XXI IMEKO World Congress “Measurement in Research and Industry”
August 30 – September 4, 2015, Prague, Czech Republic

A COMPARISON OF DC AND AC METHODS FOR CALIBRATION OF SEARCH COILS WITH A HIGH AREA TURNS VALUE

Michal Ulvr [1], Petr Kašpar [2]

[1]Czech Metrology Institute, Prague, Czech Republic, mulvr@cmi.cz
[2] Czech Technical University in Prague, Prague, Czech Republic, kaspar@fel.cvut.cz

Abstract – Search coils are widely used for measuring AC magnetic fields, and in special cases for measuring DC magnetic fields with the webermeter and for comparing magnetic flux density standards and magnetic flux standards. This paper compares the measuring equipment that is used and the accuracy that can be achieved by known DC and AC methods for calibrating search coils with a high area turns value.

Keywords: Area turns, calibration, magnetic field measurements, search coils, solenoid.

1. INTRODUCTION

Search coils (with a fluxmeter or an arithmetic mean value voltmeter) are very useful for measuring AC or DC magnetic fields. Because $\Phi = B \cdot NA$, where *NA* is area turns, search coils can be used for comparing magnetic flux density standards and magnetic flux standards. The Czech Metrology Institute (CMI) uses small multi-layer search coils with known area turns for measuring of devices for measuring saturation magnetization using the integrating method for measuring the magnetic moment or single-layer search coils for measuring an AC magnetic field up to 50 kHz. If very small AC magnetic flux density values (of the order of a few µT) on low frequencies (especially on the industrial frequency) need to be measured, a multi-layer search coil with a very high area turns value must be used. This type of coil can be used only for frequencies far below the resonance frequency. A high area turns value means high sensitivity, but also a high DC resistance value (for DC measurements) or a high impedance value (for AC measurements). The DC resistance or impedance value of this kind of coil can reach several tens of kΩ. It is important to know the area turns of these search coils with the greatest possible precision. A comparison was made of DC and AC calibration methods for multi-layer coils, and will be described here.

2. THEORY

Search coils are designed to provide a maximal approximation of the magnetic dipole. The constant of the magnetic dipole for a coil is its area turns. It is inconvenient to calculate the constant of the multi-layer search coil from its dimensions and the number of turns, as the accuracy of the constant calculation does not meet the requirements of fundamental metrology. This provides the motivation for the measurement methods described below.

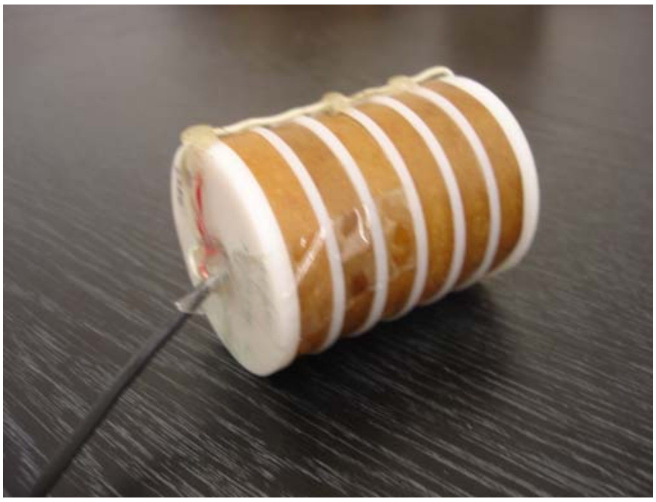

Fig. 1. Calibrated CTU search coil with a nominal value of about 90 m^2.

2.1. DC calibration methods

Generally, there are three DC calibration methods for search coils. The first two methods are based on compensating the magnetic flux from the search coil by variable mutual inductance (VMI) or by the zero differential method (ZDM), where the magnetic flux standard with a calibrated value is used [1], [2]. According to [1], the constant of the calibrated search coil can be determined as

$$K_S = \frac{K_E}{K_B}, \tag{1}$$

where K_E is the value of the variable mutual inductance, and K_B is the value of the constant of the magnetic flux density (MFD) standard. According to [2], the constant of the calibrated search coil can be determined as

$$K_S = \frac{K_E I_E}{K_B I_B} = \frac{K_E U_E R_B}{K_B U_B R_E}, \tag{2}$$

where K_E is the value of the magnetic flux standard, U_E is the voltage drop on the standard resistor R_E, U_B is the voltage drop on the standard resistor R_B and K_B is the value of the constant of the MFD standard.

