COORDINATE MEASUREMENT OF 3D LINE ON FREEFORM SURFACE

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Abstract - 3D line is defined as a line in 3D space in general position with respect to the coordinate system. However, it is impossible to consider any line in dimensional metrology without respect of underlying geometry. The paper deals with 3D line located on freeform surface and introduces a new approach to process data measured along this line by tactile coordinate measuring. Three different methods of interpretation of measured data and form error determination are described here.

Keywords: associated feature, 3D line, freeform surface, normal vector, freeform measurement.

1. INTRODUCTION

According to the ISO 10 360-6 [1], a 3D line p associated to a set of points obtained by coordinate measuring is defined by three-dimensional definition point $\mathbf{A} = (x_\mathbf{A}, y_\mathbf{A}, z_\mathbf{A})$ lying on the line and direction cosines a, b and c of the direction vector $\mathbf{d}$ of the line, see fig. 1 (angles α, β and γ between the direction vector $\mathbf{d}$ and coordinate axes x, y and z are drawn here in the given order). The direction cosines are coordinates of unit direction vector of the line, therefore, a vector equation of associated line p is given by

$$\mathbf{C}(t) = \begin{pmatrix} x_\mathbf{A} + at \\ y_\mathbf{A} + bt \\ z_\mathbf{A} + ct \end{pmatrix},\ t \in R, \tag{1}$$

where t is parameter and $\mathbf{C}(t)$ is radius vector of a point on the line.

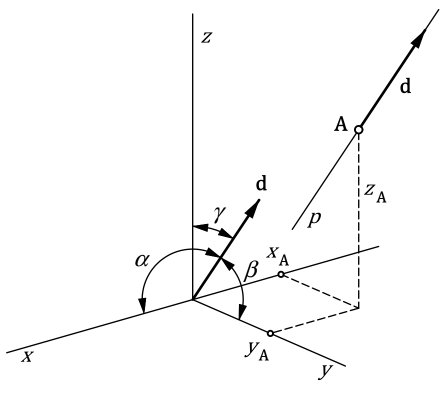

Fig. 1. To the definition of 3D line.

However, there are no single lines in practical dimensional metrology. Usually, the line is supposed to be a constructive feature (axis of symmetry, axis of revolution, pitch line, straight edge obtained by intersection of two planes, contact line of tooth surfaces of gears [2], [3], etc.) necessary for the following measuring process and analysis.

In this paper, a more general approach to the 3D line association problem is described. 3D line located on freeform surface is investigated and the underlying freeform geometry is taken into consideration when processing the measured data. The PTB (Physikalisch-Technische Bundesanstalt) Double sine standard [4], [5], [6] was selected as a suitable freeform geometry and points along 3D lines located on its freeform surface were measured by tactile probe on coordinate measuring machine. After that, the measured data sets were processed in three different ways and, depending on interpretation of measured data, different form errors were determined. As far as the authors know, metrology of 3D line located on freeform surface has not been investigated yet.

The paper is organized as follows. Section 2 describes the PTB Double-sine standard from mathematical point of view. In section 3, three possible interpretations of data measured along 3D line located on freeform surface are presented. Section 4 summarizes the achieved results and gives possible applications.

2. PTB Double-sine standard

Based on mathematical approach to the development of freeform standard, PTB Double-sine standard was designed [4] and manufactured. Double-sine surface can be expressed by explicit equation in the following form

$$z(x, y) = k \sin\left(\frac{\pi x}{l}\right) \sin\left(\frac{\pi y}{m}\right),\ (x, y) \in R^2, \tag{2}$$

where k, l and m are real constants. In particularly, constants $k = 50$ mm, $l = 100$ mm and $m = 100$ mm and range $(x, y) \in [-50 \text{ mm}, 50 \text{ mm}]^2$ were used in the case of PTB Double-sine standard shown in fig. 2. Here, the standard during the measurement by a tactile probe on SIP CMM 5 machine in the Czech Metrology Institute is shown. The equation of freeform surface is then

$$z(x, y) = 50 \sin\left(\frac{\pi x}{100}\right) \sin\left(\frac{\pi y}{100}\right). \tag{3}$$

The PTB Double-sine standard is characterized by the following very interesting properties.

- The surface curvature as well as surface normal direction is continuously changing in a great range [4].
- There are special points located on the surface such as minimum, maximum and saddle point [4].
- There are lines located on the surface [7].

