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IDENTIFICATION OF MODEL PARAMETERS OF A PARTIALLY UNKNOWN LINEAR MECHANICAL SYSTEM FROM MEASUREMENT DATA

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Abstract - The dynamic calibration of torque transducers requires the modelling of the measuring device and of the transducer under test. The transducer's dynamic properties are described by means of model parameters, which are going to be identified from measurement data. To be able to do so, two transfer functions are calculated. In this paper, the transfer functions and the procedure for the model parameter identification are presented. An example shows results of the parameter identification.

Keywords: mechanical modelling, dynamic torque calibration, linear and time invariant system

1. INTRODUCTION

Several applications with dynamic torque excitation require traceable measurement. At present, only standards and procedures for the static calibration of torque transducers exist. Static calibration is an insufficient base for an analysis of dynamic measurements in terms of measurement uncertainties and influences from dynamic signal components. To be able to describe the dynamic influences of a torque transducer on a measurement set-up and vice versa, a corresponding calibration is mandatory [1]. Therefore, a measuring device and procedures for a dynamic characterisation of torque transducers were developed in the context of a joint European research project [2].

2. DYNAMIC TORQUE MEASURING DEVICE

The measurement principle of the dynamic torque measuring device is based on Newton's second law. The product of a known static mass moment of inertia J of a body and a measured time-dependent angular acceleration $\ddot{\varphi}(t)$ equals the time-dependent torque $M(t)$

$$M(t) = J \cdot \ddot{\varphi}(t) \ . \tag{1}$$

The measuring device consists of a rotatable vertical shaft assembly, on which all essential components are arranged in series (depicted in Fig. 1). At the bottom, a rotational exciter generates a forced excitation by means of sinusoidal oscillations. The torque transducer under test (device under test, DUT) is arranged on top of the exciter between two coupling elements. These couplings are designed to be both torsionally stiff and compliant for parasitic bending moments and axial loads simultaneously. At the top, the arrangement of the DUT and the couplings is followed by an angular grating disk for the angular acceleration measurement and an air bearing to prevent axial loads acting on the drive shaft's components.

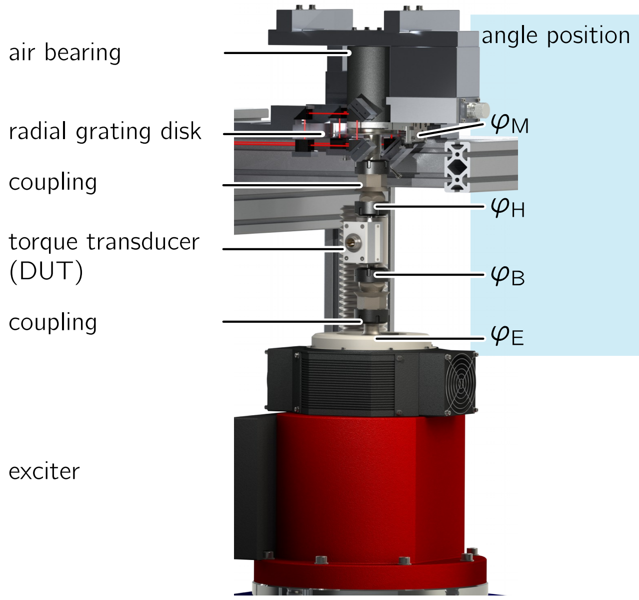

Fig. 1. Dynamic torque measuring device with the different components arranged vertically on top of the exciter.

3. MODELLING

The dynamic behaviour of the transducer under test is described by a linear time-invariant (LTI) model. It is based on the mechanical design of typical strain gauge torque transducers. The model consists of two rigid mass moment of inertia elements (MMOI) which are connected by a massless torsional spring and damper in parallel. A sole modelling of the transducer under test is not sufficient, because of the fact that the dynamic behaviour of torque transducers can be influenced by the coupled components. Torque transducers are always coupled to their mechanical environment at both sides which causes a bidirectional influence on the dynamic behaviour of the torque transducer and the coupled mechanical components (which are always arranged in some type of drive assembly).