The third and simplest calibration method for search coils (but with a much higher uncertainty value) is based on

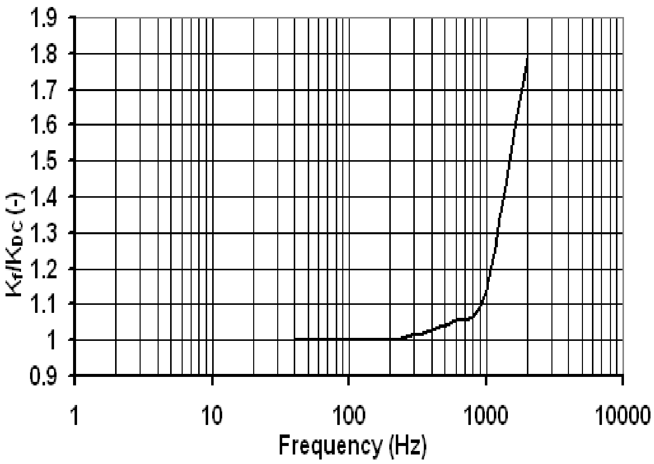

Fig. 2 Frequency dependence of the magnetic flux density standard with a nominal value of 100 mWb/A.

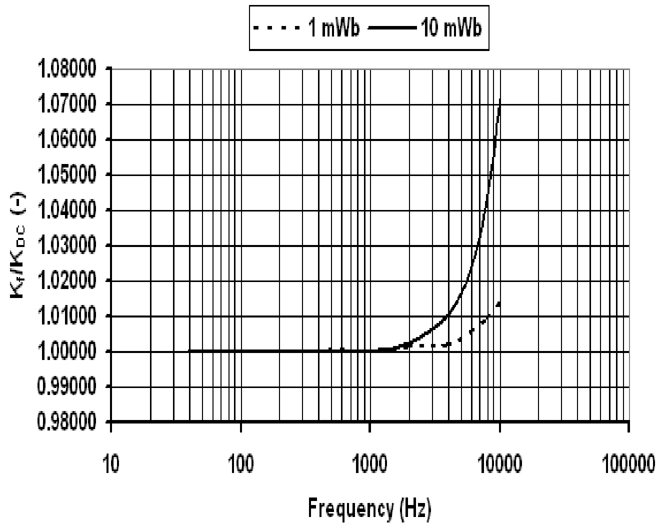

Fig. 3 Frequency dependence of the Tinsley type 4229B variable mutual inductance.

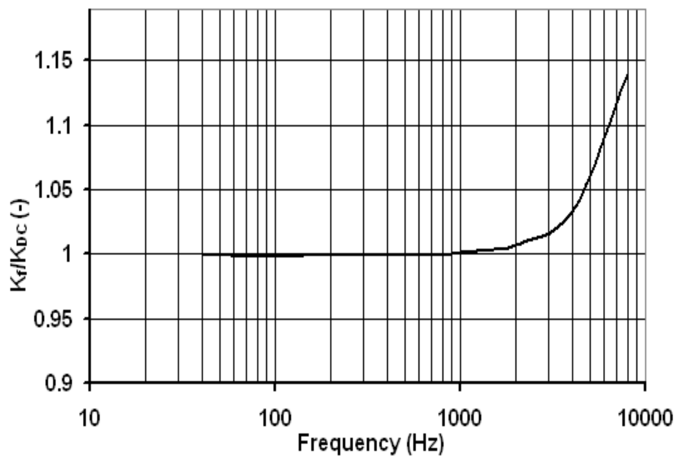

Fig. 4 Frequency dependence of the magnetic flux standard with a nominal value of 10 mWb/A.

placing the search coil in a known reference magnetic field and directly measuring the magnetic flux on the output of the search coil [3], [4]. Then the constant can be calculated as

$$K_S = \frac{\Delta\Phi R_N}{U_N K_B}, \tag{3}$$

where Φ is the magnetic flux value measured by the webermeter on the search coil output, U_N is the voltage drop on the standard resistor R_N, and K_B is the value of the constant of the MFD standard .

