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ROBUST SUBDIVISION FOR THE DISSEMINATION OF THE UNIT OF MASS

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Abstract – Usual subdivision methods are working excellent in the absence of outliers. On the other hand a small number of outliers can lead to complete wrong estimates. Several general methods are used for robust linear regression, but these methods are not used for improving the subdivision yet. The new approach which can be classified as an iteratively reweighted least square method with special weight-functions introduced in BEV provides efficient robust results by downgrading the outliers.

Keywords: subdivision, mass metrology, iteratively reweighted least square estimation, outlier

1. INTRODUCTION

Subdivision is a general method used in mass metrology to calibrate an entire or part of a set of weights against one or more reference weights. The method requires several weighings with different combinations of weights of equal total nominal mass applying adjustment calculations in order to reduce error propagation. Usually the method includes redundancy to gain confidence in the results [1].

Most of the weighing designs published so far were optimised to reduce the number of the weighings or the uncertainty of the results [2].

Robotic balances make the reduction of the amount of measurements less important. It can also be proved that the differences among the uncertainties of the results given by different traditional weighing designs are practically negligible [3].

Usual subdivision methods are vulnerable to outliers consequently they can lead to wrong estimates. There are some techniques published to detect the outliers, but there is no common handling of them.

This article propose a solution for this problem.

One of the initiations for this study was a EURAMET project (No. 1210, Best practice for dissemination of the kilogram) started in 2011.

2. THE CURRENT PRACTICE

The base model is a weighted least square method [4] that estimates:

$$\beta = (X^T W X)^{-1} X^T W Y \qquad (1)$$

where β is p dimensional vector of the estimated parameters, Y is an n-dimensional vector of observations, X is the (n,p) dimensional design matrix and W is an (n,n) dimensional diagonal matrix of positive weights w_i. The initial weights are provided by the means of the standard deviation (or the uncertainty) of the individual weighings.

$$w_{ii} = \left(\frac{\sigma_0}{s_i}\right)^2 \text{ with } \sigma_0^2 = \frac{1}{\sum \frac{1}{s_i^2}} \qquad (2)$$

The residuals can be expressed as

$$R = Y - \beta X \qquad (3)$$

In case when all the residuals are smaller than the corresponding standard deviations of the individual weighings, presumably there are no outliers. In practice it happens frequently that there are outliers. In the present of a single outlier the regression analysis usually shows more discrepancies that makes it difficult to identify the outlier.

An important characteristic of the weighing designs is that they are generally not robust. The analysis started with two weighing designs: one with 12 measurements and 4 unknowns [1] (referred later as scheme I) and one with 10 measurement and 7 unknowns [4] (referred later as scheme II). In these designs a very few outliers can lead to a wrong estimate. The complete removal of the measurements with outliers also reduces the robustness of the estimation or makes it impossible.

Several techniques were studied to handle the outliers.

2. THE ROBUST ESTIMATIONS

To solve the problems described in the previous section the principle of the robust estimations can be used.

First step is to make the initial estimation using equation (1) with the weights (2) and then calculating the residuals (3).

Second step is the investigation of the residuals and detecting the possible outliers. In case of no outliers no further step is needed.

The third step is to assign new weights to the observations (to the individual weighings).

From here an iteration of these three steps starts that continues until a stopping criterion is satisfied.

Typical robust estimates like the various M-estimators [5] are not practical here because they are not taking into account the a priori information gained from the individual weighings. They are not functioning well with a small number of observations. Simulations show that these methods might provide a robust estimates but the corresponding uncertainties are unacceptably large to use in the dissemination of the unit of mass.

Several functions were investigated to assign proper weights to the observations. Three of them were found providing good results. The weigh functions are shown in fig. 1.