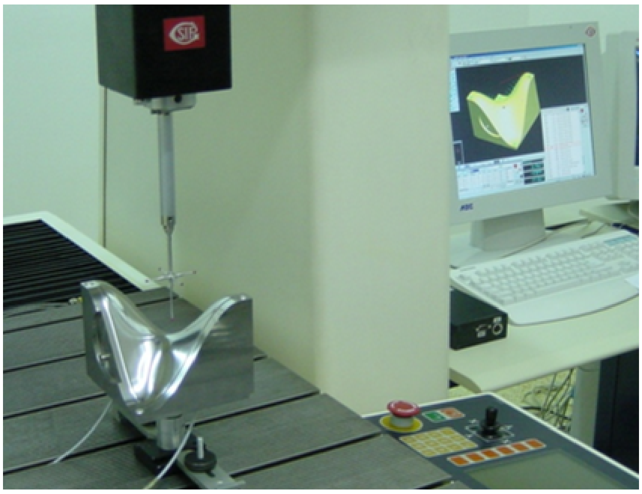

Fig. 2. Measurement of PTB Double-sine standard on SIP CMM 5 machine.

During measurement on coordinate measuring machine, the tactile probe moves in normal direction to the measured surface. The normal vector of explicit surface is given by

$$\mathbf{n}(x,y)=\begin{pmatrix} n_x(x,y)\\ n_y(x,y)\\ n_z(x,y)\end{pmatrix}=\begin{pmatrix} -\frac{\partial z(x,y)}{\partial x}\\ -\frac{\partial z(x,y)}{\partial y}\\ 1\end{pmatrix}, \tag{4}$$

i.e., the normal vector of freeform surface of the standard is

$$\mathbf{n}(x,y)=\begin{pmatrix} -\frac{\pi}{2}\cos\left(\frac{\pi x}{100}\right)\sin\left(\frac{\pi y}{100}\right)\\ -\frac{\pi}{2}\sin\left(\frac{\pi x}{100}\right)\cos\left(\frac{\pi y}{100}\right)\\ 1\end{pmatrix}. \tag{5}$$

Substituting constant value of variable x or y in (2) we obtain parametric curves (sine curves) located on double-sine surface. Specially, substituting $x=0$ or $y=0$ in (2) we obtain straight lines

$$z(0,y)=0 \text{ or } z(x,0)=0, \tag{6}$$

After substitution $x=0$ or $y=0$ in (5), the normal vectors at points along these straight lines are obtained

$$\mathbf{n}^{\mathrm{T}}(0,y)=\left(-\frac{\pi}{2}\sin\left(\frac{\pi y}{100}\right),\quad 0,\quad 1\right) \tag{7}$$

or

$$\mathbf{n}^{\mathrm{T}}(x,0)=\left(0,\quad -\frac{\pi}{2}\sin\left(\frac{\pi x}{100}\right),\quad 1\right). \tag{8}$$

It is obvious that normal vector has different direction at each point along straight lines (6) located on freeform surface (3), see fig. 3.

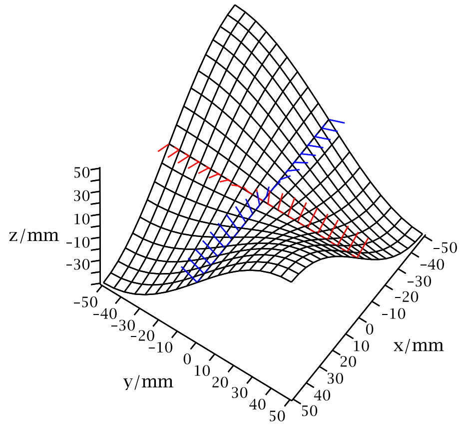

Fig. 3. Normal vectors at points along 3D lines on double-sine surface.

3. Practical experiment

Tactile measurement on SIP CMM 5 machine with maximum permissible error $(0.8+1.3L)$ μm of PTB Double-sine standard was performed. Based on SIP CMM 5 specification, calibration of the machine by means of laserail and performance of number of tests according to [8], the measurement uncertainty 1.6 μm was estimated for all measured data.