To be able to identify the model parameters of the DUT, the modelling was expanded from the model of the transducer to a model of the whole measuring device, which is the mechanical environment in the case of the calibration. The model again consists of mass moment of inertia elements, torsional springs and dampers, assuming the LTI behaviour of the measuring device. It is based on the mechanical design of the components of the drive shaft. The representation of the model components and the corresponding components of the measuring device are given in Fig. 2.

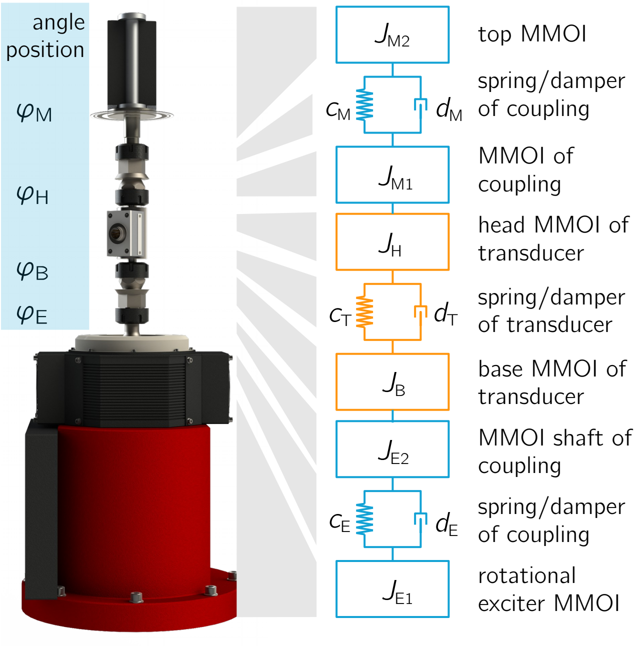

Fig. 2. Dynamic torque measuring device (left) and corresponding model representation (right) of the measuring device (blue) and the DUT (orange).

This model can be described by a set of inhomogeneous ordinary equations (ODEs). With the mass moment of inertia matrix $\boldsymbol{J}$, the torsional stiffness matrix $\boldsymbol{K}$ and the damping matrix $\boldsymbol{D}$ there follows

$$\boldsymbol{J} \cdot \ddot{\boldsymbol{\varphi}} + \boldsymbol{D} \cdot \dot{\boldsymbol{\varphi}} + \boldsymbol{K} \cdot \boldsymbol{\varphi} = \boldsymbol{M} \ . \tag{2}$$

The angle vector $\boldsymbol{\varphi}$ describes the angle excitations at different positions in the model; its derivatives $\dot{\boldsymbol{\varphi}}$ and $\ddot{\boldsymbol{\varphi}}$ represent the angular velocity and angular acceleration, respectively. The load vector $\boldsymbol{M}$ describes the excitation of the rotational exciter.

Based on this equation system, the model parameter identification will be carried out.

The excitation signals chosen are monofrequent sinusoids. With these harmonic waveforms, not all necessary angle position, angular velocity and angular acceleration data has to be derived independently or by numerical differentiation/integration but can be calculated as follows

$$\begin{aligned} \varphi(t) &= \hat{\varphi}\, \mathrm{e}^{\mathrm{i}\omega t} \ , \\ \dot{\varphi}(t) &= \mathrm{i}\omega\, \hat{\varphi}\, \mathrm{e}^{\mathrm{i}\omega t} = \mathrm{i}\omega\, \varphi(t) \ , \\ \ddot{\varphi}(t) &= -\omega^2\, \hat{\varphi}\, \mathrm{e}^{\mathrm{i}\omega t} = -\omega^2 \varphi(t) \ , \end{aligned} \tag{3}$$

where $\mathrm{i} = \sqrt{-1}$ denotes the imaginary number.

