ELASTIC BUCKLING OF 2D CELLULAR STRUCTURES

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1. Introduction

Recent experimental [1] and numerical [2] studies have uncovered a fascinating pattern switching mechanism in 2D cellular structures under uniaxial compression. A square lattice of initially circular voids transforms into an array of mutually orthogonal ellipses as a result of a buckling instability triggered at a critical stress value. This reversible, repeatable phenomenon occurs globally throughout the sample and has been observed at the sub-micron level [3]. This has led to the manufacture of tunable optical devices which utilise this geometric switching to alter photonic band gaps in periodic structures [4]. The switch also causes a counter-intuitive auxetic effect [5].

Traditional studies of 2D cellular structures focus on the role of void size relative to the lattice on the response of these structures to compression. Here, we present the results of experiments which probe the influence of this parameter on 2D cellular structures possessing circular voids. These findings are compared with a simple spring-link model which uses potential energy minimization techniques to predict the collapse of the structure, and hence the onset of pattern switching, as a result of uni-axial compression.

2. Experimental Method

2D cellular structures possessing circular voids arranged on a square lattice are manufactured using the addition-curing silicone rubber "Sil AD Spezial" (Feguramed GmbH). A mixture of two fluids is poured into moulds comprising of 16 cylindrical inclusions (diameter 8.8mm) arranged on a 4 by 4 square lattice and bound by an adjustable aluminium perimeter (Fig. 1a). This is then allowed to set for one hour before removal from the mould. By varying the inter-hole distance, the relative volume fraction of the sample is changed.

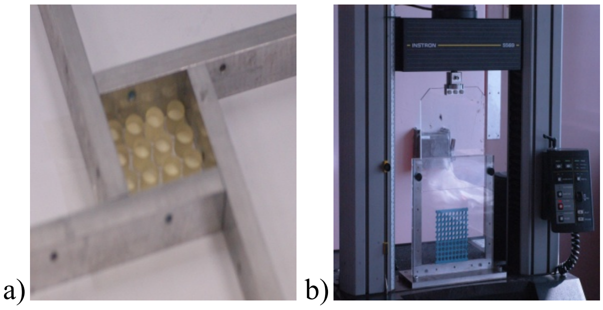

Fig. 1: a) The mould. b) A specimen stands in the housing on the "Instron 5569" machine.

Specimens are subjected to uniaxial compression tests using an "Instron 5569" machine. (Fig. 1b). The sample stands in a housing which prevents out of plane buckling and flour is used to reduce frictional effects. A perspex sheet is clamped to a 1kN load cell (#2525-806) in order to apply load to the surface of the sample.

3. Experimental Results and Analysis

A marked change in the shape of the voids is observed as the honeycomb is compressed. Initially the cells remain globally identical (Fig 2a). As the strain increases, the pattern switch to an array of mutually orthogonal ellipses is observed (Fig 2b),

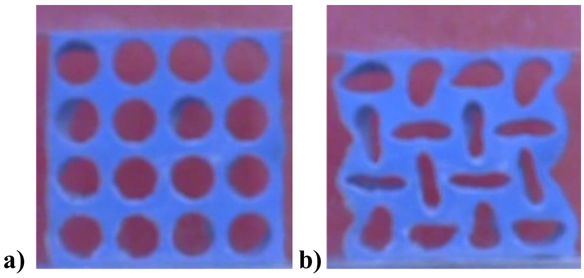

Fig 2: The honeycomb is shown prior (a) after (b) pattern switching.

The load-displacement data collected during experiments is converted into stress-strain data. The data take the form associated with cellular solids under compression, an initial linear regime which is followed by a plateau region of near-constant stress as the sample collapses. The critical stress value is determined by linear extrapolation techniques. A sample data set is shown in Figure 3.

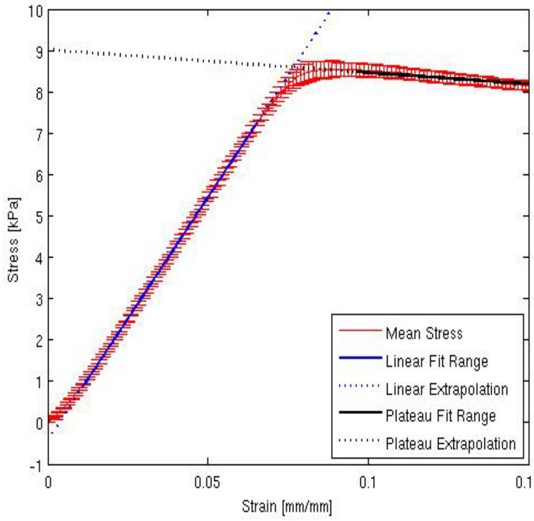

Fig. 3: The experimental stress-strain curve shows an initially linear compression regime followed by a plateau phase. The dotted lines mark linear extrapolation to find the critical stress value.

4. Discrete Model

The discrete model of the specimen is scaffold of rigid bars of length, l, equal to the inter-hole spacing. The bars are joined to one another by hinges, and where horizontal and vertical bars cross, the relative angle between the two is fixed to 90 degrees (Fig 4a). When the discrete model buckles, each rod is deflected by an angle α from its original orientation (Fig 4b).

The bending stiffness of the structure is represented by the existence of a rotational spring, stiffness ρ, at each hinge. The stiffness of each spring is calculated by considering the second moment of inertia of a tapered rod and its relationship to resistance to bending when compressed uniaxially. ρ is determined by the equation

$$\rho = E\left(\int_{\frac{-l}{2}}^{\frac{l}{2}} \frac{1}{I(x)} dx \right)^{-1}$$

where l is the inter-hole distance, E is the Young's modulus of the elastomer and $I(x)$ is the second moment of inertia of a tapered beam.

a) b)

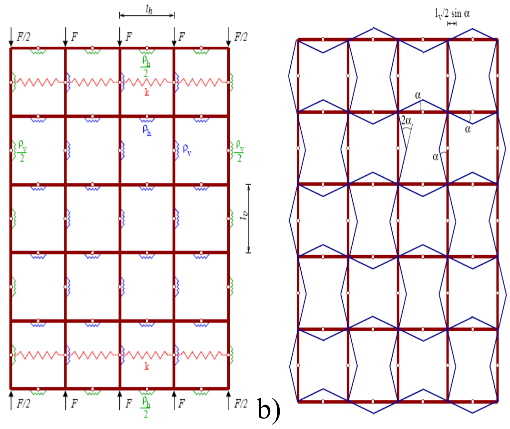

Fig. 4: a) The discrete curve is made up of crossed rigid rods and rotational springs which provide stiffness. b) The buckling of the model results in deflection of all rods about an angle α.

By considering the potential energy of the system and the stability of the trivial (α = 0) state it is possible to predict the critical stress as

$$\sigma_{CR} = \frac{8\rho}{wl^2}$$

where w is the width of the honeycomb, and ρ and l assume their previous definitions. It is possible to compare the predictions of this simple bending dominated model to those from experiments to gain further insight into the response of 2D cellular solids to compression.

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References

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