The first model (4) keeps the original weights if the residual is smaller than the corresponding standard deviation and reduce the weights if the residual is bigger.

$$w_{ii,j+1} = \left(\frac{\sigma_0}{\max(s_i;r_{i,j})}\right)^2 \text{ with } \sigma_0^2 = \frac{1}{\sum\frac{1}{\max(s_i;r_{i,j})^2}} \quad (4)$$

where j is the index of the estimation.

The next two models (5 and 6) are combining the a priori information from the individual measurements and the common approach that the results with smaller residual are better.

$$w_{ii,j+1} = \left(\frac{\sigma_0^2}{s_i^2+r_{i,j}^2}\right) \text{ with } \sigma_0^2 = \frac{1}{\sum\frac{1}{s_i^2+r_{i,j}^2}} \quad (5)$$

$$w_{ii,j+1} = \left(\frac{\sigma_0^2}{s_i^2/2+r_{i,j}^2}\right) \text{ with } \sigma_0^2 = \frac{1}{\sum\frac{1}{s_i^2/2+r_{i,j}^2}} \quad (6)$$

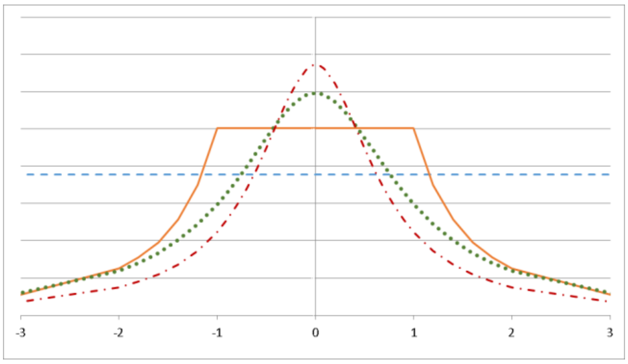

Fig. 1. Weights as a function of the residual: blue dashed line represents the initial weights; orange line the equation (4); the green dotted line the equation (5); red dash-dot line equation (6)

3. TESTING THE NEW METHODS

To evaluate the new methods a simulation based on realistic set of data was created. Actually the parameters were taken from the calibration of the last decade (10 mg – 1 mg) of a weight set using the robotic system of BEV.

For the better visibility all values expressed relative to the Maximum Permissible Error (MPE) of the class E_1.

We assume that each weighing has the same standard deviation, 3 % of the MPE.

As the worst case scenario we assume that 10 % of the measurements are having errors with a maximum 33 % of the MPE (0,001 mg).

The mass of each weight was permanently fixed to a value and results of the weighings were calculated from these values as:

$$Y' = XM + N + E \quad (6)$$

Where Y' is the vector of the calculated weighing results (observations), M is the vector of the fixed mass values, X is the design matrix, N is a vector of the (random) noise and E is a vector of random errors.

Each simulation was carried out 1000 times.

For the evaluation of the methods several parameters were assessed:

1. The uncertainty component of the subdivision method without any other components (e.g. uncertainty of the standards, uncertainty of the air buoyance) for the smallest unknown weight.
2. The ratio of the good estimates. (An estimate considered as good if it does not differ from the original fixed mass value more than the expanded uncertainty of difference).
3. The number of the undetected errors relative to the good estimates. In mass metrology control weights with known mass values are used to monitor the results. If the estimates of the control weights differ from the known mass value more than the expanded uncertainty of this difference the measurements are considered incorrect. An "undetected error" is when the control weight does not indicate incorrect measurement, but the result is wrong (see previous point).

Table 1. Performance of the scheme I without (Orig.) and with robust estimates (4 to 6.

Design	scheme I			
Original/Method	Orig.	Meth. (4)	Meth. (5)	Meth. (6)
Uncertainty	2,1 %	2,3 %	2,6 %	2,0 %
Ratio of good measurements	74,5 %	94,7 %	99,0 %	97,6 %
Undetected err/good	11,5 %	2,8 %	0,5 %	1,7 %

Table 2. Performance of scheme II without (Orig.) and with robust estimates (4 to 6.