Measurements are not possible at all angle positions given in (2) and depicted in Fig. 2. To gather the information necessary for parameter identification, the output signal of the transducer U_{DUT} is used as an indicator for the difference in the torsion angle above and below the transducer. This assumption is valid, because it is assumed that the output signal of the transducer is proportional to its torsion $\Delta\varphi_{\mathrm{HB}} = \varphi_{\mathrm{H}} - \varphi_{\mathrm{B}}$ giving

$$U_{\mathrm{DUT}}(t) \propto \Delta\varphi_{\mathrm{HB}}(t) \ . \tag{4}$$

During calibration, three measurement signals are acquired simultaneously. The signals will be processed to determine the magnitude and phase of each harmonic and monofrequent signal by means of a sine fit.

4. KNOWN AND UNKNOWN MODEL PARAMETERS

A prerequisite for the model parameter identification of the unknown parameters of the transducer under test is a sufficiently low number of unknown model parameters of the system. To this end, the necessary parameters of the measuring device were determined in advance. The properties of the measuring device will not change for different transducers, and therefore needed to be determined only once. Three auxiliary measurement set-ups for the measurement of the mass moment of inertia, torsional stiffness and rotational damping were developed [3, 4] and the corresponding properties of the measuring device's components were determined. The only model parameters to remain unknown prior to the model parameter identification are the parameters of the transducer under test.

5. TRANSFER FUNCTIONS

The three available signals acquired during calibration measurements are used to calculate two complex transfer functions of the system in the frequency domain. One transfer function $H_{\mathrm{top}}(\mathrm{i}\omega)$ describes the dynamics of the top part of the measuring device giving

$$H_{\mathrm{top}}(\mathrm{i}\omega) = \frac{\rho \cdot \Delta\varphi_{\mathrm{HB}}(\mathrm{i}\omega)}{\ddot{\varphi}_{\mathrm{M}}(\mathrm{i}\omega)} \tag{5}$$

with the (still unknown) proportionality factor ρ linking the voltage output of the transducer to its torsion (cf. (4)).

The same applies to the bottom part of the measuring device giving the transfer function

$$H_{\mathrm{bott}}(\mathrm{i}\omega) = \frac{\rho \cdot \Delta\varphi_{\mathrm{HB}}(\mathrm{i}\omega)}{\ddot{\varphi}_{\mathrm{E}}(\mathrm{i}\omega)} \ . \tag{6}$$

The two transfer functions are illustrated in Fig. 3. The underlying ODE system gives the corresponding equations for both transfer functions. The derivation of these relations is more thoroughly described in [5].

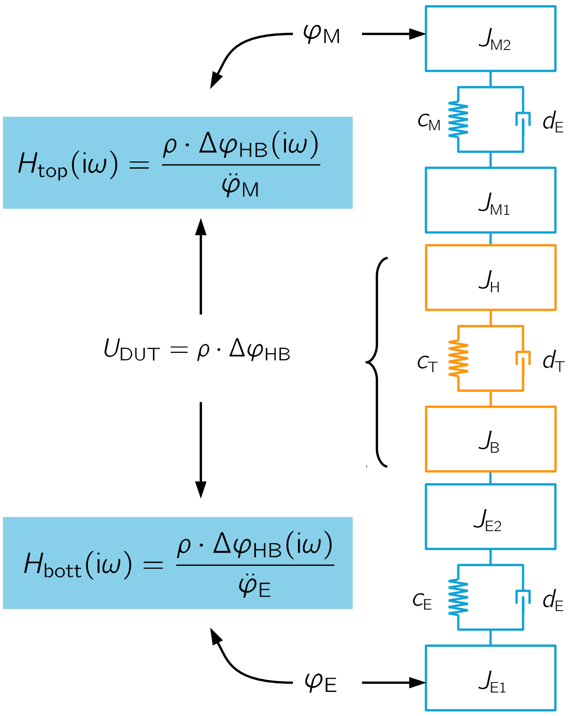

Fig. 3. The two calculated transfer functions based on the acquired measurands.

