XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 - September 4, 2015, Prague, Czech Republic

NON-LINEARITY OF SENSORS AND DYNAMIC CALIBRATION

Th. Bruns [1]

[1] Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany, thomas.bruns@ptb.de

Abstract - Sensors for the dynamic measurement of mechanical quantities or motion quantities are sometimes subject to a non-linear input-output characteristics. In the case of calibration this may result in deviations of classical calibration results using different methods or different ranges.
The contribution is analysing the effects of non-linear sensor behaviour based on a virtual acceleration sensor. With a known mathematical sensor model and numerical simulations some typical effects are described by example and a simple measurement procedure is proposed to analyse and quantify an unknown non-linearity. The effectiveness of the proposal is demonstrated by a simulated comparison with other methods and the known "true" non-linearity.

Keywords: sensor characteristics, dynamic calibration, non-linearity, harmonic distortion

1. INTRODUCTION

Sensors for the measurement of mechanical quantities (force, torque, pressure) or motion quantities (velocity, acceleration) typically are based on elastic mechanical elements (springs) whose deformation is caused by the measurand and converted to an electrical quantity, e.g. by strain gages or piezo elements. In general the design goal for such sensors is a strictly linear input-output relation, where sometimes a bias at zero input is accepted. The response of the sensor to a static input is decribed by the simple equation

$$y(x) = S_0 \cdot x + b_\mathrm{S} \tag{1}$$

which is for $b_\mathrm{S} = 0$ also known as Hook's law. However, the characteristics of the internal spring element is usually not strictly linear but deviates from this ideal behaviour for reasons of e.g. non-linearity of the material properties or geometric design or working principles, like bending. The importance of such non-linearity deviation is dependent on the precission required by the measurement application. In high precission static force calibration by dead weight machines this issue is covered to some degree by using polynomials of third order to model the sensor characteristics.

The problem of non-linear sensor characteristics has been ignored in the case of dynamic calibration so far. In the most advanced area of dynamic calibration of motion sensors, the calibration of accelerometers, the calibration procedures of ISO 16063 are facilitating sinusoidal or shock excitation of the sensor and are based on a linear model of the accelerometer. In addition to the dynamic calibration there is a methodology using constant acceleration generated by centrifuge [2].

In a prior contribution to IMEKO by Dosch and Schiefer [1] a non-linear behaviour in the area of DC-coupled accelerometers was investigated. The paper shed some light on the observation that dynamic calibrations according to ISO 16063 provided results which deviated from those gained by centrifuge calibration.

The authors investigated different methods of data analysis for the dynamic calibrations with the goal to gain consistency between shaker based calibration and centrifuge calibration.

In the following the situation will be described based on a virtual sensor and numerical simulation. Some additional effects will be demonstrated and a simple method to quantify the non-linearity will be proposed.

2. THE SENSOR MODEL

During the investigations the sensor was represented by a mathematical model based on mass-spring-damper system (c.f. Fig. 1). The non-linearity was introduced as a non-linear relation of the force (F) to displacement (x) relation, where displacement means the deformation of the spring $x = x_\mathrm{h} - x_\mathrm{b}$, .

$$F(x) = \mathrm{sgn}(x) \cdot \frac{k_0}{\alpha} \cdot \left(e^{(\alpha \cdot |x|)} - 1\right) \tag{2}$$

with $\mathrm{sgn}(x)$ being the sign of x, k_0 the stiffness at zero displacement and α a tuning parameter that was adjusted for a reasonable non-linear behaviour. For $0 < \alpha << k_0$ this equation converges to the linear model

$$F_\mathrm{lin}(x) = k_0 \cdot x \tag{3}$$

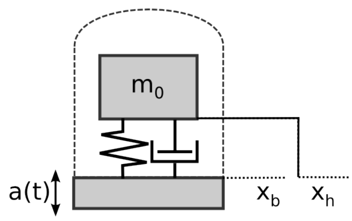

Fig. 1. sensor model

For the adjustment the following assumptions were taken:

- seismic mass $m_0 = 0,005kg$

- displacement at $a_0 = 100m/s^2$: $x_0 = 0,1\mu m$
- inertial force at $a_0 = 100m/s^2$: $F_{\text{lin}}(a_0) = 0,5N$
- relative non-linearity at $a_0 = 100m/s^2$: $F(a_0) = F_{\text{lin}}(s_0) \cdot 1,03$

