

# TRANSIENT ANALYSIS IN NON-LINEAR SYSTEMS THROUGH THE HUANG HILBERT TRANSFORM

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**Abstract** - The paper deals with the use of the Huang Hilbert Transform (HHT) for the analysis of transient signals in non linear circuits. Differently from traditional methods that decompose a signal as the sum of a priori adopted waveforms, HHT procedure decomposes the signal of interest in oscillatory modes a posteriori determined, in dependence on the signal characteristics, giving, thus, a better information about the physical characteristics of the observed system.

The performance of the proposed analysis method has been evaluated by applying the HHT to the current acquired from an actual RLC circuit during its free natural oscillation. The circuit involves a saturable inductor, therefore the HHT capability of providing physically meaningful information accounting for both non stationarity and non linearity has been appreciated.

**Keywords:** Transient analysis, non-linear systems, Huang Hilbert Transform, Empirical Mode Decomposition, Intrinsic Mode Functions, Hilbert Spectrum.

## 1. INTRODUCTION

In the analysis of electrical signals, the Fourier Transform (FT) is the widely exploited approach [1],[2]. When the signal of interest exhibits a non-stationary time evolution, the FT approach suffers from artifacts due to any spectral variation along the time observation interval [3],[4]. The Short-Time Fourier Transform (STFT) provides a time-frequency representation of the signal, by performing the Fourier transform of a window sliding on the observation interval. The drawback lies on the selection of the window width; the larger the window, the better the frequency resolution, the worse the time resolution. STFT, then, proves to be inadequate if information about the signal is not available and/or the transient signal contains both high and low frequency components [5].

Many research works in literature propose the use of Wavelet Transform (WT) for transient analysis [6], which allows to obtain a multi-resolution time-frequency signal decomposition. In this instance, however, the selection of the mother wavelet is critical, since there are not clear rules for selecting the optimal mother wavelet for a specific application.

All the aforementioned approaches, as well as other recent time frequency representations (reassigned STFT, Wagner-Ville, and so on) [7],[8],[9], are based on the assumption that the signal of interest can be accurately

expressed in terms of sum of a defined number of bases functions. The shape of the basis function, of course, is a priori determined according to the available information on the signal. As a consequence, the signal is constrained to match the model imposed by the chosen representation regardless the actual physical phenomena underlying its generation [10]. As an example, Fourier Transform uses linear superposition of trigonometric functions. Therefore, the distortion affecting a signal resulting from a nonlinear process is modeled by the presence of harmonic components, which are added in order to properly fit the signal in the chosen representation basis.

Traditional approaches, so, although provide a representation mathematically valid, suffers from a lack of physical meaning.

Moreover, the representation quality of the signal is directly dependent on the selected model. Even if preliminary knowledge about the signal should allow to select a proper model, the chosen representation will however fail if the characteristics of the signal of interest change during the observation interval (as occurs in the presence of the considered non-stationary signal).

To overcome the considered limitation and make the transient analysis independent from the specific signal of interest, the authors propose the use of Huang Hilbert Transform (HHT)[11]. Through HHT, the signal is decomposed in elementary oscillations exhibiting different shapes, which are a posteriori determined, according to the signal characteristics [12]. Differently from the traditional approaches, the representation model is tailored on the analyzed signal, thus allowing a deeper understanding of the physical phenomena underlying the examined process.

## 2. THEORETICAL BACKGROUND

### The Intrinsic Mode Functions

In the traditional Fourier analysis, the frequency is defined as fundamental characteristic of sine or cosine function; the frequency can be estimated if at least one full oscillation is observed. Of course, such a definition cannot be applied to non-stationary signal, whose amplitude and frequency rapidly change during the observation window. For this class of signals, an extension of the concept of frequency, able to describe the local characteristics of a time-varying signal, is required. At this aim, in signal processing theory the term instantaneous frequency has been introduced. The instantaneous frequency is the

generalization of the definition of frequency, i.e., it is the rate of change of the phase angle  $\theta$  at time  $t$  [13].