6. MODEL PARAMETER ESTIMATION

The model parameter identification is carried out based on the two transfer functions. Each acquired measurement data point is a sample with random deviations. For the identification of the parameters of the DUT with evaluated uncertainties and known deviations, the occurrence of these statistical influences has to be taken into account. A transfer function $H(\mathrm{i}\omega)$ describes the relation between input $X(\mathrm{i}\omega)$ and output $Y(\mathrm{i}\omega)$ and follows

$$Y(\mathrm{i}\omega) = H(\mathrm{i}\omega) \cdot X(\mathrm{i}\omega) \quad . \tag{7}$$

For the derived frequency responses from the measurement data, a best set of model parameters needs to be chosen to fulfil this relation as well as possible. However, due to the measurement uncertainty deviations, which will always occur, an exact measurement of $X(\mathrm{i}\omega)$, $Y(\mathrm{i}\omega)$ will not be possible. The measured data will always be disturbed randomly due to the measurement uncertainty. The vectors of measurement data $\boldsymbol{X}_\mathrm{M}(\mathrm{i}\omega)$, $\boldsymbol{Y}_\mathrm{M}(\mathrm{i}\omega)$ are therefore multivariate random vectors with estimated state of knowledge probability density functions (PDF). Figure 4 depicts the relation of the input, output, measured quantities and model transfer function.

The more information is taken into account for the model parameter estimation, the more reliable the outcome can be. Based on the chosen estimator, knowledge about the input quantities, the output quantities, and about the parameters to be identified prior to the estimation (so-called *a priori* knowledge) can be considered.

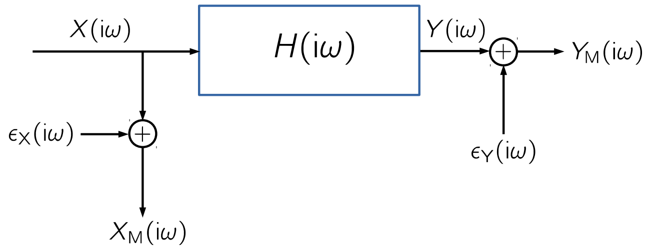

Fig. 4. Schematic illustration of a transfer function and its relation to input and output.

Three typical estimators are compared regarding their demand of knowledge of the different quantities in Table 1. A '+' indicates necessary information about the distribution of a quantity; a '−' indicates that it is not necessary to know about the distribution.

Table 1. Requirements of different estimators.

knowledge about →	distribution of input quantities	distribution of the (unknown) parameters
Bayes estimator	+	+
maximum likelihood estimator	+	−
least squares estimator	−	−

The classical, frequentist approach to model parameter estimation takes no uncertainty contributions in terms of PDFs of input quantities into account. Therefore, the estimated set of parameters consists of the estimated values, but not of information about their distribution. The least squares (LS) estimator and the maximum likelihood estimator are frequentist estimation approaches.

The *least squares estimator* minimises the squared sum of residuals of a complex model function $\boldsymbol{G}$ and of the measurement data $\boldsymbol{X}_\mathrm{M}$, $\boldsymbol{Y}_\mathrm{M}$ for n data points giving

$$\hat{\boldsymbol{\theta}}_2 = \underset{\hat{\boldsymbol{\theta}}_2}{\operatorname{argmin}} \sum_{i=1}^{n} \Big((X_{\mathrm{M},i}(\mathrm{i}\omega), Y_{\mathrm{M},i}(\mathrm{i}\omega)) - \boldsymbol{G}(\mathrm{i}\omega, \boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) \Big)^2 \quad . \tag{8}$$

In the case of the model parameter estimation of torque transducers, the model function consists of a vector of parameters of the measuring device $\boldsymbol{\theta}_1$, which is known, and the vector of unknown parameters of the transducer under test $\hat{\boldsymbol{\theta}}_2$, which is estimated. No weighting of the different values is applied with this estimator.