The null indicator for the VMI method or the ZDM method must be able to integrate the difference of the two voltage impulses, which can be time shifted, while their processes can be different. In cases when the effect of time shifting cannot be eliminated by the null indicator, the time constants of the secondary circuit (magnetic flux density standard and search coil) must be aligned. For all three methods, a high input resistance high-sensitivity webermeter must be used, due to the high resistance of the search coil, or it is necessary to use a webermeter, which allows the coil resistance to be used as the webermeter input resistance.

2.2. AC calibration methods

Generally, VMI and ZDM can be used for AC calibrations of search coils. The frequency range is limited by the magnetic flux standard, or especially by the variable mutual inductance that is used. At CMI, we have a Tinsley variable mutual inductance (type 4229B) and a special coil with double winding (mutual inductance) with a nominal value of 10 and 100 mWb/A as a magnetic flux standard. We need to know the frequency dependence of the variable mutual inductance and the magnetic flux standard. The frequency dependence was measured by comparison with a Sullivan universal inductance bridge (type A.C.1100) from 40 Hz up to 10 kHz. The measured frequency dependence results are shown in Fig. 3 and Fig. 4. It is obvious that variable mutual inductance and the magnetic flux standard with a nominal value of 10 mWb/A can be used up to 1000 Hz. If a magnetic flux standard with a nominal value of 100 mWb/A is needed, the system can be used up to 250 Hz only (according to Fig. 2). And, if AC calibrations are required, the lock-in amplifier or an oscilloscope in XY mode is used as a null indicator.

Of course, the simplest method for AC calibration of a multi-layer search coil is based on placing the search coil in the center of an MFD standard, and the output voltage of the search coil is measured by a digital multimeter. The MFD standard is powered from a generator and an amplifier, and the current I through the MFD is measured as the voltage drop U_N on the standard resistor R_N, using an Agilent 3458A digital multimeter. Assuming a sinusoidal waveform, the adjusted root mean square value for the magnetic flux density B_{RMS} inside the MFD is then calculated from a measured current I and the constant of the magnetic flux density standard that is used. Constant K_S of the calibrated search coil is then calculated as

$$K_S = \frac{U_{RMS}}{2\pi B_{RMS} f}, \tag{4}$$

where U_{RMS} is the RMS value of the output voltage of the search coil, measured using a Agilent 3458 digital multimeter. B_{RMS} is the adjusted value of the magnetic flux density inside the MFD, and f is the frequency measured by an HP 53131A digital counter.

There is another possible way to determine the constant of the multi-layer search coil. If there is no digital multimeter for measuring the mean output voltage value, a digital oscilloscope with mathematical functions (integration of the input signal) can be used. The search coil output is connected to the oscilloscope, and if the voltage has a sinusoidal waveform, integration on this signal can be applied. Then the amplitude of the magnetic flux from one period of the integration waveform can be measured. The constant of the search coil can then be calculated from

$$K_S = \frac{\Phi_m R_N}{K_B U_N \sqrt{2}}, \tag{5}$$

where Φ_m is the amplitude of the magnetic flux measured from one period, and K_B is the calibrated constant of the MFD. The use of an oscilloscope is not very accurate, so this method should be used only for indicative measurements.

Because the impedance of the coil is negligible in relation to the input resistance of the measuring device (the oscilloscope or the multimeter), it is necessary to make a correction to the measured value of the output voltage. The actual voltage value induced in the search coil is greater than the measured voltage value. The impedance of the coil to the input resistance of the device consists of a voltage divider, so it is necessary to multiply the value of K_S by the correction factor k, which is to be determined as

$$k = \frac{R_{in} + R_{sc}}{R_{in}}, \tag{6}$$

where R_{in} is the input resistance of the multimeter or oscilloscope, and R_{SC} is the resistance of the search coil.

As an alternative to measurements with an oscilloscope can also take advantage of the direct measurement method of the AC magnetic flux effective or maximal value from the search coil described in 2.1 DC calibration methods. However, it is necessary to use a webermeter, that is able to measure the AC magnetic flux. However, most of commercially available webermeters are not able to measured the sinusoidal waveform. The constant of the search coil can then be calculated according to (5).