$$f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad (1)$$

The instantaneous phase in equation (1), can be determined if the amplitude and frequency information of the signal of interest  $x(t)$  is expressed in complex form. In fact, the complex signal:

$$z(t) = x(t) + iy(t) \quad (2)$$

can be expressed in polar form:

$$z(t) = a(t) e^{i\theta(t)} \quad (3)$$

where:

$$\begin{aligned} a(t) &= \sqrt{x^2(t) + y^2(t)} \\ \theta(t) &= \tan^{-1} \left( \frac{y(t)}{x(t)} \right) \\ f(t) &= \frac{1}{2\pi} \frac{d\theta(t)}{dt} \end{aligned} \quad (4)$$

obtaining, thus, the instantaneous amplitude, phase and frequency of  $x(t)$ . As a matter of practice, the concept of phasor adopted for stationary data has been generalized, representing the signal as a phasor with time-varying amplitude ( $a(t)$ ) and frequency ( $f(t)$ ) given by equation (4).

Several methods have been proposed in literature for expressing a real signal in complex form. Among them, Gabor demonstrated that the introduction of the analytic signal provides the definition closest to the physical meaning of instantaneous phase [14]. In particular, the Gabor's procedure takes advantage from the Hilbert transform, which, under defined assumptions, provides the quadrature component of  $x(t)$  so that the resulting complex signal:

$$z(t) = x(t) + iH[x(t)] \quad (5)$$

is an analytical signal that exhibits a spectrum identical to that of  $x$  for positive frequencies and zero for the negative frequencies.

As aforementioned, the signal  $z(t)$  in equation (5) is analytic only if determined conditions are met. From equation (3) it can be noticed that if a signal is time-varying, its changes can be expressed through an amplitude modulation (i.e. through  $a(t)$ ) as well as a phase modulation (i.e. through  $\theta(t)$ ). The analytical signal procedure leads always to a solution characterized by slow variations of amplitude and fast variations of phase [15],[16]. As a consequence, a time-varying signal  $x$  can be reliably represented by its analytic function only if its amplitude and phase have different rate of change; more specifically, the signal has to satisfy the Bedrosian condition [17], which requires that the spectrum of  $a(t)$  has to exhibit frequencies lower than that of the spectrum of  $\theta(t)$  and the two spectra have not to be overlapped.

Another issue related to the given definition of

instantaneous frequency has to be taken into account. It is expected that, for each instant, the signal frequency can be expressed by means of only one value. At this aim, the term monocomponent signal has been introduced. Despite an exact definition of monocomponent is still under study, this term intuitively means that the signal is represented, locally, by a ringing wave that can be characterized by only one value of frequency.

The Huang theory [18] defines a class of functions, the Intrinsic Mode Functions (IMFs), for which the constraints discussed above are met and, so, the instantaneous frequency has physical meaning everywhere.

An IMF is a function that satisfies two conditions assuring that it is a monocomponent function without superimposed ringing: (i) in the whole observation interval, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (ii) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero [19].

The Huang-Hilbert Transform (HHT) is based on the decomposition of a signal in a finite number of IMFs, which benefit from the following characteristics:

- the instantaneous frequency is always defined
- there are not constraints on their shape;
- can be non-stationary.

In particular, the last two items makes the IMFs free to adapt their evolution to the specific signal of interest, thus making possible to highlight the inherent phenomena underlying the signal itself. In fact, the name Intrinsic Mode Function is adopted because it represents the oscillation mode embedded in the observed signal.

### The Empirical Mode Decomposition

The Empirical Mode Decomposition (EMD) is the process aiming at find out the significant IMFs of the signal of interest. It is based on the sifting procedure, which extracts, from the empirical data, each one of the oscillatory modes characterizing the signal.

Firstly, the local extrema of the input signal  $x(t)$  (shown in Fig.1) are detected; the upper and lower extrema are fitted through cubic splines, obtaining, thus, the upper and lower envelopes, shown with the green and red lines in Fig.1. Being  $m_1$  the mean of the envelopes (the black line in Fig.1) the difference between  $x$  and  $m_1$  is evaluated:

$$h_1(t) = x(t) - m_1(t) \quad (6)$$

The result  $h_1$  is shown in the left plot of Fig.1.