Two steps were used to determine coordinate system of the standard. Firstly, 6 surface points in three mutually perpendicular planes were measured and 3-2-1 alignment model was applied. Secondly, 64 surface points uniformly distributed along the double-sine surface of the standard were measured and best-fit transformation was calculated. Then, two sets of theoretically exact points along two 3D lines located on double-sine surface were generated in such a way that for the given initial and terminate point of 3D line on CAD model and the required distance between two consecutive points (0.25 mm), 401 positions and corresponding normal vectors for each 3D line (6) were calculated by Metrolog software implemented in control system of SIP CMM 5 machine.

After theoretical data sets generation, the tactile measurement was performed and two sets of data measured with respect to the CAD model of the freeform surface were obtained. The behavior of deviations along both 3D lines is similar, therefore only one set of theoretical points given by

$$\mathbf{P}:\left\{\mathbf{P}_i=\left(-50+\tfrac{i}{4},0,z(-50+\tfrac{i}{4},0)\right)\right\}_{i=0}^{400}, \tag{9}$$

obtained by substitution $y=0$ in (3) is considered in this paper in order to keep the sufficient clearness of the pictures below.

The theoretical data set (9) and normal direction given by (8) were used as input data for tactile CAD based measurement. Thus, measured data set

$$\mathbf{M}:\{\mathbf{M}_i=(x_{\mathbf{M}_i},y_{\mathbf{M}_i},z_{\mathbf{M}_i})\}_{i=0}^{400} \tag{10}$$

was obtained. Finally, the measured data set was analyzed from the three following different points of view.

1. The normal distances of all measured points from the CAD model of the free form surface of the standard were evaluated and form error FE_{SRF} with respect to the freeform surface were determined. This approach corresponds to the freeform measurement commonly used.

2. The normal distances of all measured points from the CAD model of theoretical 3D line located on the freeform surface of the standard were evaluated. Form error FE_{LINE} in this case was determined with respect to this theoretical 3D line.

3. Least squares method (LSM) was used to fit 3D line through the measured points and form error FE_{LSM} with respect to this LSM 3D line as well as characteristic parameters of LSM associated line were determined.

3.1. Form error FE_{SRF}

All points from the theoretical data set $\mathbf{P}$ were measured as well as evaluated with respect to the CAD model of the freeform surface in this case. Evaluation of form error FE_{SRF} consists in the following steps.

- The set of measured points $\mathbf{M}$ was orthogonally projected on the freeform surface to obtain nominal data set

$$\mathbf{N} : \{\mathbf{N}_i = (x_{\mathbf{N}_i}, y_{\mathbf{N}_i}, z_{\mathbf{N}_i})\}_{i=0}^{400} \tag{11}$$

  of points located on the freeform surface. Point $\mathbf{N}_i$ lies at intersection of normal line of the theoretical double-sine surface and the theoretical double-sine surface, see fig. 4. Normal line passes through the measured point $\mathbf{M}_i$ perpendicularly to the tangent plane of the theoretical surface. Tangent plane is given by two tangent lines of two parametric curves passing through the point $\mathbf{N}_i$.

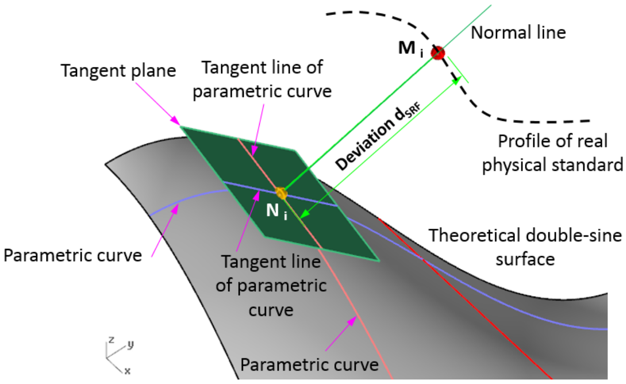

Fig. 4. Deviation d_{SRF} evaluated with respect to the freeform surface of the PTB Double-sine standard.

- The oriented deviations d_{SRF} as signed distances between the measured points $\mathbf{M}$ and the corresponding orthogonally projected points $\mathbf{N}$ were evaluated

$$d_{\text{SRF}} : \{d_{\text{SRF}_i} = \\ = \sqrt{(x_{\mathbf{M}_i} - x_{\mathbf{N}_i})^2 + (y_{\mathbf{M}_i} - y_{\mathbf{N}_i})^2 + (z_{\mathbf{M}_i} - z_{\mathbf{N}_i})^2}\}_{i=0}^{400}$$

  Positive sign of the deviation is in the case

$$z_{\mathbf{M}_i} > z(x_{\mathbf{M}_i}, y_{\mathbf{M}_i}),\ i = 0, 1, \ldots, 400, \tag{12}$$

  and negative sign in the case

$$z_{\mathbf{M}_i} < z(x_{\mathbf{M}_i}, y_{\mathbf{M}_i}),\ i = 0, 1, \ldots, 400. \tag{13}$$

  The behaviour of these deviations along 3D line located on freeform surface of PTB Double-sine standard is depicted in figs. 5 and 6.