The maximum likelihood function ℓ for a vector of samples $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T$ with the sample's PDF

$p(x_i, \boldsymbol{\theta})$ for the estimation of the parameters $\boldsymbol{\theta}$ is the joint PDF of all samples giving

$$\ell(\boldsymbol{x}, \boldsymbol{\theta}) = p(\boldsymbol{x} \mid \boldsymbol{\theta}) = \prod_{i=1}^{n} p(x_i, \boldsymbol{\theta}) \ . \qquad (9)$$

For the dynamic torque application described and assuming normally distributed input quantities, the maximum likelihood function leads to

$$\ell(\mathrm{i}\omega, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{X}_\mathrm{M}, \boldsymbol{Y}_\mathrm{M}) \propto \prod_{i=1}^{n} e^{-\frac{((X_{\mathrm{M},i}(\mathrm{i}\omega), Y_{\mathrm{M},i}(\mathrm{i}\omega)) - \boldsymbol{G}(\mathrm{i}\omega, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2))^2}{2\,u_i^2}} , \qquad (10)$$

with the measurement data $X_{\mathrm{M},i}$, $Y_{\mathrm{M},i}$, the corresponding measurement uncertainties u_i, and the model function $\boldsymbol{G}$. The likelihood function is maximised for the parameter estimation. The estimated parameters $\hat{\boldsymbol{\theta}}_2$ are given by

$$\hat{\boldsymbol{\theta}}_2 = \arg\max_{\hat{\boldsymbol{\theta}}_2} \Big(\ell(\mathrm{i}\omega, \boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2, \boldsymbol{X}_\mathrm{M}, \boldsymbol{Y}_\mathrm{M}) \Big) \ . \qquad (11)$$

For independent and normally distributed input quantities, the maximum likelihood estimator can be reduced to a weighted least squares (WLS) estimator [6]. Then, the parameter estimation can be carried out with the standard uncertainties of the measurement data u_i as follows

$$\hat{\boldsymbol{\theta}}_2 = \operatorname*{argmin}_{\hat{\boldsymbol{\theta}}_2} \sum_{i=1}^{n} \frac{\Big(\boldsymbol{X}_{\mathrm{M},i}(\mathrm{i}\omega), \boldsymbol{Y}_{\mathrm{M},i}(\mathrm{i}\omega) - \boldsymbol{G}(\mathrm{i}\omega_i, \boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2) \Big)^2}{u_i^2} \qquad (12)$$

applying one of the widely available WLS estimators. For the given application, the induced errors are based on the different measurement quantities and need to be quantified by a measurement uncertainty evaluation. Based on this analysis, a correct weighting of the input measurement channels can be carried out.

The uncertainties of the estimated parameters cannot be calculated directly for the frequentist approaches. The calculation of lower uncertainty limits or of confidence intervals is not applicable for the dynamic torque calibration, because the influences of the uncertainties of the parameters of the measurement ($p(\boldsymbol{\theta}_1)$) are invisible for the estimator.

Therefore, the uncertainty evaluation of the estimated parameters will be carried out according to the recommendations of the *Guide to the expression of uncertainty in measurement* [7] (GUM) and its Supplement 1 [8], respectively. A Monte Carlo simulation with all input PDFs will be carried out to evaluate the uncertainty of the estimated parameters.

Differently from the frequentist approach, a parameter estimation based on Bayes' statistics assumes all parameters to be uncertain. Based on Bayes' theorem, the a posteriori PDF follows from the a priori PDF, the evidence's PDF and the likelihood (cf. equation 9) giving

$$p(\text{a postriori}) = \frac{\text{likelihood} \cdot p(\text{a priori})}{p(\text{evidence})} \ . \qquad (13)$$