Ideally,  $h_1$  should be an IMF, since its construction seems to have been made to satisfy all the requirements of an IMF. Actually, after the first sifting, a soft swell can be amplified and lead to new local extrema. This occurrence is experienced when the signal  $x$  is affected by low amplitude riding waves which are lost in the initial examination, but are

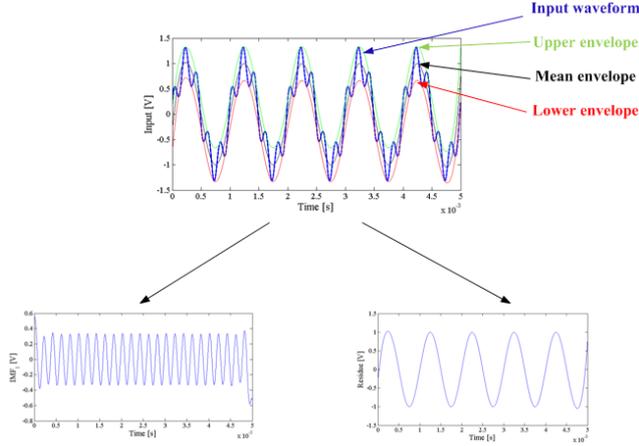


Fig. 1. Example of sifting procedure.

recovered after repeated sifting. If  $h_1$  is not an IMF, thus, the sifting process is repeated on  $h_1$  (here expressed as  $h_{1(1)}$  to evidence that it represents the first trial of  $h_1$ ). Again, the difference between the input and the mean of the envelope is determined:

$$h_{1(2)}(t) = h_{1(1)}(t) - m_{1(2)}(t) \quad (7)$$

The process is iterated until the resulting waveform:

$$h_{1(k)}(t) = h_{1(k-1)}(t) - m_{1(k)}(t) \quad (8)$$

satisfies the condition of an IMF.

In this instance, the first IMF  $c_1$  of  $x$  has been determined. The residue, shown in Fig.1-d and evaluated as:

$$r_1(t) = x(t) - c_1(t) \quad (9)$$

still contains modes of  $x(t)$  and it is treated as the new input for a new sifting process.

The procedure is iterated, obtaining a number  $n$  of different IMFs and it is stopped when one of the following criteria is met: (i) the component,  $c_n(t)$ , or the residue,  $r_n(t)$ , becomes smaller than a predetermined value; (ii) the residue,  $r_n(t)$ , is a monotonic function and the extrema cannot be detected. If the input data are characterized by a trend, the residue  $r_n(t)$  provides that trend.

After the EMD procedure, the input signal can be expressed as the sum of  $n$  empirical modes and a residue:

$$x(t) = \sum_{j=1}^n c_j(t) + r_n(t) \quad (10)$$

The intrinsic characteristic of the EMD due to the sifting process and confirmed by the resulting IMFs of Fig.??, is the capability of ordering the IMFs in terms of their oscillation rate from the highest to the lowest [20]. Typically the first IMFs (characterized by the highest values of oscillation rate) accounts for noise contribution. This way, EMD can also be adopted as a data-adaptable denoising method by combining the IMFs exhibiting the

lowest oscillation rate.

When the EMD is implemented, two practical aspects have to be taken into account. First, the end effect caused by the spline fitting. Cubic spline can exhibit large swing close to the edge of the observation window, which are propagated by the sifting process and can corrupt the whole decomposition. Different solutions, in literature, have been recently proposed in order to mitigate this effect [21],[22].

The second issue is related to the proper number of iteration of the sifting. The iterations of the sifting process aim to make the wave profiles symmetric with respect to the zero mean. Nevertheless, if too many repetitions are carried out, the resulting IMF tends to align the local maxima and the local minima, becoming a constant-amplitude waveform. As a consequence, amplitude fluctuations that could carry physical information about the observed signal, are lost. Therefore, a criterion for the sifting process to stop is required. Huang proposed a limitation on the standard deviation SD computed from two consecutive sifting results; in particular, the sifting process is stopped if:

$$SD = \int_T \frac{h_{1(k)}^2(t) - h_{1(k-1)}^2(t)}{h_{1(k-1)}^2(t)} dt < \epsilon \quad (11)$$

where  $T$  is the whole observation window. Typical values of  $\epsilon$  are within the range 0.2-0.3.

### The Huang Hilbert Spectrum

Thanks to the adopted definition of the IMFs, their instantaneous frequency as expressed in equation (1) turns out to be physically meaningful.

