XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 - September 4, 2015, Prague, Czech Republic

Recursive Frequency Estimation Technique for Synchrophasor Measurement Applications

Pasquale Cuccaro, Daniele Gallo, Carmine Landi, Mario Luiso, Gianmarco Romano

Dept. of Industrial and Information Engineering, Second University of Naples, Aversa (CE), Italy
Email: gianmarco.romano@unina2.it

Abstract - The proper and stable operation of a power grid relies on a efficient voltage and current phasor measurement instrumentation. A key feature of such an instrumentation is the accuracy of frequency measurement. The scope of this paper is to present a technique to track frequency variations of voltage and current phasors. We propose a recursive algorithm based on the Maximum Likelihood estimation of the phase; preliminary results show that its performance is comparable to the Cramer-Rao Lower Bound and can track phase variations over time.

Keywords: synchrophasor measurement, frequency measurements, power system measurement, maximum likelihood, recursive estimation

1. Introduction

Smart Grid Operation Economies today are dependent on reliable and secure power grids. Growing public sensitivity to electrical supply disruptions reflects the increasing dependence of countries on reliable and efficient electricity supplies. Security is the ability of a power system to withstand sudden disturbances such as electric short circuits, e.g., the preservation of interconnected system operation in the presence of severe disturbances. In particular, transient changes in the grid, such as increased real or reactive loads, high power transfers, or the loss of generation or transmission facilities can create a temporary imbalance between the power supply and demand, producing variation in the voltage and current waveforms [1–4]. In this context, having the capability to measure dynamic variations of voltage and current synchronized phasors is becoming a necessity, in order to guarantee proper operation of the power grid, for utilities. There are several research groups working in field of synchrophasor measurement [5–7]. The main feature of a synchrophasor measurement technique is the ability to track, in an accurate and fast way, frequency variations.

Many exact or approximate solutions can be found in the literature regarding the estimation problem in Gaussian noise only. The classical approach is to derive a maximum likelihood estimator because it has appealing asymptotic properties. More specifically, it can achieve asymptotically the Cramer-Rao lower bound (CRLB), a theoretical limit on the variance of the estimator and then on the accuracy of the estimate. The ML estimator for a complex single-frequency sinusoid is known and is given by the frequency that maximize the periodogram computed over the observed samples [8–10]. In many applications, frequency tracking is also needed and recursive and/or iterative algorithms constitute good solutions that can adaptively provide the current running estimate such as the digital phase-locked loop (DPLL) [11, 12] .

In this paper we propose a recursive algorithm that performs frequency estimation based on the maximum likelihood of the phase. Simulations shows that performances, measured in terms of variance of the estimator for a given SNR, in most cases are very close to the CRLB.

2. System Model

We consider the complex representation of a sinusoidal signal embedded in noise in the discrete-time domain, i.e.

$$v\left(n\right)=Ae^{j\left(2\pi fnT+\phi\right)}+w\left(n\right), \quad (1)$$

where A is the magnitude of the signal of interest, f is the frequency to be estimated, T is the sampling period, ϕ is the initial phase and $w\left(n\right)$ is a complex Gaussian noise with zero mean and variance σ^2. The same signal model can also be written in terms of normalized frequency $fT=\nu$ as

$$v(n)=Ae^{j(2\pi\nu n+\phi)}+w\left(n\right), \qquad n=m-M,\dots,m \quad (2)$$

We assume that $M+1$ consecutive samples of the sequence $v\left(n\right)$ are available for processing at each time instant and without loss of generality we can assume that the time index n is in the range $[m-M,m]$, which defines the observation window whose size is then $M+1$; m is the time index of phase estimate.

The complex equivalent can be obtained from the real waveform, for example, with a quadrature demodulator in the digital domain, as suggested, even though not required, by the standard [13, 14] and shown in Fig. 1. The choice of the frequency ν_q is arbitrary, though a common choice is the nominal frequency of the signal under observation, e.g. 50 Hz. Note that in this case the nominal frequency is $\nu=0$ of the complex representation and in practice it measures just the frequency offset with respect to the nominal frequency of the real signal that can take on values in the range of few hertz, e.g. between -5 and 5 Hz. The low-pass filters cut the unwanted double-frequency terms and may also cut the harmonics.