Design	scheme II			
Original/Method	Orig.	Meth. (4)	Meth. (5)	Meth. (6)
Uncertainty	3,6 %	3,8 %	4,9 %	4,1 %
Ratio of good measurements	64,4 %	88,9 %	96,7 %	96,0 %
Undetected err/good	4,3 %	8,4 %	3,3 %	3,9 %

Table 1 and 2 show that using the original evaluation both designs have advantages over the others in some parameters.

Scheme I provides higher number of good measurements, but far more undetected errors than scheme II.

Using robust estimate both designs provide better performance regarding the number of the good measurements on the penalty of a minor uncertainty increase. Analysing the undetected errors the scheme I shows a remarkable improvement while the scheme II could not benefit significantly from these methods.

4. NOVEL WEIGHING DESIGN

The weighing designs investigated in the previous section do not provide reliable results due to the following reasons:

their degree of freedom is small or the number of control weights is not enough to detect the errors.

Due to the fact that not all the balances in BEV are capable to perform the measurements according to scheme I this design is not applicable in BEV. The robustness of the scheme II was found not satisfactory. To override these problems a new weighing design (scheme III) was developed in BEV (Table 3). This weighing design is assumed to perform better according to the mentioned parameters since it includes more measurements and more control weights (Table 4).

Table 3. The new weighing design. The use of the weights can be as standard (S), control weight (C) and Test weight (T); n.v stands for the nominal value of the weights

use	S	C	T	C	T	T	C	T	C
n.v.	10	10	5	5	2	2	2	1	1
1	–	+							
2	–		+	+					
3		–	+	+					
4			–	+					
5	–			+	+	+		+	
6		–	+		+		+		+
7			–		+	+		+	
8			–		+	+			+
9					–	+			
10					–			+	+
11						–		+	+
12								–	+
13								–	+
14				–		+	+	+	
15						–	+		
16							–	+	+

Table 4. Comparison of weighing designs.

Scheme	I	II	III
Number of measurements	12	10	16
Number of control weights	1	3	4
Number of measurements with the standard	2	2	3
Minimum number of measurements with unknown weights	4	4	6
Degree of freedom	7	3	8

4. THE PREFORMANCE OF THE NOVEL WEIGHING DESIGN

After the positive results of the initial evaluation using the weight function (4) the novel design showed the highest number of correct measurements with the lowest ration of undetected errors. The other methods developed later (5-6) show also a great benefit of the robust estimation.

Remarkably using method (5) scheme I delivers less undetected errors while strangely method (4) increases the undetected errors by scheme II.

Method (6) greatly reduce the uncertainty by the novel design with no practical consequences by the other parameters.

Table 5. Performance of the novel weighing design without (Orig.) and with robust estimates (4 to 6).

Design	scheme III			
Original/Method	Orig.	Meth. (4)	Meth. (5)	Meth. (6)
Uncertainty	3,4 %	3,6 %	3,8 %	2,1 %
Ratio of good measurements	54,2 %	97,8 %	98,3 %	97,5 %
Undetected err/good	4,8 %	1,1 %	0,9 %	1,1 %

Since the scheme I cannot be used in BEV the novel design with practically any of the robust estimates (5-7) ensures a very reliable result.

4. CONCLUSIONS

Three weighing designs were analysed simulating a worth case scenario with random errors. With the traditional evaluation method the random errors cause 25 % to 45 % wrong final results. Even if control weights are used to recognise the errors 4 % to 11 % of the errors remain hidden which leads to incorrect final results. The use of robust estimates with the recommended special weight-functions radically improves the performance of the weighing designs. The number of errors are reduced typically to 3 % to 5 % while the undetected errors are reduced to 0,5 % to 3 %. The less robust design hardly profits at all.

The design used in BEV provides good results. The research for an even better design can be a future goal since the scheme I showed in some parameter better results than expected at the beginning of this work.

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