With such assumptions one can easily derive the parameters for equation 2 as

- $k_0 = 5 \cdot 10^6 \frac{N}{m}$
- $\alpha \approx 588292 \frac{1}{m}$

The resulting force vs. displacement characteristics are displayed in fig. 2

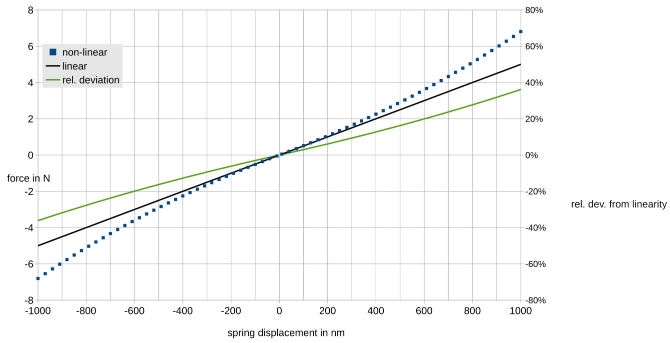

Fig. 2. Force vs. displacement characteristics of the non-linear model (blue points), linear approximation at zero (black line) and relative deviation of the two (green triangles)

The output $y(\ddot{x})$ of the accelerometer as a function of the applied acceleration was modelled proportional to the actual displacement as

$$y(x) = S_0 \cdot x\,(a(t)) \qquad S_0 = 1,0 \cdot 10^7 \tag{4}$$

with

$$y_{\text{lin}}\left(x(100m/s^2)\right) = 1.0 \tag{5}$$

Where y_{lin} presumes the linear spring model of the sensor. The non-linear model ist then introduced into the equation of motion of this single-mass oscillator in the form

$$m_0 \cdot \ddot{x}_{\text{h}} + d \cdot (\dot{x}_{\text{h}} - \dot{x}_{\text{b}}) + F(x_{\text{h}} - x_{\text{b}}) = 0 \tag{6}$$

Where x_{h} is the displacement of the seismic mass and x_{b} is the displacement of the base or the shaker armature, say.

With this virtual set-up of an accelerometer and the numerical tools for solving ordinary differential equations the investigation of non-linear sensors is possible in virtual experiments. In the subsequent sections several aspects of the non-linearity will be considered in order to have a reference to estimate the influence, the properties of a linear sensor will be considered.

3. The resonant frequency

As a first example the resonant frequency of the sensor may be considered. For the simple mechanical model described here the approximated resonant frequency for small damping factor ($d = 40$ Ns/m) is

$$f_{\text{res}} = \frac{1}{2\pi}\sqrt{k_0/m_0} \approx 5,00 \text{ kHz} \tag{7}$$

This approximation holds for strictly linear models or in the non-linear case for small strains of the sensing element, i.e. low acceleration levels.

With the virtual sensor one can investigate what happens to the resonant frequency for high accelartion levels. For this purpose the equation of motion was solved for purely sinusoidal exitation for a range of frequencies and amplitudes. Just like for a usual calibration, the output signal according to Eq. (4) was analysed using sine approximation and the derived amplitude devided by the input acceleration amplitude to get the magnitude of sensitivity. Figure (3) shows the resulting magnitude of sensitivity frequency response curves for four different excitation levels.

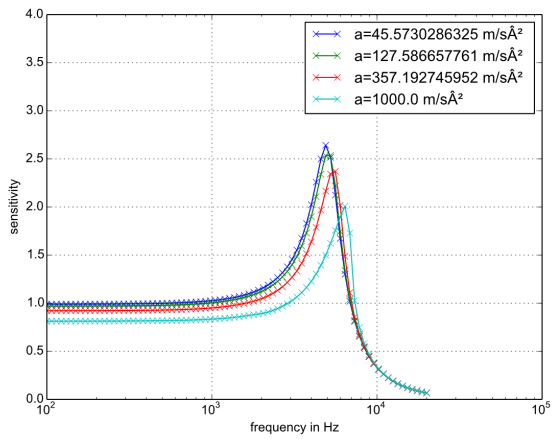

Fig. 3. magnitude of the transfer function of the virtual accelerometer at four different vibration levels

With relation to the resonant frequency it can be observed that the frequency of the maximum shifts to the right for increasing vibration acceleration levels. This can be explained by the progressive characteristics of the sensor's non-linear spring. Its effect for higher acceleration level is comparable with an increased effective stiffness of the spring. According to the simplified equation (7) an increased stiffnes results in a higher resonant frequency.

