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THE USE OF PRONY’S METHOD TO DETECT CHANGES IN THE TECHNICAL CONDITION OF A PRESSURE TRANSDUCER

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Abstract – The use of Prony’s method to detect changes in the technical condition of a pressure transducer is described. The technique is based on spectral analysis of transducer output signal and estimating its frequency response by Prony’s method. Ways to increase the accuracy of Prony’s method have been proposed and algorithm for transducer output signal processing based on modified Prony’s method is presented.

Keywords: pressure transducer, fault detection, Prony’s method, sampling rate, frequency response

1. INTRODUCTION

Current scientific research in the field of intelligent sensors is closely related to their fault detection and diagnosis techniques as well as ways to improve the quality of measurements and to make them self-validating [1-2]. Although several self-validating pressure sensor prototypes have been proposed (see, for example, [3] or [4]), still there is a need of methods and techniques of detecting changes in the condition of sensor structure that may be caused by a diaphragm, weld joint or rod damage, change in other technical parameters.

To develop a self-validating sensor one should propose a method to diagnose its possible faults affecting the results of a measurement. The attractive way is to use additional information contained in its output signal. Methods of fault detection and diagnosis based on the signal model are described in [5].

In [4] a technique to detect changes in the condition of a strain-gage pressure transducer based on analyzing its output signal and estimating frequency response was proposed and examined. The methodology described is consecutive excitation, recording a response, spectral analysis and estimation of the condition. As an excitation mean ultrasonic pulse is used. The spectrum considered is up to 100 kHz. The study of the transducer finite-element model and experimental results have shown that it is possible to detect faults by analyzing the output signal of the transducer.

But still there are unsolved problems. The key technical challenge is to estimate the parameters of a pressure transducer output signal (such as frequencies or damping factors) with very high accuracy in order to be able to detect small shifts.

The conventional method of spectral analysis is the Fourier transform which suffers from various shortcomings, including the resolution limit depending on the response duration. Prony’s method is an alternative to the Fourier transform widely applied to spectral analysis. It provides high-accuracy estimation for amplitudes, damping factors, frequencies and phases of signals.

Unfortunately, in practice there are some disadvantages with Prony’s method because it is very sensitive to noise. Therefore special techniques are needed to analyze real signals with Prony’s method. Moreover, the accuracy of the method depends on parameters choices, such as the model order, the sampling rate of the signal and the number of samples to be processed.

In this paper the ways to improve the performance of Prony’s method are analyzed, and an algorithm for frequency response estimation based on modified Prony’s method is presented. The layout of the paper is as follows. Section 2 provides a brief description of ways to improve Prony’s method. In section 3 the algorithm for frequency response estimation based on modified Prony’s method is presented. Results from simulations are discussed in section 4. Finally, conclusions and future plans are stated.

2. WAYS TO IMPROVE THE PERFORMANCE OF PRONY’S METHOD

Prony’s method is based on fitting a sum of exponential terms to the measured data and is well-known tool for estimating signal parameters because many physical phenomena are described by equations whose solution is an exponential decay [6]. The description of the conventional procedure of Prony’s method can be found in [7] or [8].

2.1. Well-known techniques for improving the accuracy of Prony’s method

Prony’s method in its original form is very sensitive to noise. In practice, techniques can be used to improve its performance. Firstly, more than $2n$ experimental noisy points can be used to determine $2n$ parameters with the use of the least squares method.

Another approach (described in [6]) is to use Prony’s method first with model order p larger than the expected number of exponential components q. The result is a set of p exponentials that are candidates for the signal components. The next step is to determine a smaller subset of exponentials which gives the best fit to the observed data using the least squares criterion. To determine the number of exponential components in TLS-Prony’s method Singular Value Decomposition is used (TLS stands for Total Least Square).

One further approach is to use both forward and backward linear prediction polynomial zeros and to separate genuine and noisy zeros by comparing the two sets of results.