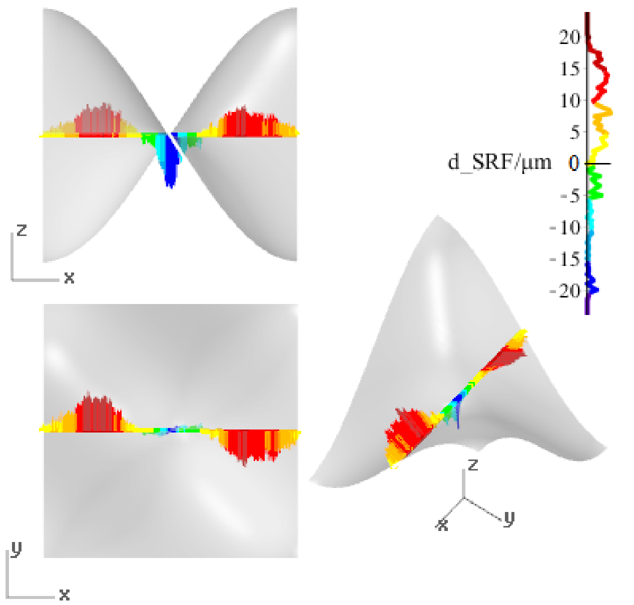

Fig. 5. Deviations d_{SRF} along 3D line with respect to the freeform surface of the PTB Double-sine standard.

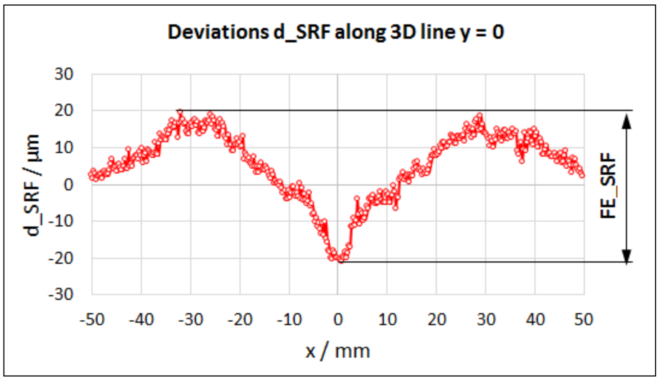

Fig. 6. Graph of deviations d_{SRF} along 3D line with respect to the freeform surface of the PTB Double-sine standard.

- The form error

$$FE_{\text{SRF}} = [\min(d_{\text{SRF}}), \max(d_{\text{SRF}})]$$

was determined as the only characteristic of the measured feature. In particularly, the obtained range of form error in this case was

$$FE_{\text{SRF}} = [-21.1, 19.6]\ \mu\text{m}.$$

3.2. Form error FE_{LINE}

In this case, all points from the theoretical data set were measured with respect to the CAD model of the freeform surface and evaluated with respect to the theoretical line located on the freeform surface in the following way.

- The set of measured points $\mathbf{M}$ was orthogonally projected on the theoretical 3D line located on the freeform surface and nominal data set

$$\mathbf{L} : \{\mathbf{L}_i = (x_{\mathbf{L}_i}, y_{\mathbf{L}_i}, z_{\mathbf{L}_i})\}_{i=0}^{400} \tag{14}$$

of points located on the theoretical 3D line was obtained, see fig. 7. Point $\mathbf{N}_i$ lies at intersection of theoretical 3D line located on the freeform surface of Double-sine standard and normal plane of this theoretical 3D line. Normal plane passes through the measured point $\mathbf{M}_i$.