Another option is to calculate the constant K_S from the frequency dependence of the coil constant, which can be determined aproximately from [5]

$$K_{SAC} = K_{SDC}\left(1 + \frac{f^2}{f_r^2}\right), \tag{7}$$

where K_{SDC} is the search coil constant (m^2), f is the frequency of the MFD generated in the MFD standard (Hz), and f_r is the resonance frequency value of the search coil (Hz). The change of K_{SAC} from K_{SDC} is 1 % for $f/f_r = 0.1$, and 4 % for $f/f_r = 0.2$. Formula (7) is then valid only in its approach for small changes of frequency f from resonance frequency f_r.

3. UNCERTAINTY ANALYSIS

3.1. Uncertainty analysis of the DC methods

Three DC methods for search coil calibration were described above. The type B relative uncertainty of the direct magnetic flux measurement method can be determined as

$$u_{B1} = \sqrt{u_\Phi^2 + u_{RN}^2 + u_{KB}^2 + u_V^2 + u_d^2 + u_h^2}, \tag{8}$$

where u_Φ is the standard uncertainty of the magnetic flux value measurement, u_{RN} is the uncertainty of the standard resistor, u_V is the uncertainty of the voltage drop measured on the standard resistor R_N, u_{KB} is the uncertainty of the constant of the MFD that is used, u_d is the uncertainty of the directional dependence measurement of the search coil, and u_h is the uncertainty of the influence of homogeneity inside the MFD in the volume of the search coil. The uncertainty of the measured voltage on the standard resistor is dependent on the specification of the digital multimeter that is used. The value of u_V can lie in the order of thousandths to hundredths of one percent. The maximum uncertainty value of the standard resistor usually varies in units to tens of ppm, so it can be neglected, because the other uncertainties are essentially higher. The uncertainty value of the MFD constant depends on the calibration method that is used - at CMI, we use direct comparison with the national coil standard or some of the NMR methods (forced precession or flowing water - nutation method). This means that the value of u_{KB} can lie in the order of hundredths of one percent. The uncertainty of the measured magnetic flux value depends on the webermeter that is used. The value of u_Φ can usually lie in the order of tenths of one percent. The value of the measured magnetic flux from the search coil output also depends on the direction in which this coil is inserted into the magnetic field. In principle, the value depends on cos φ, where φ is the angle between the axis of the search coil and the direction of the magnetic flux density vector. The search coil must be set to the position where cos φ = 1, which is the maximum value from the output of the search coil. For example, there is a difference of about 0.02 % from the true value of the measured voltage for φ = 1°, there is a difference of about 0.06 % for φ = 2°, and so on. This means that the value of u_d can lie in the order of hundredths to tenths of one percent. The value of u_h can lie in the order of tenths of one percent [6].

The type B relative uncertainty of the VMI method can be determined as

$$u_{B2} = \sqrt{u_{EV}^2 + u_{KB}^2}, \tag{9}$$

where u_{EV} is the uncertainty of the variable mutual inductance value and u_{KB} is the uncertainty of the constant of the MFD that is used, as previously. The variable mutual inductance value is determined by direct comparison with the national magnetic flux standard. This means that the value of u_{EV} can lie in the order of hundredths of one percent.

The type B relative uncertainty of the ZDM method can be determined as

$$u_{B3} = \sqrt{u_{EN}^2 + u_{KB}^2 + u_{RB}^2 + u_{UB}^2 + u_{RE}^2 + u_{UE}^2}, \tag{10}$$

where u_{EN} is the uncertainty of the magnetic flux standard value, u_{RB} is the uncertainty of the standard resistor R_B and u_{RE} is the uncertainty of the standard resistor R_E; u_{UB} is the uncertainty of the voltage drop measured on standard resistor R_B, and u_{UE} is the uncertainty of the voltage drop measured on standard resistor R_E; and u_{KB} is the uncertainty of the constant of the MFD, as previously. The magnetic flux standard value is determined by direct comparison with the national magnetic flux standard. This means that the value of u_{EN} can lie in the order of hundredths of one percent. The uncertainty value of the standard resistors usually varies in units to tens of ppm, as previously. The value of u_{UB} resp. u_{UE} can lie in the order of thousandths to hundredths of one percent, as previously.