A second effect, which can be observed is a general decrease in the sensitivity. As the output was defined to be proportional to the spring deformation (c.f. Eq. 4), the increased effective stiffness leads to a decreased effective deformation of the spring and therefore a decrease of the effective output level, as compared to a constant stiffness, i.e. a linear sensor.

Thus, both effects observed for the transfer function are easily understood.

4. Analysing the non-linearity

In [1] the form of the non-linear characteristics is modelled by a polynomial and a methodology is described, how to derive the polynomial coefficients from a sequence of calibration measurements. The question is, whether this complex process is, in fact, necessary.

A simpler approach is to consider an analysis of the time series signals available from a single calibration run. Figure(4) shows the charts of the output signal of the virtual sensor over the current acceleration like an oscilloscope in x-y-mode. The three plots show the results for an acceleration amplitude of 1000 m/s at three different frequencies.

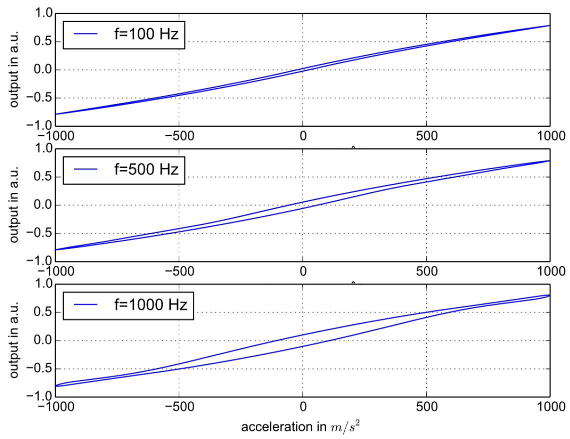

Fig. 4. Output of the sensor vs. momentary input acceleration for mono-frequent sinusoidal excitation at three different frequencies.

The non-linear response is clearly visible in all three plots. However, for increasing frequencies the gap around zero becomes larger. It is apparent that the non-linearity detectable in Fig. (4) is a consequence of the non-linarity of the internal spring of the sensor defined in Eq. 2. How the two relate and in how far a quantitative evaluation is possible with a single measurement will be discussed in the full paper.

5. comparison to other methods

In metrology the comparability of results is always a major concern. Especially when the compared results are derived with different methods. For the results derived in the previous section it is therefore quite relevant, how they compare to those derived with calibration methods described in [1]. In addition to this methodology comparison, the virtual transducer offers the opportunity to compare the results with the well-defined and known true non-linearity given by the mathematical model of Eq. (2). As the typical analysis of non-linearities in calibration makes use of polynomial approximations, for lack of better knowledge, we will use the Taylor-series expansion of Eq. (2) for that comparison purpose.

The formulas and results will be given in the full paper.

6. summay and conclusion

This contribution analyses the effects of non-linearity based on an example mathematical model of a virtual accelerometer. It describes some effects observed in dynamic measurements with such a sensor and explains their causes in a heuristic way.

For the quantification of the non-linear behaviour a single measurement methodology is proposed and the relation of the outcome to the intrinsic non-linearity of the sensor model is described.

The issues and effects described here can be transfered to other sensor systems like force, torque or pressure transducers, as well. Accordingly the proposed methodology can be adapted to other measurands considered in dynamic measurements.

Although issues like harmonic distortion seem to compromise the output of non-linear transducers, the utilization of such systems even in presission measurements appears feasible.

REFERENCES

[1] J. Dosch, M. Schiefer , *Vibration-based Methods for Evaluating Linearity of DC-Coupled Vibration Accelerometers*, IMEKO 22nd TC3, 12th TC5 and 3rd TC22 International Conferences, Cape Town, South Africa, 2014.

[2] ISO 5347-7:1993, *Methods for the Primary Calibration of Vibration and Shock Pick-ups Part 7 Primary Calibration by Centrifuge*, ISO, Geneva, 1993