2.2. Techniques for improving the accuracy of Prony's method proposed in this research

All the ways mentioned in the previous subsection are good, but improve the performance of Prony's method insufficiently. In this subsection some additional techniques for improving the accuracy of Prony's method will be presented and discussed.

First, pre-processing of a transducer signal is used to estimate appropriate values of parameters (number of frequencies, their rough values, signal-to-noise ratio, etc.) Using additional parameters (such as *eps* – parameter that separates genuine and noisy zeros within the forward – backward linear prediction algorithm) gives better results if we know how to choose values of these parameters. The right values can be determined by numerical calculations. When analyzing real signals sample (or test) signals may be used to determine the right values.

Second, as mentioned in [9] and [10], the accuracy of the frequencies estimated by Prony's method strongly depends on the sampling rate of the analyzed signal. In [8] we presented the algorithm for choosing the optimal sampling rate of a signal to be processed by Prony's method based on minimization of condition number of matrices in Prony's method. Using the Euclidean norm and solving problem in general form, one can obtain analytical formula for the optimal sampling rate.

If a signal contains n frequencies $f_1<\ldots<f_n$ then the optimal sampling rate can be expressed as:

$$Fsopt = 4 \cdot n \cdot \frac{\sum_{k=1}^{n} f_k^2}{\sum_{k=1}^{n} (2k-1) f_k} \quad (1).$$

Third, algorithm of combining frequency estimates obtained from different parts of a signal can be improved by taking into consideration signal-to-noise ratio of each part of the signal. Signal-to-noise ratio of a signal is decreasing, so it influences on estimates. It is better to find resulting estimate of a frequency as weighted average of M estimates from M segments of the signal by using signal-to-noise ratio of each segment (SNR_j) as weights:

$$\hat{f} = \frac{\sum_{j=1}^{M} (SNR_j \cdot \hat{f}_j)}{\sum_{j=1}^{M} SNR_j} \quad (2).$$

3. ALGORITHM FOR FREQUENCY RESPONSE ESTIMATION BASED ON MODIFIED PRONY'S METHOD

As a result of implementation of aforementioned techniques we developed an algorithm for frequency response estimation based on modified Prony's method (see Fig. 1).

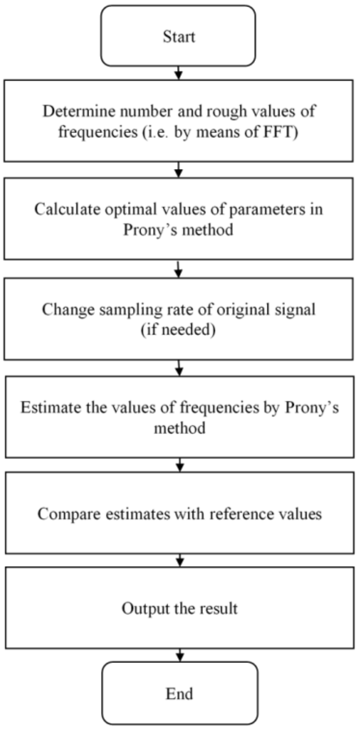

Fig. 1. Algorithm for frequency response estimation

4. SIMULATION AND RESULTS

4.1. A case of original sampling rate

Real output signal can be expressed as

$$x(t) = \sum_{j=1}^{k} A_j \, e^{-\alpha_j t} \cos(2\pi f_j t + \varphi_j) + r(t), \quad (3)$$

where j – index of an exponential component of the signal, A_j – magnitude of j component, α_j – damping factor, f_j – frequency value, φ_j – initial phase, $r(t)$ – noise, k – model order.

In the case of discrete signals (3) will transform into following expression:

$$x[n] = s[n] + r[n], \quad (4)$$

where $s[n] = \sum_{j=1}^{k} A_j \, e^{-\alpha_j \Delta T n} \cos(2\pi f_j \Delta T n + \varphi_j)$ – samples of the informative part of the signal, that carries useful information concerning the characteristics of a transducer (ΔT – sampling period), $r[n]$ – additive white nose.