There is another advantage with the low-frequency complex equivalent representation: the CRLB, which is proportional to the sampling frequency, becomes smaller as the

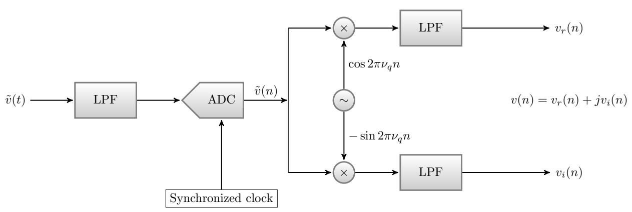

Figure 1: Quadrature demodulator to get the low-frequency complex equivalent

sampling rate decreases and then greater accuracies can be achieved. If frequencies in the range $[-5, 5]$ Hz are to be estimated the sampling rate can be as small as 100 Hz, for example.

3. Recursive Frequency estimation

The frequency is estimated as the derivative of the instantaneous phase estimates, according the definition of instantaneous frequency given in [13]. Rather than directly estimating such a derivative we approximate it with the first differences of the sequence of the estimated phases. Hence, we formulate the frequency estimation problem as a phase only estimation problem. We develop here a recursive algorithm that, given the observed sequence $v(n)$, estimates the phase ϕ at time m and to this end we adopt the maximum likelihood (ML) criterion.

The ML estimate of the phase of a sinusoid in Gaussian noise is a well-known result that can be found, for example, in [8]. We review here the main steps of the derivation of the closed form solution upon which our proposal for a recursive search is based. The ML estimate of the phase is the phase that maximize the likelihood function

$$\mathcal{L}(\phi; \mathbf{v}) = \frac{1}{(2\pi\sigma^2)^{\frac{M+1}{2}}} \exp\left\{-\frac{1}{\sigma^2}\sum_{n=m-M}^{m}\left|v(n) - Ae^{j(2\pi\nu n+\phi)}\right|^2\right\}. \quad (3)$$

Equivalently, after taking the logarithm of the likelihood function and dropped all the terms that do not depend on ϕ, the problem is to find the ϕ that maximize

$$\lambda(\phi; \mathbf{v}) = \Re\left\{\sum_{n=m-M}^{m} v(n) \cdot Ae^{-j(2\pi\nu n+\phi)}\right\}. \quad (4)$$

The standard way to get the closed-form solution is to take the first derivative with respect to ϕ and setting it to zero. The result is

$$\hat{\phi}_{ML} = \arctan\frac{\Im\left\{\sum_{n=m-M}^{m} v(n)\, e^{-j2\pi\nu n}\right\}}{\Re\left\{\sum_{n=m-M}^{m} v(n)\, e^{-j2\pi\nu n}\right\}}. \quad (5)$$

The ML estimate is obtained as the angle of the complex quantity computed as the sum of the samples of the product between the observed signal and a local generated complex sinusoid. Eq. (5) could be used to get the estimate at time instant m, however, we propose to employ the gradient search method to iteratively search for the solution.

In the gradient descent search the maximim is obtain through the iteration

$$\phi_{k+1}(m) = \phi_k(m) + \mu \left.\frac{\partial\lambda}{\partial\phi}\right|_{\phi=\phi_k(m)} \quad (6)$$

where μ is a step size parameter which regulates the speed of updating at each iteration and then the speed of convergence. Specifically, the iterative update equation is given by

$$\phi_{k+1} = \phi_k + \mu\Im\left\{\sum_{n=m-M}^{m} v(n) \cdot Ae^{-j(2\pi\nu n+\phi_k)}\right\}, \quad (7)$$

where we dropped the time index m for ease of notation. Until now, there is no advantage over the closed form solution. However, we can use (7) to formulate a recursive algorithm that tracks the instantaneous phase. In order to track phase variations we consider sliding observation windows with given size and partial overlap. For each observation window an iterative search according the (7) is performed with the phase at the first iteration given by the phase estimate obtained with the previous observation window. Such a choice leads to a recursive formulation of the ML phase estimation algorithm that for each observation window requires only small adjustments so that small variations of the phase can be tracked. The partial overlap between successive observation windows controls the rate of the updates of estimated phases and the resolution in time instants over phase variations. A trade-off must be achieved as the greater is the overlap the greater is the rate of update. On the other hand small or no overlap may decrease the rate of update of phase estimation with loss of estimation of very quick phase variations.

