional space of the *r* (in the present case *r* = 8) random variables. The expanding point should be coincident with the MPP, i.e. the point where the most suitable approximation is obtained. An iterative procedure is required for its determination: at the first step the expanding point is defined by the mean values of the involved variables.

$$Y(\underline{U}) = Lg(N) \cong = g(\underline{a}) + \sum_{i=1}^{r} \left.\frac{\partial g}{\partial U_i}\right|_{(\underline{a})} (U_i - a_i) \quad (4)$$

The Eq. (4) shows the first order

approximation of the function g, relating the inputs to the output. Previous studies [7] showed that, despite the non linear models in LCF, a linear approximation is sufficiently accurate, and very efficient. Each derivative term can be determined by the forward finite difference approach [3], approximating each derivative as a ratio of finite differences. The following step consists in the determination of the failure function h, i.e. of the boundary between failure and safe conditions. As shown in Eq. (5), where N_0 represents the number of cycles for which the probability of failure is computed, it is posed in the form of a polynomial equal to zero.

$$h(\underline{U}) = Lg(N) - Lg(N_0) = 0 \Leftrightarrow \Leftrightarrow h(\underline{U}) = Y(\underline{U}) - Lg(N_0) = 0 \qquad (5)$$

The further step consists in a change of the reference system: the random variables are replaced by reduced ones $\underline{u}$, according to Eq. (6). The new coordinates have a zero mean value and a standard deviation normalized to 1.

$$u_i = \frac{U_i - \mu(U_i)}{STD(U_i)} \qquad (6)$$

$$\begin{cases} h(\underline{u}^*) = 0 \\ \beta = \min\|\underline{u}^*\| \end{cases} \qquad (7)$$

$$p_f = \phi(-\beta) \qquad (8)$$

The final step consists in the determination of the safety index β, by solving [7] the constrained system in Eq. (7), with the unknowns β and $\underline{u}^*$. The vector $\underline{u}^*$ is the previously mentioned MPP and has the meaning of indicating the most critical state for the studied failure mode. It is related to the combination of values of the random variables, which may potentially lead to failure and are at the same time the closest to the respective mean values. The procedure is repeated by choosing the determined MPP as a new expanding point, until convergence of the results. The probability of failure p_f is finally determined by Eq. (8), where ϕ is the standard normal distribution function.

5. Results and Discussion

The CDF curve (Figure 4) of the CRR life was point by point determined by the described procedure: p_f was computed for different values of N_0 over a wide range.

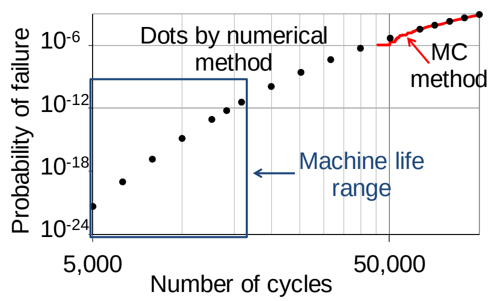

Fig. 4. *P. of failure p_f in the machine life*

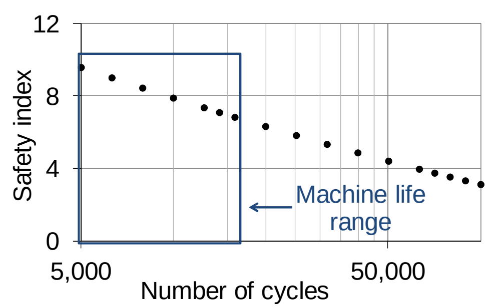

Fig. 5. *Safety index β in the machine life*

As a reference, in order to compare results and performance, a MC simulation was carried out with 10^6 iterations. The processing was supported by custom Matlab routines (release 7.9.0.529 (R2009b)). The MC simulation lasted about three hours and was not suitable to estimate probabilities lower than 10^{-6}. Whereas, the AMV based numerical approach was very efficient: in just 10 s the p_f level at each N_0 value could be estimated, as just two iterations were generally sufficient to get the convergence of the yields. The comparison of results for $p_f > 10^{-6}$ showed a very good agreement, the numerical approach made it possible to easily estimate even lower probabilities in the machine life range. The diagram in Fig. 5 shows the trend of the safety index β. It can be remarked that it assumes values greater than 6.5 in the machine life range. These values appear to be acceptable, if compared to reference values in [1], [5]. The estimated p_f is in the order of 10^{-18} after 6,000 cycles, of 10^{-14} after 10,000 cycles and of 10^{-11} at full life. It is important to remark that these values are in the typical range of the probability of failure of highly loaded components under LCF, whose breakage may have very serious consequences. For instance in [5] values in the range 10^{-12} – 10^{-8} were found with reference to a turbine disc for an aeronautical engine. It must be added that the computation of so low probabilities by the MC method would require too a long computational time (presumably years).

6. Conclusions

The subject of this paper consisted in the determination of the probability of failure of a turbogenerator CRR under LCF. The following points may be emphasized.

An original and efficient method was used to provide a probabilistic analysis based on LCF data, integrating experimentation scattering in the numerical model.

The obtained results proved to be accurate and acceptable, after a comparison with those of a MC simulation and with literature reference values.

The here considered failure mode is not the only one for turbogenerator components under LCF. The presented results must be integrated with those in [7], concerning another critical failure mode, i.e. rotor cracking at the shrink-fit location.

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