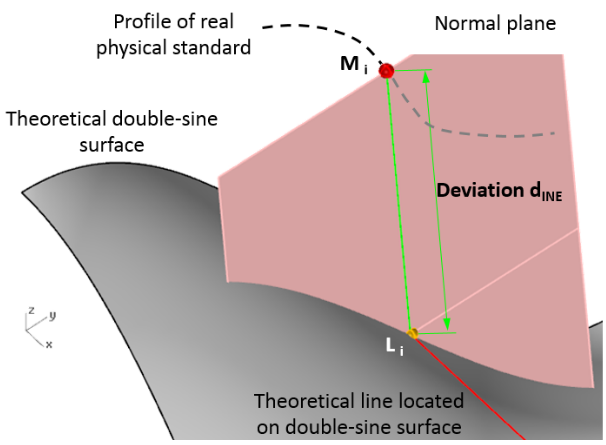

Fig. 7. Deviation d_{LINE} evaluated with respect to the theoretical 3D line located on theoretical double-sine surface of the PTB Double-sine standard.

- The oriented deviations d_{LINE} as signed distances between the measured points $\mathbf{M}$ and the corresponding orthogonally projected points $\mathbf{L}$ were evaluated

$$d_{\text{LINE}} : \{d_{\text{LINE}_i} = \\ = \sqrt{(x_{\mathbf{M}_i} - x_{\mathbf{L}_i})^2 + (y_{\mathbf{M}_i} - y_{\mathbf{L}_i})^2 + (z_{\mathbf{M}_i} - z_{\mathbf{L}_i})^2}\Big\}_{i=0}^{400}$$

The rules for the sign of the deviation are given by (12) and (13). The behaviour of these deviations along 3D line located on PTB Double-sine standard is shown in figs. 8 and 9.

- The form error

$$FE_{\text{LINE}} = [\min(d_{\text{LINE}}), \max(d_{\text{LINE}})]$$

was determined as the only characteristic of the measured feature or – in combination with the previously given procedure – as an additional characteristic to the freeform measurement commonly used. In particularly, the obtained range of form error in this case was

$$FE_{\text{LINE}} = [-23.5, 23.0]\ \mu\text{m}.$$

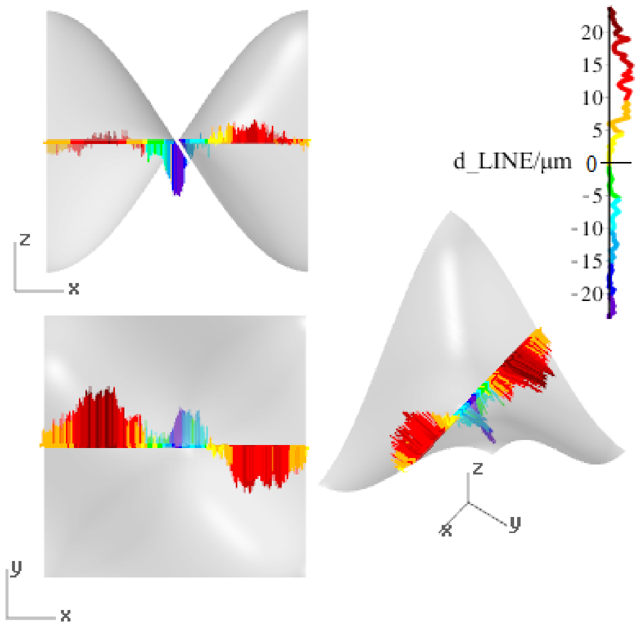

Fig. 8. Deviations d_{LINE} along 3D line with respect to the theoretical 3D line located on the freeform surface of the PTB Double-sine standard.

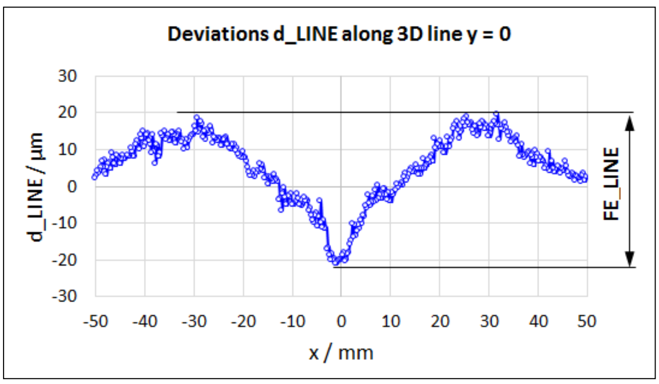

Fig. 9. Graph of deviations d_{LINE} along 3D line with respect to the theoretical 3D line located on the freeform surface of the PTB Double-sine standard.