3.2. Uncertainty analysis of the AC methods

Six AC methods for search coil calibration have been described in this paper. The type B relative uncertainty of the VMI and ZDM method for AC calibration can be calculated according to (9) and (10). However, the change in the constant value of the magnetic flux standard or the variable mutual inductance from the DC constant value and the other influences on the 50 Hz must be taken into account. The value of u_{UB} resp. u_{UE} can also lie in the order of hundredths of one percent.

The type B relative uncertainty of the RMS output voltage measurement method can be calculated as

$$u_{B4} = \sqrt{u_k^2 + u_{Brms}^2 + u_{Urms}^2 + u_d^2 + u_h^2}\ , \tag{11}$$

where u_{rms} is the uncertainty of the RMS voltage measurement, u_{Brms} is the uncertainty of the RMS magnetic flux density value sets inside the MFD, u_k is the uncertainty of the influence of the correction factor k and u_d and u_h have the same meaning as previously. The uncertainty of the u_{rms} depends on the digital voltmeter that is used. Its value can lie in the order of hundredths of one percent. The value of u_k can lie in the order of tenths on one percent. The value of u_{Brms} can lie in the order of hundredths to tenths on one percent. The type B relative uncertainty of the method with a digital oscilloscope or webermeter can be calculated as

$$u_{B5} = \sqrt{u_{\Phi m}^2 + u_k^2 + u_{KB}^2 + u_{RN}^2 + u_{UN}^2 + u_d^2 + u_h^2}\ , \tag{12}$$

where $u_{\Phi m}$ is the standard uncertainty of the AC magnetic flux maximal value measurement, and u_k, u_{KB}, u_{RN}, u_{UN}, u_d and u_h have the same meaning as previously. Most widely-used oscilloscopes have 8-bit A/D converters, representing 256 levels. The effective number of bits (ENOB) is in fact lower than the number given in the specification. When using signal averaging, the number of bits can be raised by 1 – 2 bits. Because we assume that the measurement is at low frequency (e.g. 50 Hz), it can be calculated with effective bit number ENOB = 8 bit by switched-on averaging. The sampling error then corresponds to the last significant bit LSB, which is 0.4 % FS. This means that the value of $u_{\Phi m}$ can lie in the order of tenths of one percent, when the digital oscilloscope is used. The accuracy of AC magnetic flux measurement by a webermeter is 1 %, when the LakeShore Model 480 webermeter is used. This means that the value of $u_{\Phi m}$ can lie in the order of tenths of one percent, as previously.

The type B absolute uncertainty of the method when constant K_S is calculated from the frequency dependence of the coil constant can be determined as

$$u_{B6} = \sqrt{\left[\left(1 + \frac{f^2}{f_r^2}\right) u_{KDC}\right]^2 + \left(-\frac{2K_{SDC} f^2}{f_r^3} u_{fr}\right)^2} \tag{13}$$

where u_{KDC} is the uncertainty of the DC coil constant, and u_{fr} is the uncertainty of the resonance frequency determination. The value of u_{KDC} depends on the method used for the DC coil calibration. Methods and uncertainty analysis for DC calibration were described above. Generally, the value of u_{KDC} can lie in the order of tenths of one percent. The resonance frequency value of the search coil can be determined by an RLC meter or in the frequency independent Helmholtz coils, where the frequency value of the current through the Helmholtz coils is changing and the maximum value of the output voltage from the search coil is measured. The value of u_{fr} can lie in the order of tenths of one percent.

4. EXPERIMENTAL RESULTS

A calibrated search coil with a nominal value of about 90 m^2 was designed and constructed at the Department of Measurement, Faculty of Electrical Engineering of the Czech Technical University in Prague (see Fig. 1). The frame of the CTU coil is made from Teflon. It is 50 mm in diameter, 68 mm in length, and the winding is divided into 6 sections to reduce the parasitic capacity of the winding. The CTU coil has 71500 turns of enamelled copper wire 0.056 mm in diameter. The DC resistance is 59.8 kΩ, the inductance is 90 H, and the resonance frequency value is 1200 Hz. This search coil was experimentally calibrated by the DC and AC methods described above.