More precisely, for each observation window a local complex exponential signal must be generated. Then, to compute the derivative in (7) the product of the observed signal and the local generated complex sinusoid must be performed. Finally, the imaginary part of the sum of the samples of the product in the observation window is used as increment to update the current estimate of the phase. After a given number of iterations the observation window is shift by a given number of samples, ranging from simply one sample to the number of samples that constitute the observation window.

A scheme of the algorithm is shown in Fig. 2. Note that in the algorithm three time indexes are involved leading to a multirate implementation with three underlying clocks. The time index n represents the sampling period index for the observed signal; the time index m represents the time at which the estimate is computed and corresponds to a time interval that it is an integer multiple of the sampling period; the index k denotes the iteration index given the observation window, i.e. given m.

In order to preserve the continuity of the local generated signal across multiple consecutive observation win-

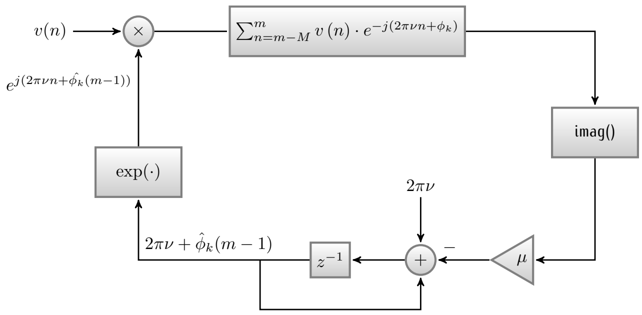

Figure 2: Recursive phase estimation for a given observation window at time index m.

dows, the generation of the local signal must be done imposing the phase continuity of the argument of the exponential at the beginning of the window. Within one window the generation of the samples of the local complex sinusoid can be done recursively, as

$$s(n+1;m) = Ae^{j(2\pi\nu(n+1)+\phi)} = s(n;m)\, e^{j2\pi\nu},$$

where $n = 0, 1, \ldots, M$ denotes the index within the window that starts at time index $m - M$. For each observation window we impose the constraint $s(0;m+1) = s(p;m)$, where p is the number of samples the window is shifted.

The proposed algorithm relies on the following tunable parameters: 1) observation window size M; 2) overlap between two successive observation windows O or, equivalently, the number of samples the observation window is shifted; 3) number of iterations for each observation window K; 4) step size μ;

4. Relationship to Digital PLLs

One of the solution to the problem of estimation of frequency is to employ a phase locked loop (PLL) to track the phase of the signal $v(n)$, whose generic scheme is shown in Fig. 3. The digital PLL of type I implements the recursive relation

$$\phi(n) = \phi(n-1) + \mu_d e(n-1), \qquad (8)$$

where μ_d is a step parameter and $e(n)$ is a error signal derived by the the phase error at time n. The error signal is proportional to the first difference of the sequence of phase estimates and as such represents a scaled estimate of the instantaneous frequency. The phase detector computes the phase error as phase difference between the observed signal and a local generated complex sinusoid, though some approximations may be used to reduce the computational complexity. The error signal $e(n)$ is then typically computed with a proportional-integrator filter. Eq. (8) represents the update relationship governing the iterative algorithm and the recursive generation of the phase of the local oscillator (NCO). At convergence the phase error is zero and the error signal is constant, i.e. the instantaneous phase is linear with time and its slope is just the frequency estimate.

The proposed algorithm resembles the iterative structure of the digital PLL (see for example [11, 12]). Indeed

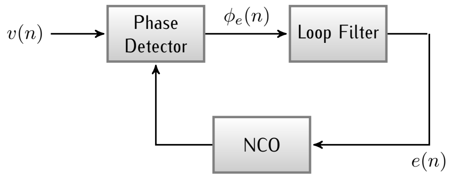

Figure 3: Digital Phase Locked Loop

Lindsey *et al.* recognized the connections between the DPLL and the MAP phase estimate [15] and algorithms based on this idea has been developed over the time, as for example in [16]. Our algorithm presents some important differences: 1) the idea of sliding window with tunable step size parameter; 2) inner iterations to improve the estimate of the phase for a given observation window. It is worth to note that the DPLL use a different loop filter that tracks the loop to zero phase error while the recursive phase estimation algorithm does not require a phase error block (and hence, no phase extraction is needed).