3.3. Form error FE_{LSM}

In this case, all points from the theoretical data set were measured with respect to the CAD model of the freeform surface and evaluated with respect to the 3D line associated by means of LSM to the measured data. This approach consists in the following steps.

- The LSM 3D line was associated to the measured points and the characteristic parameters of LSM 3D line were evaluated according to (1), i.e. coordinates of definition point

$$\mathbf{A} = (50.0070, -0.0128, 0.0015) \text{ mm}$$

  and direction vector

$$\mathbf{d} = (-1.0000, 0.0003, 0.0001) \text{ mm}.$$

  Note, that the associated LSM 3D line does not lie on the freeform surface.

- The set of measured points $\mathbf{M}$ was orthogonally projected on the associated LSM 3D line and nominal data set

$$\mathbf{S} : \{\mathbf{S}_i = (x_{\mathbf{S}_i}, y_{\mathbf{S}_i}, z_{\mathbf{S}_i})\}_{i=0}^{400} \qquad (15)$$

  of points located on this line was obtained, see fig. 10. Point $\mathbf{S}_i$ lies at intersection of LSM 3D line and normal plane of this LSM 3D line. Normal plane passes through the measured point $\mathbf{M}_i$.

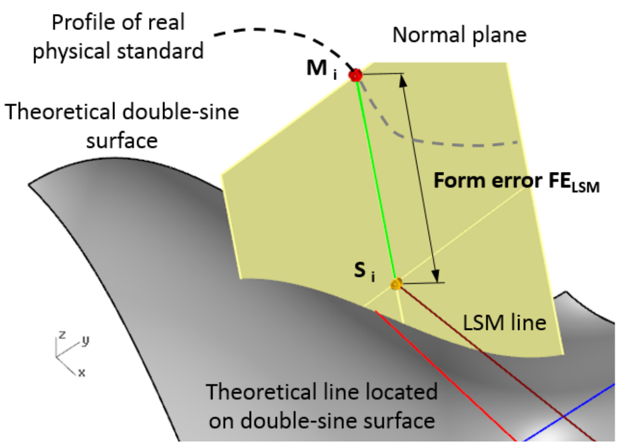

Fig. 10. Deviation d_{LSM} evaluated with respect to the LSM 3D line associated to the set of points measured along 3D line located on freeform surface of PTB Double-sine standard.

- The oriented deviations d_{LSM} as signed distances between the measured points $\mathbf{M}$ and the corresponding orthogonally projected points $\mathbf{S}$ were evaluated

$$d_{\text{LSM}} : \{d_{\text{LSM}_i} =$$

$$= \sqrt{(x_{\mathbf{M}_i} - x_{\mathbf{S}_i})^2 + (y_{\mathbf{M}_i} - y_{\mathbf{S}_i})^2 + (z_{\mathbf{M}_i} - z_{\mathbf{S}_i})^2}\Big\}_{i=0}^{400}$$

  The rules for the sign of the deviation are given by (12) and (13). The behaviour of these deviations along LSM 3D line associated to the points $\mathbf{M}$ measured along 3D line located on PTB Double-sine standard is shown in figs. 11 and 12.

- The form error

$$FE_{\text{LSM}} = [\min(d_{\text{LSM}}), \max(d_{\text{LSM}})]$$

  was determined as the next characteristic of the measured feature. In particularly, the obtained range of form error in this case was

$$FE_{\text{LSM}} = [-21.0, 14.6] \ \mu\text{m}.$$

- Moreover, the angle

$$\alpha = 0.004°$$

  formed by the theoretical and associated LSM 3D line was evaluated. This angle deviation can be considered as the additional characteristic of the measured feature.

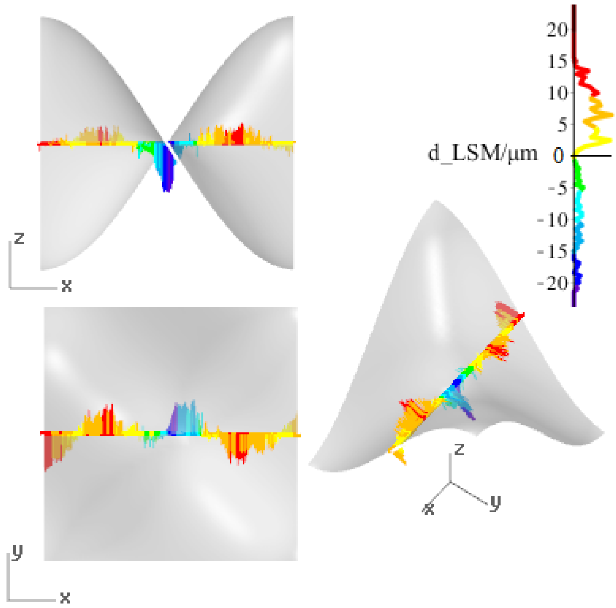

Fig. 11. Deviations along 3D line with respect to the LSM 3D line associated to the set of points measured along 3D line located on freeform surface of PTB Double-sine standard.