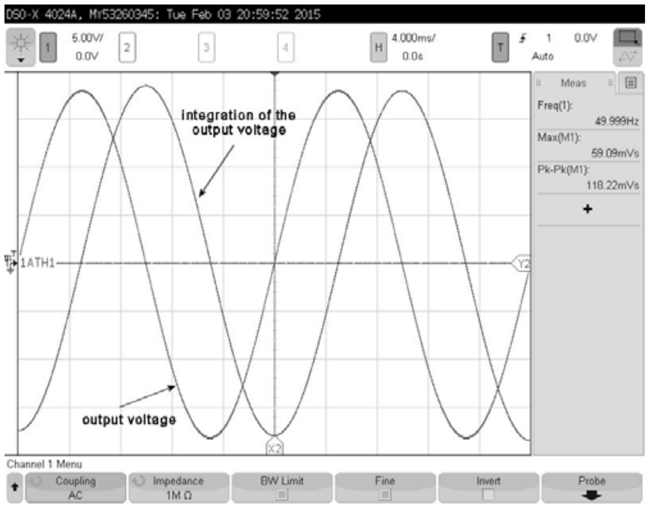

Fig. 5 The output voltage signal and its integration waveform from the DSO-X 4024A oscilloscope.

4.1. DC calibrations results

The CTU coil was calibrated by the VMI and ZDM methods, and also by the method with direct measurement of the magnetic flux using a webermeter. An LDJ 702P webermeter was used as a null indicator in the VMI and ZDM methods. When the magnetic flux from the coil is measured directly, a webermeter with high input resistance must be used. However, most commercially produced webermeters have an input resistance value of 1 or 10 kΩ. Exceptions include the Laboratorio Elettrofisico Digital Flux type and the Lake Shore Model 480 webermeter with input impedance of 100 kΩ. For our measurements, we used a Model 480 webermeter and a Magnet-Physik EF 5 type webermeter. If the coil connected to the EF 5 or Model 480 type webermeter has a resistance value higher than 10 kΩ, the input resistance can be manually changed to the value of the coil resistance, and the coil resistance becomes the input resistance of the webermeter. A Helmholtz type solenoid No. 1201 with a calibrated constant value of 0.105497 mT/A was used for the VMI and ZDM methods. A massive multi-layer Helmholtz coils with a calibrated constant value of 1.944092 mT/A was used for direct magnetic flux measurements, using a Model 480 and an EF 5 type webermeter. The results of the DC calibrations of the CTU coil for all four methods, including the expanded uncertainty value, are presented in Table 1.

Table 1. DC calibration results for the CTU search coil.

Method	Constant value Ks (m^2)	Expanded uncertainty (%)
VMI	91.23	0.12
ZDM	91.05	0.15
webermeter EF 5	91.30	0.56
webermeter Model 480	91.00	0.56

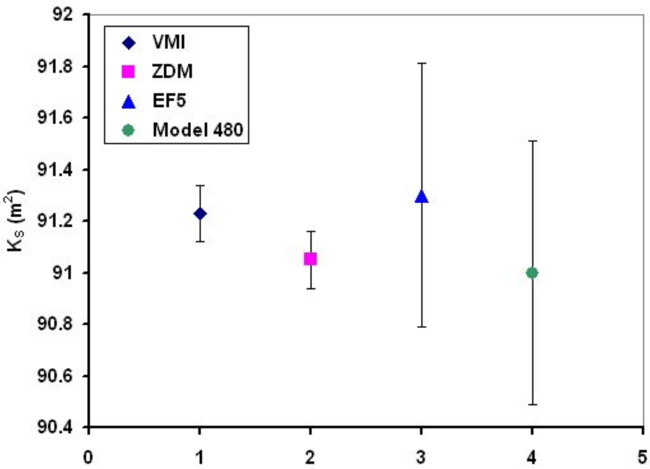

Fig. 6 DC calibration results for the CTU search coil with the expanded uncertainties values.

4.2. AC calibration results

The CTU coil was also calibrated for a frequency of 50 Hz. The SR 830 lock-in amplifier was used as a null indicator and also as a generator for the VMI and ZDM method. The calculations from the frequency dependence, the mean voltage measurement, the VMI method, or using the oscilloscope and webermeter for measuring the amplitude of the magnetic flux (see Fig. 5) are in good