If each IMF is processed through the Hilbert Transform and expressed in polar form, neglecting the residue, the input signal  $x(t)$  can be denoted in the form:

$$x(t) = \sum_{j=1}^n a_j(t) e^{i\theta_j(t)} = \sum_{j=1}^n a_j(t) e^{i \int_T \omega_j(t) dt} \quad (12)$$

By comparing equation 12 with the traditional Fourier expansion:

$$x(t) = \sum_{j=1}^{\infty} a_j e^{i\omega_j t + \phi_j} \quad (13)$$

it can be observed that the HHT is a generalization of the Fourier transform for non-stationary signals, for the offered opportunity of decomposing the signal in components with time-varying amplitude and frequency.

In order to obtain a global interpretation of the signal behavior, the amplitudes and the instantaneous frequencies of each IMF have to be combined to represent the so called Hilbert Amplitude Spectrum  $H(\omega, t)$ , which is a three dimensional plot where the amplitude is shown with respect to the time frequency plane.

### 3. EXPERIMENTAL SETUP

In order to better appreciate the HHT performance when applied to time-varying signals in non-linear systems,

the experimental setup in Fig.2 has been realized. The station involves: a  $40 \mu F$  capacitor, a DC power supply providing a voltage  $E=400$  V to charge the capacitor, a  $0.1 \Omega$  shunt resistor for the measurement of the circuit current, a 240 mH magnetic core inductor characterized by 3 A rated current, and, finally, a Single Pole, Double Throw switch (SPDT) that turns the capacitor from the charge condition to the discharge one. Both the capacitor voltage and the voltage drop on the standard resistor have been acquired by means of a digital oscilloscope set for digitizing 50000 samples with a frequency rate equal to 125 kS/s. The capacitor is firstly charged to the 400 V voltage by connecting the SPDT to the terminal 1. When the SPDT is switched on the terminal 2, the capacitor is connected to a series RLC and discharges through damped oscillations at the natural RLC frequency, theoretically, of 56 Hz.

The waveforms acquired by the oscilloscope are shown in Fig.3. As expected, the voltage shows the oscillations of an under damped circuit. Also the current exhibits damped oscillation, but, compared with the voltage, the current waveform better evidences the non linearity of the circuit. In particular, the acquired current shows a first distorted portion due to the saturation of the magnetic core of the inductor; successively, the current decays, making the magnetic core operate in the linear zone of its characteristic.

The HHT, so, has been applied to the current signal and eight IMFs have been obtained. For a better display of the waveforms, in Fig.4 only the four IMFs, characterized by the higher energy, are shown.

The advantages related to the EMD decomposition are clearly highlighted and can be summarized as follows. First, the start of the discharging transient is plainly identified by  $IMF_1$  and  $IMF_5$  that, accounting for the fastest oscillation rates, isolate the contact bounce of the switch. Next,  $IMF_7$  highlights the inductor saturation, since it reveals a non-sinusoidal component only in the first 50 ms, i.e. when the current overcomes the inductor rated current. Finally,  $IMF_8$  accounts only for the damped sinusoidal

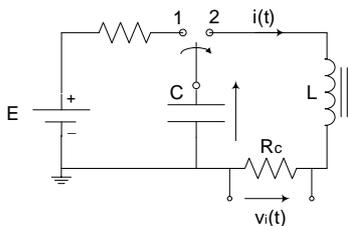


Fig. 2. Experimental set up and its circuitual model.

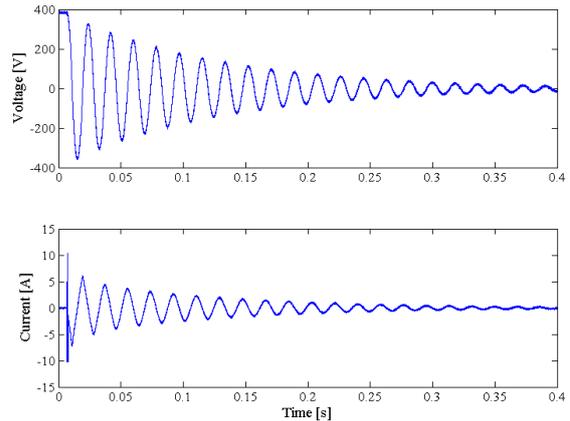


Fig. 3. Acquired signals.

oscillations of the RLC circuit; this way the signal, as an example, can be processed in order to obtain the RLC circuit characteristics, regardless fast transients, noise, and non-linearity originally affecting the acquired signal. The estimated Hilbert amplitude spectrum, then, is shown in Fig.5.