For the parameter estimation of torque transducers, the latter equation leads to

$$p(\boldsymbol{\theta}_1, \hat{\boldsymbol{\theta}}_2 \mid \boldsymbol{X}_\mathrm{M}, \boldsymbol{Y}_\mathrm{M}) = \frac{\ell(\mathrm{i}\omega, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{X}_\mathrm{M}, \boldsymbol{Y}_\mathrm{M})\, p_0(\boldsymbol{\theta}_1)\, p_0(\boldsymbol{\theta}_2)}{p(\boldsymbol{X}_\mathrm{M})\, p(\boldsymbol{Y}_\mathrm{M})} . \qquad (14)$$

It becomes obvious that the measurement uncertainty is inherent in a parameter estimation by means of a Bayes' estimator, including uncertainty contributions of the parameters of the measuring device $p_0(\boldsymbol{\theta}_2)$, the contributions of the uncertainty of the measurement data, and contributions of prior knowledge of the parameters to be estimated $p_0(\boldsymbol{\theta}_1)$. The prior distribution of the parameters to be estimated does not need to be known exactly prior to the estimation (although this is mentioned accordingly in literature [6]), instead reasonable initial PDFs should be available.

Despite all of its advantages, the Bayes' estimator is still rarely used for parameter estimation of mechanical systems. It requires much more effort than LS approaches to be implemented, because it has to be developed individually for each application, whereas for the least squares based approaches, an application of the widely available LS algorithms is possible. This effort is the reason why this paper focuses on the least squares approaches in a first step. However, it is planned to develop a Bayes' estimator for the application to dynamic measurements of mechanical quantities in the future.

7. IMPLEMENTATION OF THE MAXIMUM LIKELIHOOD ESTIMATOR

The parameter estimation based on the measurement data is carried out by using the open source scientific computing software GNU Octave. The measurement uncertainties for the three measurement channels have been estimated prior to the implementation of the maximum likelihood estimator. The measurement uncertainties are assumed to be independent and normally distributed.

Therefore, the maximum likelihood estimator is implemented as a weighted least squares estimator (cf. (12)). The weighting of the different data points of each channel is based on two conditions:

1. The measurement uncertainties of the three acquired signals are assigned to the measurement values. The combined uncertainties u_comb of the two transfer functions are calculated by means of a quadratic summation giving $u_\mathrm{comb} = \sqrt{u_1^2 + \cdots + u_n^2}$ based on the uncertainty contributions of the measurement channels.

2. The uncertainty of each measurement value is taken into account by evaluating the covariance matrix of the sine approximation applied to the time series data. The variances for the approximated parameters (the diagonal elements of the covariance matrix) are normalised and inverted for the weighting.

The two resulting matrices are multiplied element by element to obtain the weighting matrix for the WLS estimator.

The equations representing the model parameters within the transfer functions are nonlinear in their parameters. Therefore, only iterative algorithms for nonlinear regression were applicable. To avoid complex numbers in the results for the model parameters, it was necessary to constrain the algorithm to real numbers in the parameter vector. To this end, one set of approximated parameters was calculated by means of the real ($\Re$) and imaginary parts ($\Im$) of the two inverse transfer functions giving $\Re(H_{\mathrm{top}}^{-1}(\mathrm{i}\omega))$, $\Re(H_{\mathrm{bott}}^{-1}(\mathrm{i}\omega))$, $\Im(H_{\mathrm{top}}^{-1}(\mathrm{i}\omega))$, $\Im(H_{\mathrm{bott}}^{-1}(\mathrm{i}\omega))$.

8. RESULTS

The analysis of the feasibility of the parameter estimation was carried out with one 10 N·m shaft type torque transducer. The transducer is a passive strain gauge transducer, the bridge signals are transmitted by means of slip rings. The signal conditioning electronics, data acquisition systems, and filters have been calibrated dynamically prior to the measurements. The electrical influences of these components in the measurement chain have been compensated for [9].

Figure 5 shows the magnitude and phase responses of the complex transfer functions of the top and of the bottom of the measuring device $H_{\mathrm{top}}(\mathrm{i}\omega)$ and $H_{\mathrm{bott}}(\mathrm{i}\omega)$ for both, measurement data and estimation. A good agreement between the measurement data and the outcome of the regression can be found. However, the measurement results show that the deviations from the expected values of the top transfer function increase for frequencies higher than the resonance frequency of the system. This is caused by an increasing decoupling of the top part of the shaft assembly for frequencies beyond the resonance frequency.