5. Simulations

The frequency estimator algorithm based on the proposed phase recursive estimation has been tested with Monte-Carlo simulations to get the performances under additive white Gaussian noise and to study the effect of the parameters that can be tuned in the proposed algorithm. We assume an ideal front-end and, hence, that the frequency to be estimated is in the range $[-5, 5]$ Hz. Performances are evaluated in terms of the variance of the frequency estimator, computed averaging over 1000 estimations in steady-state regime. The benchmark is the CRLB, though it represents a lower bound only for feed-forward estimation. For all simulations we set the step size $\mu = 1/(M+1)$.

First, in Fig. 4 we show the results for the proposed estimator for sampling frequency equal to 1000 and 100 samples/seconds, observation window size $M = 4$, observation window step size of one sample and no inner iterations. The initial frequency of the recursive estimator has been set to zero. The frequency to be estimated is equal to 51.23456789 Hz that, after the front-end processing, is shifted to 1.23456789 Hz. In all our simulations we found that performances do not depend on the frequency to be estimated as long as it is in the range of interest specified before, at least.

In Fig. 4 we find the confirmation that by decreasing the sampling frequency the accuracy for a given SNR increases. For example, at SNR=90dB the variance is 10^{-6} for sampling rate set to 1000 samples/seconds while for 100 samples/seconds the variance becomes 10^{-8}. It is worth to note that the proposed estimator get performances slightly better than the corresponding CRLB. This is due to the iterative nature of the proposed algorithm and remember that the CRLB refers to the feed-forward estimation limit.

We have repeated the simulations with different sizes of the observation window to study the increase in perfor-

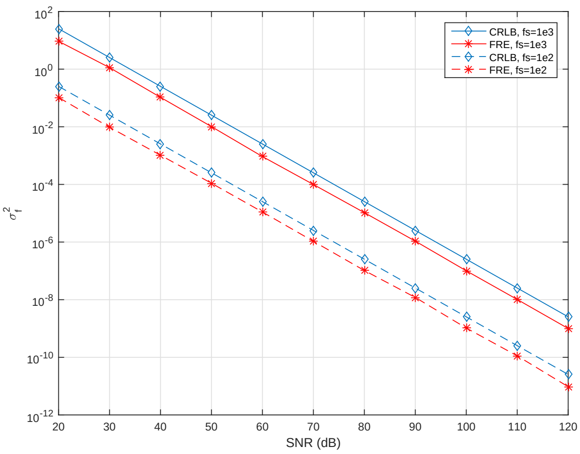

Figure 4: Estimated variance of the proposed frequency recursive estimator (FRE) versus signal-to-noise ratio (SNR).

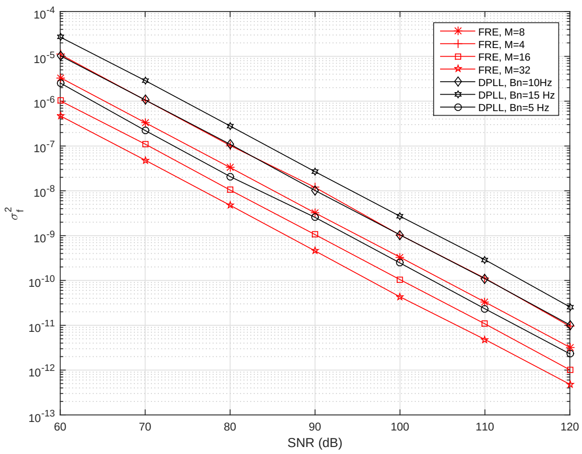

Figure 6: Estimated variance of the proposed frequency recursive estimator and DPLL versus signal-to-noise ratio (SNR).

mances that can be obtained. In Fig. 5 results for $M = 4, 8, 16, 32$ are shown with the corresponding CRLBs. We see that the variance decreases as the size increases, as expected. However, the difference with respect to the corresponding CRLB increases as well, since the variance becomes greater than the corresponding CRLB as M increases. This is not surprising as the proposed estimator is based on the ML estimation of the phase only.

