DETERMINATION OF THE PROBABILITY OF FAILURE OF TURBOGENERATOR ROTORS BASED ON LCF EXPERIMENTATION

Giorgio Olmi

DIEM Department, Engineering Faculty, University of Bologna, Viale del Risorgimento, 2, 40136 Bologna, Italy

Corresponding author: giorgio.olmi@unibo.it

1. Introduction

Turbogenerator rotors are a typical example of large components experiencing low cycle fatigue (LCF) [1]. Every machine switch-on and switch off corresponds to a LCF cycle, for a total amount of 10,000-15,000 cycles in the whole machine life. In [1], an experimental study was presented, concerning the characterization of two materials for rotor and coil retaining ring (CRR) manufacturing.

The design task of rotors and CRRs involves serious safety issues: incidents caused by unexpected failures may lead to rotor explosion with catastrophic effects. For this reason, the structural analysis must be integrated by the estimation of the probability of failure in the machine life stages, to be fulfilled by a suitable probabilistic method. However, there are very few papers in literature (e.g. [2-3]), tackling this issue. The object of this paper is to show a suitable methodology to quantify the safety of a rotor, starting by the knowledge of LCF experimental data and of the nominal loads on the shrink-fit coupling with the CRR.

2. Methodology and results

The fatigue curves determined in [1] where processed for the computation of the fatigue strength (σ'_f) and ductility (ε'_f) coefficients and of the fatigue exponents (b, c). In [4] it was shown that a statistical model [5] can be used for the determination of the standard deviations of the aforementioned material parameters. In Fig. 1 the fatigue curve of the rotor material is sketched together with is lower and upper bounds, to account for the worst scenario of twice the standard deviation. Afterwards, a log-normal distribution was presumed for σ'_f and ε'_f and a normal one for b and c [6-7]. A similar procedure was adopted for the determination of the normal distributions of the static and plastic coefficients of plasticity (K, K') and related hardening exponents (n, n'). The mean values (μ) and the standard deviations (σ) of the eight random variables are summarized in Table 1.

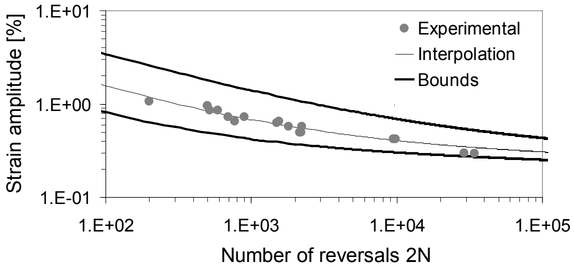

Fig. 1: **Fatigue curve together with its lower and upper bounds**

Table 1: **A summary of the eight random variables**

Variable	μ	σ
$Lg(\sigma'_f) = U_1$	2.948	0.010
$Lg(\varepsilon'_f) = U_2$	-0.830	0.122
$b = U_3$	-0.043	0.003
$c = U_4$	-0.546	0.039
$K = U_5$	1013 MPa	28.1 MPa
$n = U_6$	0.059	10^{-4}
$K' = U_7$	890 MPa	20 MPa
$n' = U_8$	0.052	$4\cdot 10^{-4}$

The determination of the probability of failure can be performed by running a Monte Carlo simulation, however this method is computationally expensive for problems with a high reliability. Alternative approximated methods, such as AFOSM or AMV [6-7] are based on a polynomial approximation of the functional relationship h between the inputs (listed in Table 1) and the output variable, the logarithm of the expected life *Lg (N) = U₀*.

$$Lg(N) = U_0 = h(\underline{a}) + \sum_{i=1}^{8} \left.\frac{\partial h}{\partial U_i}\right|_{(\underline{a})} (U_i - a_i) + + \frac{1}{2}\sum_{i=1}^{8} \left.\frac{\partial^2 h}{\partial U_i^2}\right|_{(\underline{a})} (U_i - a_i)^2 \qquad \textbf{Eq. 1}$$

The vector $\underline{a} = (a_1, \ldots a_8)$ indicates the expanding point: its components are initially selected as the mean values of the eight variables.