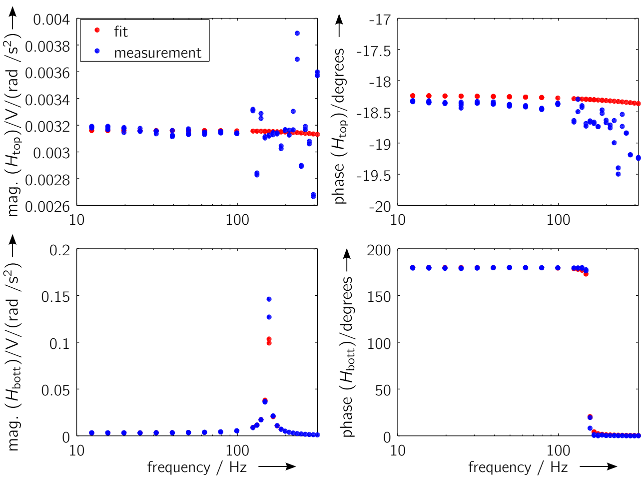

Fig. 5. Measurement data (blue) and fit result (red) for $H_{\mathrm{top}}(\mathrm{i}\omega)$ (top) and $H_{\mathrm{bott}}(\mathrm{i}\omega)$ (bottom) in magnitude and phase.

The real and imaginary parts of the two inverse transfer functions are given in Fig. 6. These are the transfer functions, with which the parameter estimation is carried out.

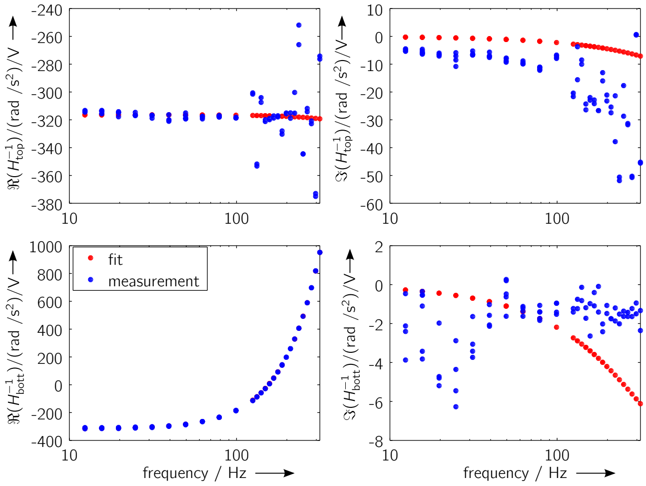

Fig. 6. Measurement data (blue) and fit result (red) for real ($\Re$) and imaginary ($\Im$) parts of $H_{\mathrm{top}}^{-1}(\mathrm{i}\omega)$ (top) and $H_{\mathrm{bott}}^{-1}(\mathrm{i}\omega)$ (bottom).

The real parts of the two transfer functions show a very good agreement of measured values and fit, however, the imaginary parts reveal discrepancies between observed values and the regression data. The identified model parameters are reasonable and agree with the data given in the specifications for the transducer.

9. SUMMARY

A model-based approach enables the identification of the dynamic properties of torque transducers from measurement data. The model is linear and time-invariant and consists of known model properties of the measuring device and unknown model properties of the transducer under test. The parameter identification is carried out using the acquired measurement data which was corrected for influences from signal conditioning electronics and the data acquisition system. Three signals are acquired during the calibration measurements. Based on this data, two complex transfer functions are calculated. A nonlinear regression is carried out based on the two transfer functions to estimate a set of common parameters. These parameters describe the dynamic behaviour of the torque transducers under test. The discrepancies in the imaginary parts of the two transfer functions require further investigation.

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