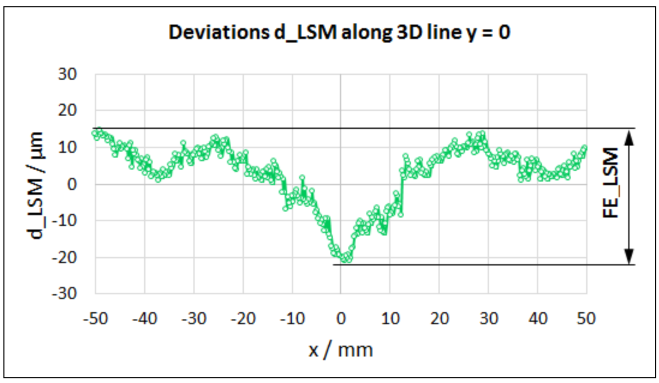

Fig. 12. Graph of deviations d_{LSM} along 3D line with respect to the LSM 3D line associated to the set of points measured along 3D line located on freeform surface of PTB Double-sine standard.

4. CONCLUSIONS

Three different interpretations of data obtained by tactile CAD-based measurement along 3D line located on freeform surface are introduced in this paper.

The first approach corresponds to the freeform measurement commonly used, i.e. the points are measured as well as evaluated with respect to the CAD model of the freeform surface. The range of form error is given by minimum and maximum orthogonal distance of measured points from the freeform surface. In the second interpretation, the orthogonal distances of measured points from the theoretical 3D line located on freeform surface are evaluated and corresponding form error is determined. In the third approach, the LSM line is associated to the set of measured points and orthogonal distances of measured points from this LSM line are evaluated. Here, it is possible not only determine the form error with respect to the LSM line but also definition parameters of associated line and its angular deviation from the theoretical one, see tab. 1, where all evaluated characteristics are summarized.

Table 1. Form error of 3D line located on freeform surface.

Reference feature	Characteristics
CAD model of freeform surface	$FE_{\text{SRF}} = [-21.1, 19.6]\ \mu$m
CAD model of theoretical 3D line located on freeform surface	$FE_{\text{LINE}} = [-23.5, 23.0]\ \mu$m
LSM 3D line associated to the measured points	$FE_{\text{LSM}} = [-21.0, 14.6]\ \mu$m $\mathbf{A} = (50.0070, -0.0128, 0.0015)$ mm $\mathbf{d} = (-1.0000, 0.0003, 0.0001)$ mm $\alpha = 0.004°$

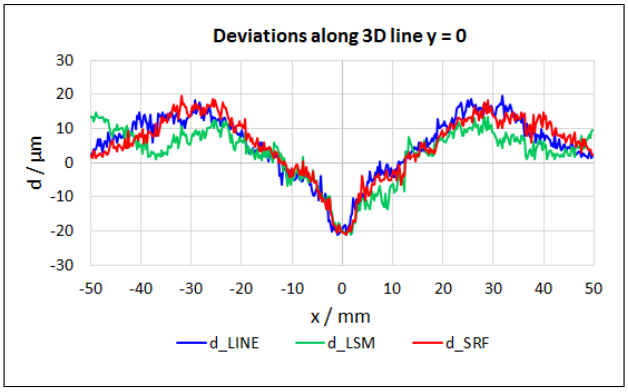

Fig. 13. Deviations along 3D line located on freeform surface of PTB Double-sine standard.

Behaviour of all three deviations d_{SRF}, d_{LINE} and d_{LSM} is shown in fig. 13. The correspondence between all the three different interpretations of data measured along 3D line located on freeform surface is obvious.

The latter two suggested ways of interpretation of data measured along 3D line located on freeform surface can be used for example in geometrical alignment where an origin point (intersection of two 3D lines in this case) and two-axis directions (the directions given by two 3D lines) are needed to define a new coordinate system [9].

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