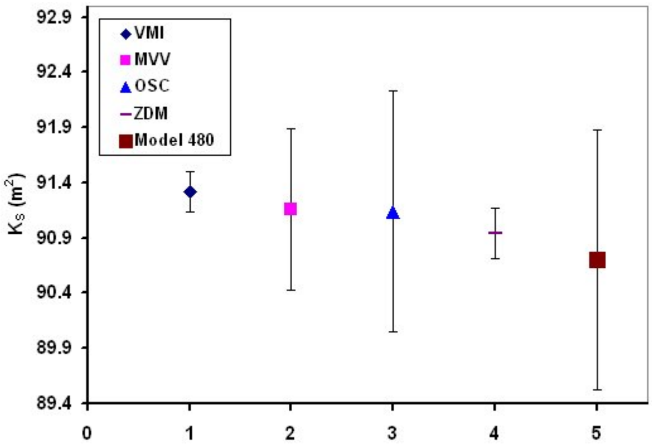

Fig. 7 AC calibration results for the CTU search coil with the expanded uncertainties values (MVV - mean voltage value, OSC - oscilloscope).

agreement, according the results presented in Table 2. A DSO-X 4024A type oscilloscope and a Model 480 type webermeter were used for this type of measurement. However, the oscilloscope is rather tricky to use. There is much greater uncertainty when an oscilloscope is used than when the direct mean voltage value is measured. In addition, the voltage value of the input signal is limited. The maximal value of the magnetic flux measured from integration of the input signal is also dependent on the displayed periods of the waveform. The value of K_S = 91.05 m^2 determined by the ZDM method was used as the value for K_{SDC} for calculating from the frequency dependence according to (7). The expanded relative uncertainty of the result calculated from (7) is very small (about 0.01 %). However, the uncertainty value is not listed in Table 2, because it is an approximate formula.

Table 2. AC calibration results for the CTU search coil.

Method	Constant value Ks (m^2)	Expanded uncertainty (%)
VMI	91.32	0.22
ZDM	90.94	0.25
webermeter Model 480	90.70	1.3
mean voltage value	91.16	0.8
oscilloscope	91.14	1.2
frequency dependence	91.20	x

5. CONCLUSIONS

DC and AC methods for calibration of search coils with a high area turns value at low frequencies have been described and compared. The differences in the DC calibrated values are smaller than 0.35 % for the CTU search coil. The differences of the AC results (VMI method, ZDM method, mean voltage measurement, measurement of the magnetic flux using an oscilloscope and a webermeter, and calculation from the frequency dependence) are also in relatively good agreement. The differences are smaller than 1.1 %. A detailed uncertainty analysis of each method has also been described. The calibration results and the uncertainty analysis show that the VMI method or the ZDM method is best for the DC search coil calibration. These methods are also the most suitable for AC calibration of the search coils in terms of the uncertainty value. However, an additional circuit is needed for compensation of the losing factor of the induced voltages in the secondary circuit. Neverthelesss, for practical measurements, the most widely-used and easiest method is to measure the RMS value of the search coil output voltage. All methods described here could also be used for search coils with smaller area turns values.

REFERENCES

[1] M. Ulvr: "A system for calibration of search coils with area turns up to 100 m^2", Proc. of CPEM 2012, July 2012.

[2] J.V. Afanasjev et al: "Sredstva izmerenij parametrov magnitnogo polja", Leningrad, Energia 1979.

[3] M. Buzzio: "Fabrication and calibration of search coils", CERN Accelerator School CAS 2009: Specialised Course on Magnets, June 2009.

[4] M. I. Green: "Search coils", CAS - CERN Accelerator School: Measurement and Alignment of Accelerator and Detector Magnets, pp.143-174, April 1997.

[5] V. E. Černyšev: "Rasčot častotnoj pogrešnosti měr magnitnoj indukcii", Trudy metrologičeskich institutov SSSR 140 (200), Leningrad, Izd. standartov 1972 (in Russian).

[6] M. Ulvr: "Calibration of magnetic field meters up to 50 kHz at CMI", Proc. of 19th IMEKO TC-4 Symposium, Barcelona, 2013.

[7] S. G. Lee, J. G. Bak, DeRac Son: "Effective Area Calibration and Dynamic Characteristic Measurements of KSTAR Magnetic Field Probes", *Journal of the Korean Physical Society*, Vol. 49, December 2006.

[8] Toshihide Tosaka, Isamu Nagano, Satoshi Yagitani: "Development of a Magnetic Field Measurement System using a Tri-axial Search Coil", Proc. of 2005 International Symposium on Electromagnetic Compatibility, August 2005.