To examine the proposed algorithm simulation was performed. Model signals consist of 7 damped sinusoidal components with frequency values from 10 to 50 kHz. Number of samples of each observation is 2500. Signal-to noise ratio was varying from 10 to 5 dB (see (5)). The initial value of sampling rate is 500 kHz.

The parameters of Prony's method being under investigation are:

p – model order in Prony's method (should be much greater than expected number of exponential components),

q – expected number of exponentials (for real signal is twice greater than the number of frequency components),

N_samp – number of samples of the signal being analyzed (must be greater than 2p),

eps – parameter that separates genuine and noisy zeros within the forward – backward linear prediction algorithm and controls choosing right roots,

Nobs – number of observation of the signal taken into consideration,

fs – sampling rate,

SNR – signal-to-noise ratio.

Evaluation of estimating error is performed using relative error of frequency estimate (6)

$$\delta f = \frac{|f - f_0|}{f_0} \cdot 100\%, \qquad (6)$$

where $\hat{f}$ – frequency estimate by Prony's method, f_0 – reference value of the frequency.

Results of simulation and applying Prony's method to frequency response estimation are shown on Figures 2-5. In first case sampling rate is not changed. Colored lines correspond to estimates from Prony's method, dashed line – estimate from Fast Fourier Transform (FFT), solid line – limit for relative error, *p* is model order.

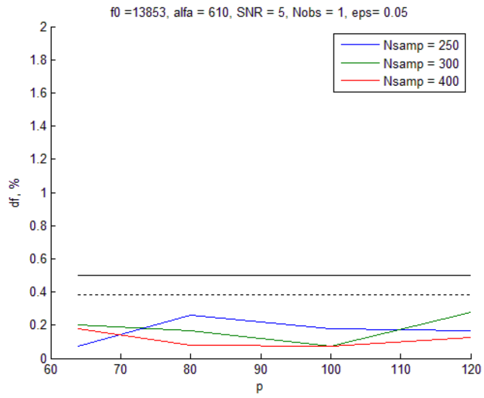

Fig. 2. Frequency estimate relative error in the case of original sampling rate (f0 = 13853)

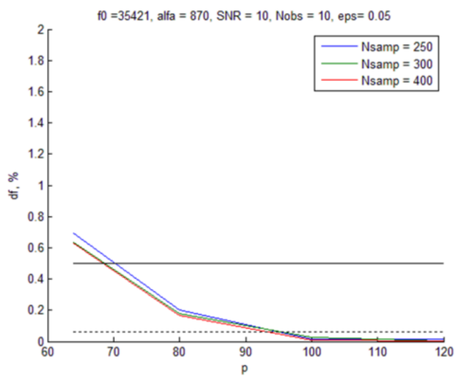

Fig. 3. Frequency estimate relative error in the case of original sampling rate (f0 = 35421)

4.2. A case of optimal sampling rate

In this case sampling rate is changed as described in subsection 2.2. According to (1) for the set of frequencies in Hz – 13853, 17690, 27690, 29600, 32600, 35421 and 44310 – the optimal value of sampling rate is 107680 Hz.

Taking into account that new signal is obtained from original by resampling, initial sampling rate should be a multiple of the new sampling rate. The closest appropriate value is 100 hHz.

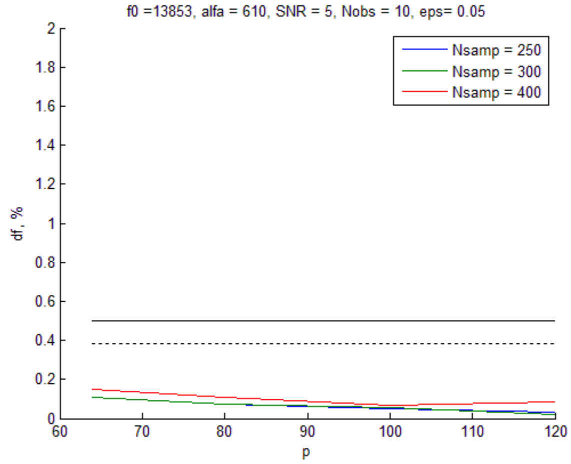

Fig. 4. Frequency estimate relative error in the case of optimal sampling rate (f0 = 13853)

