DIC MEASUREMENTS – A COMPARISON OF DIFFERENT METHODS TO EVALUATE THREE-DIMENSIONAL DEFORMATION STATES

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1. Introduction

More and more optical 3d deformation measurement systems are used to analyse material and component behaviour. Obvious advantages in the analyses - especially at high strain deformation states, where nonlinearity caused by material behaviour and kinematics, anisotropy or inhomogeneity plays a role - are pushing these methods. Nevertheless DIC-based systems only serve to observe surfaces of specimens or components. For the determination of a full 3d deformation state, there is a need of information about a material point behind the observed surface. Due to this fact it is only possible to get either deformation states relative to a local coordinate system of a surface element, or to determine the full 3d deformation state relative to a global coordinate system by making assumptions in thickness direction. The aim of this paper is to give a general view of how deformation states can be evaluated based on raw point coordinates.

2. Alternative one (global system)

Consider an arbitrary continuum body embedded in a 3-dimensional Euclidean space in different configurations at different times (Fig. 1). The position of its particles in the reference configuration (at time t=0) is described by the vector X_i. x_i refers to the position of the particle i in the current configuration (at time t). Here the points A, B and D are related to particles at the surface of the continuum (black). Point C is referred to a particle inside of the body (red). Assuming a linear displacement function, the deformation gradient F(X,t)=Grad u+I can be written as follows. Here x, y and z are related to the axes X_1, X_2 and X_3. Indices in capital letters are the components of the point in the reference configuration, lower case letters refer to the current configuration.

$$\boldsymbol{F}(\boldsymbol{X},t) = \begin{bmatrix} \frac{(x_b - x_a)}{(x_B - x_A)} & \frac{(x_d - x_a) - (x_D - x_A)}{(y_D - y_A)} & \frac{(x_c - x_a) - (x_C - x_A)}{(z_C - z_A)} \\ \frac{(y_b - y_a) - (y_B - y_A)}{(x_B - x_A)} & \frac{(y_d - y_a)}{(y_D - y_A)} & \frac{(y_c - y_a) - (y_C - y_A)}{(z_C - z_A)} \\ \frac{(z_b - z_a) - (z_B - z_A)}{(x_B - x_A)} & \frac{(z_d - z_a) - (z_D - z_A)}{(y_D - y_A)} & \frac{(z_c - z_a)}{(z_C - z_A)} \end{bmatrix}$$

A constant F can only map a parallelepiped to another parallelepiped. This means that the edges will always be lines, and opposed edges are always parallel. That is why only four points are needed to completely describe the volume element.

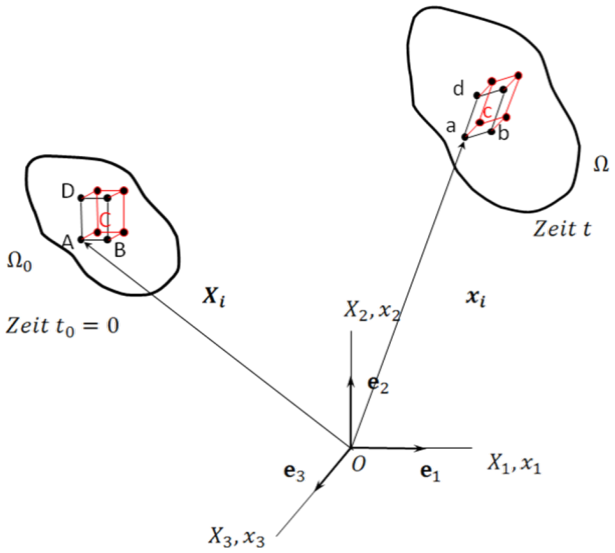

Fig. 1: continuum body

As you can see, six out of nine components of F can directly be calculated. With these two columns it is possible to directly obtain four components of the right Cauchy Green Tensor $C=F^TF$.

The deformation gradient can also be seen as a linear mapping of a vector. This mapping must hold for all three vectors that describe the parallelepiped. Two of the vectors ($\overrightarrow{\boldsymbol{AB}}$, $\overrightarrow{\boldsymbol{AD}}$) are known in both configurations, so there are six equations but nine unknowns. To derive the missing three equations it is necessary to make a hypothesis about the position of the points C and c.

A simple strategy is to assume that the vector $\overrightarrow{\boldsymbol{AC}}$ is perpendicular to $\overrightarrow{\boldsymbol{AB}}$ and $\overrightarrow{\boldsymbol{AD}}$ and has the

length t. We then further restrict ourselves by stating that $\overrightarrow{\boldsymbol{ac}}$ is perpendicular to $\overrightarrow{\boldsymbol{ab}}$ and $\overrightarrow{\boldsymbol{ad}}$. In fact, this assumption would completely agree with the realistic situation if no shear deformation in thickness direction took place.