In the first portion of the discharge transient, the spectrum exhibits a frequency oscillation that, after about 50-60 ms, settles down to 55 Hz, which is the circuit natural frequency. As a matter of practice, the circuit non linearity, that in a traditional Fourier analysis would be modeled through the addition of harmonic components, here is represented by a fluctuation of the instantaneous frequency, that better models the actual physical evolution of the system [23]. In fact, the frequency fluctuation accounts for the rate of change of the instantaneous phase, that, in a linear oscillating circuit, is supposed to be constant. The period of frequency oscillation turns out to be the half of the period of the current signal; such a behavior agrees with the inductor saturation phenomena, that occur at the current peak values, i.e. twice in a period.

As regard the current amplitude, that can be appreciated through the trace color of the Hilbert spectrum, it becomes negligible after about 250 ms, agreeing with the

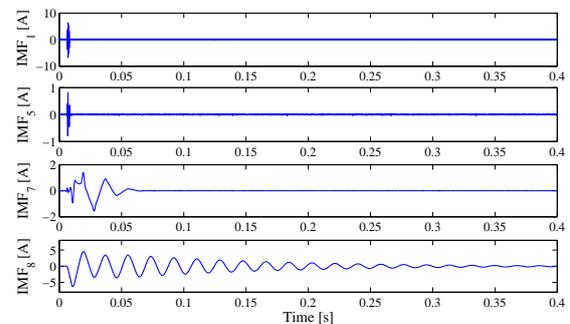


Fig. 4. IMFs extracted by the circuit current.

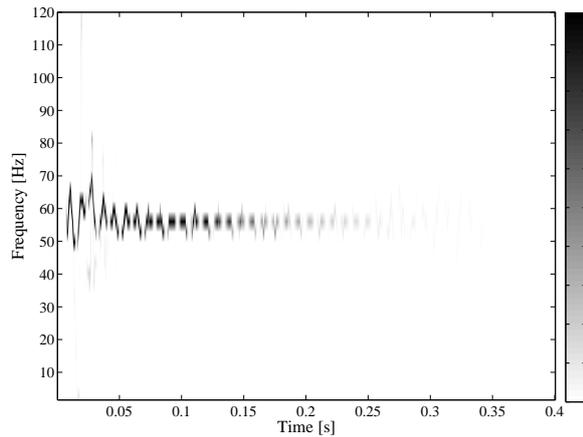


Fig. 5. Hilbert Amplitude Spectrum of the current.

time evolution behavior of the current signal shown in Fig.3.

#### 4. CONCLUSION

In the paper, the use of Huang Hilbert Transform for the analysis of non-stationary signals acquired from non-linear systems has been proposed.

The Empirical Mode Decomposition, in fact, allows to recognize specific features characterizing the signal of interest, contrary to approaches based on the signal decomposition based on apriori defined waveform set, which cannot provide information about the observed physical phenomena since they try to adapt the signal of interest to the selected class of waveforms.

In the proposed approach, HHT assures the following advantages: (i) noticeable both time and frequency resolutions, (ii) preliminary information about the signal of interest is not required, (iii) the signal is decomposed in elementary oscillations (IMFs) depending on the analyzed signal but not known apriori, allowing to reliably reconstruct the instantaneous frequency of non-stationary signals, whatever its evolution.

The HHT performance has been assessed through experimental tests; in particular a free oscillating RLC circuit characterized by the presence of a non linear inductor has been exploited in order to obtain a current signal exhibiting both non-stationary and non-linear behavior. The noticeable capability of estimating the instantaneous frequency evolution versus time of HHT has been appreciated. The experimental tests have evidenced, in particular, the most appealing characteristic of HHT, which is the rare opportunity of extracting information about the physical phenomena (i.e. periodic saturation of an inductor) originating the signal under test.

In conclusion, a non-stationary signal can be expressed through the sum of IMFs as well as the sum of harmonic components or wavelet functions; in all these analyses, the signal of interest can be recovered by adding the

obtained components. However, this equivalence is merely mathematical and not associated to the physical phenomena. Fourier harmonics, in fact, as well as Wavelet coefficients, appears because of the model adopted to represent the signal of interest; on the contrary, HHT is not based on a model of the input signal, but is a data-driven representation, assuring, thus, components that are determined by the actual behavior of the observed system.

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