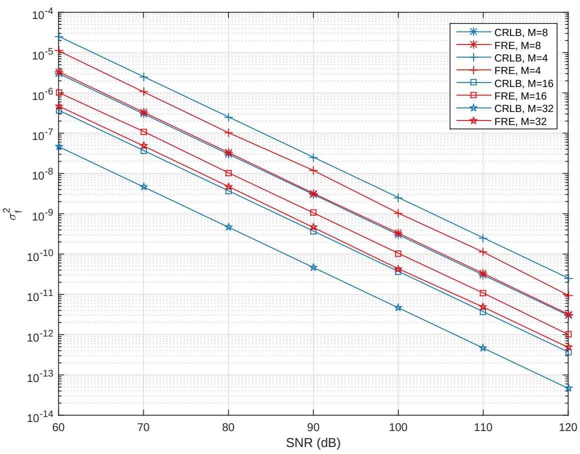

Figure 5: Estimated variance of the proposed frequency recursive estimator versus signal-to-noise ratio (SNR).

Finally, we have made a comparison between the proposed recursive estimator and the DPLL. The DPLL has been implemented according the scheme in [12] and the frequency is estimated as first difference of the estimated phases in steady-state regime. The damping factor has been set to $1/\sqrt{2}$ and the initial frequency to zero. Several equivalent noise bandwidths have been selected, $B_n = 5, 10, 15Hz$, and for both algorithms we set the same sampling frequency $f_s = 100Hz$. Results in Fig. 6 show that the variance in case of the DPLL increases as the equivalent bandwidth increases, as expected.

6. Experimental results

In order to experimentally test the proposed algorithm, a measurement station has been set up. It is composed by a synthesized function generator (Yokogawa FG300 with frequency resolution 1 µHz e frequency accuracy ±20 ppm, Amplitude accuracy ±(0.8% of setting + 14 mVrms), Harmonics (maximum of 2nd to 5th harmonic components) 100 kHz -55 dBc max), a data acquisition board (NI 9215 measurement range ±10 V, Simultaneous sampling, maximum sampling frequency 100 kHz, 16-bit resolution) and a Personal Computer (PC). Its block scheme is represented in Fig. (7)

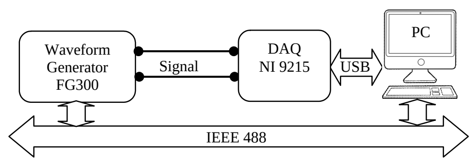

Figure 7: block scheme of the adopted automated measurement system

The measurement software has been developed in LabView environment and the execution of repetitive measurement was made automatic. The tests were executed generating sinusoidal signals, at 50 Hz and 51 Hz, using different window sizes and different sampling frequencies, that are 500 Hz, 1 kHz, 5 kHz and 10 kHz. For each test, several frequency measurements were carried out and one value of variance was calculated over one thousand measurements. In particular, in the first group of tests, signal frequency was set to 50 Hz. Six different values for window size, expressed in samples, were chosen from 2 to 50. Fig. (8) shows four different curves, one for each used sampling frequency; each curve represents the variance of frequency measurements vs window size, expressed in samples. It can be seen that variance goes from 10^{-3} Hz^2 (at 1 kHz sampling frequency and window size equal to 2) to less than $10^{-6} Hz^2$ (at 500 Hz sampling frequency and window size equal to 50).

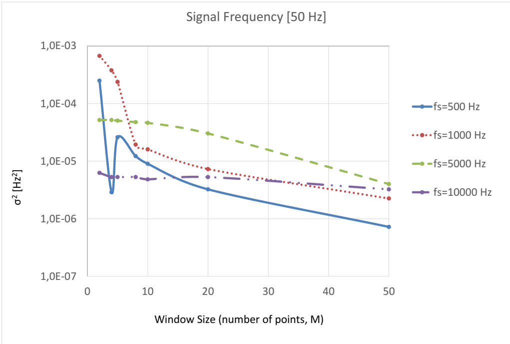

Figure 8: variances on signal frequency estimation, at 50 Hz, vs. window size (samples) using different sampling frequencies

In particular, in the second group of tests, signal frequency was set to 50 Hz. Six different values for window size, expressed in milliseconds, were chosen from 4 to 100. Fig. (9) shows four different curves, one for each used sampling frequency; each curve represents the variance of frequency measurements vs window size, expressed in milliseconds. It can be seen that variance goes from less than 10^{-3} Hz^2 (at 1 kHz sampling frequency and window size equal to 4 ms) to less than 10^{-7} Hz^2 (at 10 kHz sampling frequency and window size equal to 100 ms).