Above assumptions lead to

$$\boldsymbol{X}_3 := \frac{\boldsymbol{X}_1 \times \boldsymbol{X}_2}{\|\boldsymbol{X}_1 \times \boldsymbol{X}_2\|}\, t, \boldsymbol{x}_3 := \|\boldsymbol{X}_3\| \frac{\boldsymbol{x}_1 \times \boldsymbol{x}_2}{\|\boldsymbol{x}_1 \times \boldsymbol{x}_2\|} \lambda_{\boldsymbol{x}_3}$$

where $\lambda_{\boldsymbol{x}_3}$ represents the stretch in thickness direction.

A plane strain state is given, when the material vector does not change its length ($\boldsymbol{\lambda_{x_3}} = \mathbf{1}$).

For other assumptions, one first has to calculate the first two eigenvalues $\boldsymbol{\epsilon_{1,2}}$ of C out of the first four directly computed components of C.

These are the first two squares of the main stretches. Assuming incompressibility the third principal stretch is calculated by $\boldsymbol{\lambda_{x_3}} = \frac{\mathbf{1}}{\sqrt{\epsilon_1}\cdot\sqrt{\epsilon_2}}$

Or the stretch in thickness direction could be equal to the minor stretch value for orthogonal material behavior $\boldsymbol{\lambda_{x_3}} = \sqrt{\boldsymbol{\epsilon_2}}$.

Using one of these choices it is possible to obtain the last missing vector to complete the linear system of equations.

Advantages of this procedure are the easy handling of assumptions and only very few calculation steps.

3. Alternative two (local systems)

A second alternative to validate a three dimensional deformation state of surfaces can be done by creating local coordinate systems. This can be achieved by reducing the 3- dimensional problem by one dimension. There are only three points needed to compute the full 2d deformation gradient. The local coordinate systems for both configurations must be parallel to the tangential surface, as shown in Fig. 2.

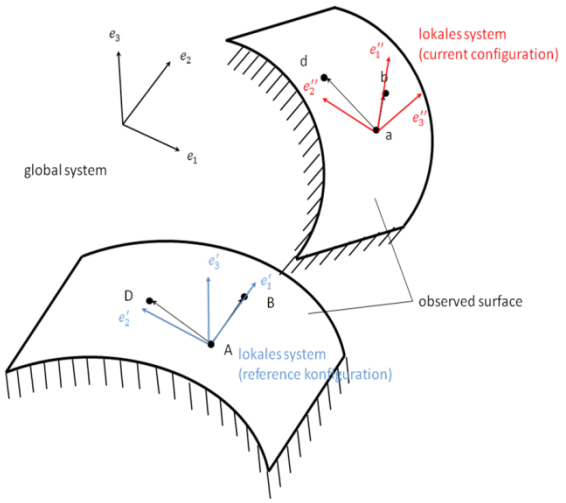

Fig. 2: definition of local systems

It is useful to create orthonormal local systems. One can imagine that only the direction of the local 3- direction is obvious - the normal to the tangential surface. The choice of the local 1- and 2-axis defines the basis for the right Cauchy-Green tensor in each point. Obviously these directions vary from point to point. To compute quantities which refer to only one system one has to transform the basis of the local deformation gradient. To do so, one has to expand local $\boldsymbol{F}$ by one dimension, because the 3- dimensional transformation matrices are 3 x 3, the local deformation gradient is 2 x 2. By expanding the local $\boldsymbol{F}$, the assumption in thickness direction comes into play. When one expands $\boldsymbol{F}$ like

$$F'^{,''}_{ab} = \begin{bmatrix} F'^{,''}_{ab\,11} & F'^{,''}_{ab\,12} & 0 \\ F'^{,''}_{ab\,21} & F'^{,''}_{ab\,22} & 0 \\ 0 & 0 & \lambda_{\boldsymbol{x}_3} \end{bmatrix}$$

the same assumptions are taken as described before. Please note that one needs to switch both bases of F by

$$\boldsymbol{F} = F'^{,''}_{ab}\, T'_{ia} T''_{jb}\, \boldsymbol{e}_i \otimes \boldsymbol{e}_j$$

with

$$\boldsymbol{e}'_{\mathrm{a}} = \mathrm{T}'_{\mathrm{ia}}[\boldsymbol{e}_{\mathrm{i}}], \boldsymbol{e}''_b = T''_{jb}[\boldsymbol{e}_j]$$

The advantage of this alternative is that both, local and global quantities can be calculated. Both alternatives lead under the same assumptions to the same global values of all kinematic tensors.