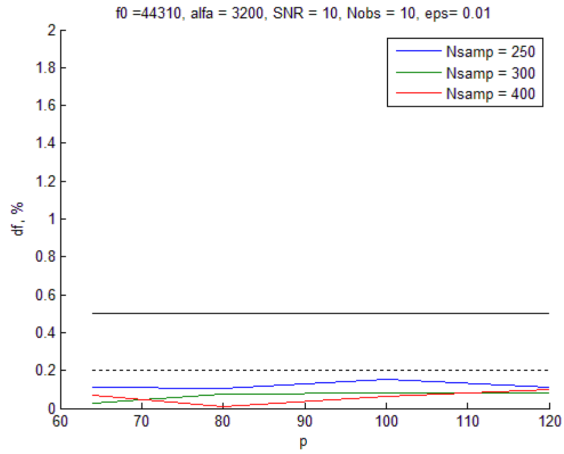

Fig. 5. Frequency estimate relative error in the case of optimal sampling rate (f0 = 44310)

As it can be seen, using optimal sampling rate notably improves the performance of the algorithm based on Prony's method. It improves both the number of frequencies that are estimated with sufficient accuracy, and accuracy of estimates of each frequency.

5. EXPERIMENTAL RESULTS

Experiments were performed by the technique described in [4]. A pressure transducer weld damage influence on frequency response of the transducer was examined.

To evaluate the efficiency of proposed algorithm test frequency estimates were obtained. Test samples of a sinusoidal signal with known frequency value was added to the samples of real signals. 10 observations were taken and each time the frequency value of the test signal was changed by 0.1% from initial value – from 16542 Hz to 16691 Hz. Maximum change is 0.9%. Results of frequency estimation are shown on Figure 6.

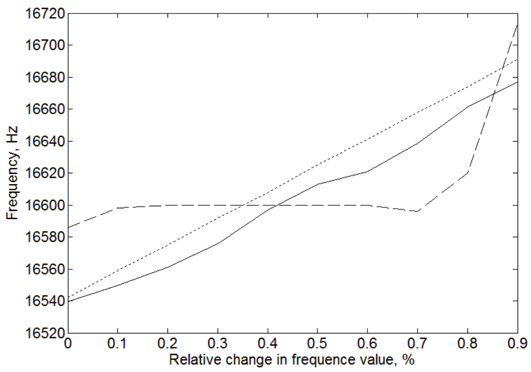

Fig. 6. Results of estimation of the change in frequency with initial value 16542 Hz in a real signal by Prony's method (solid line) and by FFT (dashed line). Dotted line – genuine value of the frequency.

Analyzing the results shown on Figure 6 one can conclude that proposed algorithm based on Prony's method gives accurate estimates of frequency and is more sensitive than FFT, especially for small changes of frequency – smaller than 0.7% - in that case FFT does not work. Applying the algorithm to analyzing real signals when weld damage occurred we obtained estimates of change in frequency values which is the result of the defect.

Table 1 – Estimates of change in frequency values of a signal before and after a transducer weld damage

Frequency value before the defect	Frequency value after the defect	Relative change (in %)
17643 Hz	17525 Hz	0,67 %
26878 Hz	26753 Hz	0,46 %
45440 Hz	45275 Hz	0,36 %

Prony's method is more sensitive than FFT, so it can determine the influence of the weld damage on frequency response of the transducer. Change in three frequencies (17643 Hz, 26878 Hz and 45440 Hz) was registered – see Table 1.

6. CONCLUSION

In this paper, ways to improve the performance of Prony's method are described, and algorithm for frequency response estimation based on modified Prony's method is presented. Numerical calculations and experimental results show that the proposed algorithm based on modified Prony's method can be used to detect changes in the technical condition of a pressure transducer.

7. REFERENCES

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