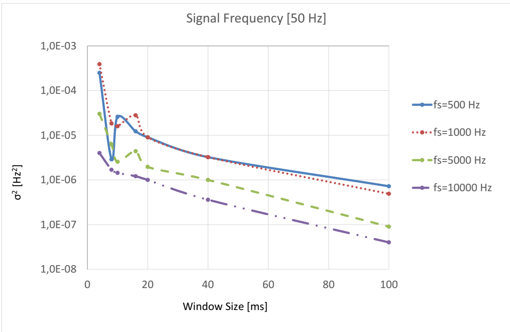

Figure 9: variances on signal frequency estimation, at 50 Hz, vs. window size (time) using different sampling frequencies

In particular, in the third group of tests, signal frequency was set to 51 Hz. Six different values for window size, in samples, were chosen from 2 to 50. Fig. 10 shows four different curves, one for each used sampling frequency; each curve represents the variance of frequency measurements vs window size, expressed in samples. It can be seen that variance goes from 10^{-3} Hz^2 (at 1 kHz sampling frequency and window size equal to 2) to 10^{-6} Hz^2 (at 500 Hz sampling frequency and window size equal to 50).

In particular, in the fourth group of tests, signal frequency was set to 51 Hz. Six different values for window size, expressed in milliseconds, were chosen from 4 to 100. Fig. (11) shows four different curves, one for each used sampling frequency; each curve represents the variance of frequency measurements vs window size, expressed in

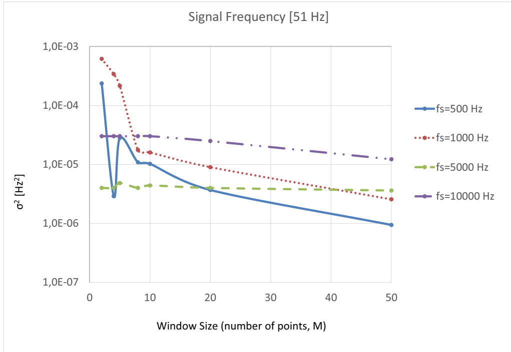

Figure 10: variances on signal frequency estimation, at 51 Hz, vs. window size (samples) using different sampling frequencies

milliseconds. It can be seen that variance goes from less than$10^{-3}Hz^2$ (at 1 kHz sampling frequency and window size equal to 4) to less than $10^{-7}Hz^2$ (at 10 kHz sampling frequency and window size equal to 100 ms).

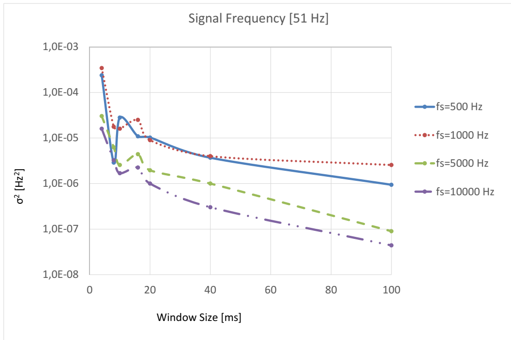

Figure 11: variances on signal frequency estimation, at 51 Hz, vs. window size (time) using different sampling frequencies.

7. Conclusions

This paper presents a technique to track frequency variations of voltage and current phasors. We propose a recursive algorithm based on the Maximum Likelihood estimation of the phase; simulations show that its performance is comparable to the Cramer-Rao Lower Bound and can track phase variations over time. and Experimental results show that, with proper choice of sampling frequency and observation window, variances on frequency measurement as low as 10-7 Hz2 can be reached even in real situations.

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