The derivative terms were determined by fitting procedure [6]. The following step consisted in rearranging Eq. 1 in the form of a failure function g, as in Eq. 2, where N_p indicates a generic life duration, for which the failure probability is estimated.

$$g(U_1,\ldots,U_8) = Lg(N) - Lg(N_p) = 0 \qquad \textbf{Eq. 2}$$

After introducing the reduced variables ($u_1,\ldots,u_8$), a safety index β was computed by solving the constrained problem (Eqq. 3-4).

$$u_i = \frac{U_i - \mu_{Ui}}{\sigma_{Ui}} \qquad \textbf{Eq. 3}$$

$$\begin{cases} g(\underline{u}^*) = 0 \\ \beta = \min \sqrt{\sum_{i=1}^{8} u_i^{*2}} \end{cases} \qquad \textbf{Eq. 4}$$

Finally, the probability of failure p_f was computed by applying Eq. 5, where ϕ is the standard normal distribution function.

$$p_f = \phi(-\beta) \qquad \textbf{Eq. 5}$$

The described procedure was then iterated until β convergence: the vector *($\underline{u}$*)* was considered as the expanding point at the following iteration. The machine safety was finally quantified, by computing the safety index and the probability of failure at different stages of the machine life (Fig. 2).

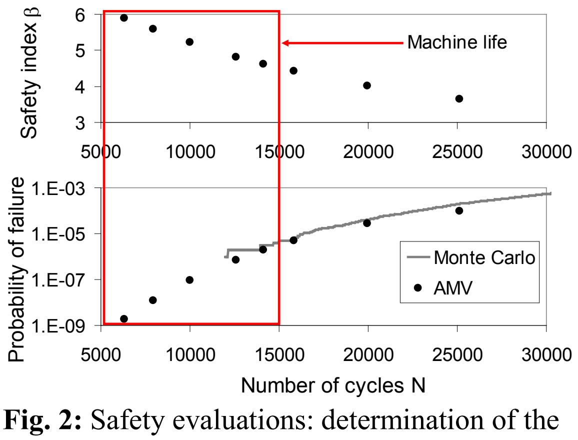

Fig. 2: Safety evaluations: determination of the safety index and of the probability of failure

3. Discussion and conclusive remarks

The following observations can be made with reference to the calculation procedure and the determined results.

- Considering K, n, K', n' as random variables was very important, to account for the scatter of static and cyclic curves and for the random local strain history, even under a deterministic nominal load.
- The proposed numerical procedure proved to work well: both first-order and second-order models were developed and both led to convergence after few iterations, with negligible computational times.
- The Monte Carlo method proved to be inefficient to calculate the probability of failure in the machine life range, but was used anyway to validate the results outside this range with a very good agreement.
- The determined values for p_f and β at the end of machine life, respectively $5\cdot10^{-6}$ and 4.4 are both acceptable with reference to the safety requirements of several structures under fatigue, even in the nuclear field [8].

References

[1] Olmi G., Freddi A., LCF characterization with sensitivity analyses of materials for turbogenerator coil retaining rings and rotors, Proc of the 27th DAS Symp (2010)
[2] Zhao Y.X., A methodology for strain-based fatigue reliability analysis, Reliab Eng Syst Saf, Vol. 70 (2000), 205–213
[3] Liu C.L., Lu Z.Z., Xu Y.L., Yue Z.F., Reliability analysis for low cycle fatigue life of the aeronautical engine turbine disc structure under random environment, Mater Sci Eng A, 395 (2005), 218–225
[4] Olmi G., Freddi A., Fatica oligociclica su cappe e rotori di turboalternatori: prove sperimentali, valutazione dell'anisotropia dei materiali, analisi di sensitività sui modelli di comportamento, Proc of the 39th AIAS National Conference (2010)
[5] Doyle J.F., Modern Experimental Stress Analysis, John Wiley and Sons Ltd., West Sussex, England (2004)
[6] Wu Y.T., Wirsching P. H., Advanced reliability method for fatigue analysis, J Eng Mech, ASCE, Vol. 110, 4 (1984), 536–553
[7] Wirsching P.H., Torng T.Y., Martin W.S., Advanced fatigue reliability analysis, Int J Fatigue, Vol. 13, 5 (1991), 389–394
[8] Avrithi K., Ayyub B.M., A Reliability-Based Approach for Low-Cycle Fatigue Design of Class 2 and 3 Nuclear Piping, J Press Vessel Technol, Vol. 132 (2010), 051202-1–6