XXI IMEKO World Congress "Measurement in Research and Industry"
August 30 – September 4, 2015, Prague, Czech Republic

THE USE OF SINGULAR VALUE DECOMPOSITION OF MATRICES FOR EXTRACTION OF SIGNALS FROM RADAR DATA

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Abstract – The research reported in this paper is related to the ultra-wide-band radar technology that may be employed in care services for elderly people. Four methods, based on singular value decomposition of matrices, designed for extraction of useful signals from clutter-corrupted data, acquired by means of an ultra-wide-band radar sensor, have been compared. The uncertainty of estimation of echo parameters, performed using the extracted signals, has been used as the criterion of comparison.

Keywords: UWB radar, healthcare, clutter, singular value decomposition

1. INTRODUCTION

The European and North-American populations are aging quickly. The problem of organised care over elderly people, especially those suffering dementia is, therefore, of increasing importance. Hence the demand for research on new technologies that could be employed in care services for such people. Its primary objective is to examine the applicability of various sensor systems for non-invasive monitoring of the movements and vital bodily functions, such as heart beat or breathing rhythm, of elderly persons in their home environment. There are three main categories of monitoring techniques already applied in care practice – wearable, environmental, vision-based – and two emerging categories: depth-camera-based (*cf.* the 2014 review paper [1]) and radar-based techniques (*cf.*, for example, the documents [2-8]). This paper is devoted to the latter ones, more precisely to the application of an ultra-wide-band (UWB) monostatic radar system for monitoring elderly and disabled people as described in [9]. Four methods for the extraction of useful signals from data acquired by means of such a radar system are compared with respect to the uncertainty of estimation of the position of a moving object.

2. RESEARCH PROBLEM STATEMENT

The measurement data from an ultra-wide-band (UWB) impulse radar sensor can be modelled with the following equation:

$$x_n(t_i) = x_{s,n}(t_i) + x_{c,n}(t_i) + \varepsilon_n(t_i) \quad \text{for} \quad n = 1, ..., N \qquad (1)$$

where $i = 1, ..., I$, and:

- I is the number of data sequences acquired in the consecutive time moments t_i;
- N is the number of data in each sequence;
- $x_n(t_i)$ is the n-th data point in the sequence acquired at the time moment t_i;
- $x_{s,n}(t_i)$ is the (useful) component representative of the echo directly caused by a moving object to be located;
- $x_{c,n}(t_i)$ is the (unwanted) component representative of the sum of echoes reflected from other objects, hereafter referred to as *clutter*;
- $\varepsilon_n(t_i)$ is the component corresponding to noise, modelled by a zero-mean random variable.

The signal extraction is, therefore, equivalent to clutter suppression, and consists in finding an estimate $\hat{x}_{s,n}(t_i)$ of the signal $\tilde{x}_{s,n}(t_i)$, defined as:

$$\tilde{x}_{o,n}(t_i) \equiv x_{s,n}(t_i) + \varepsilon_n(t_i) = x_n(t_i) - x_{c,n}(t_i) \quad \text{for} \quad n = 1, ..., N \qquad (2)$$

The clutter is generated by two phenomena: the reflections from static objects, such as walls or furniture, and the secondary reflections between static objects and moving objects. While the first of them can be represented by a single data sequence, independent of t_i, the latter may change whenever an object is moving in the observed area. This is why the simplest approach of signal extraction – namely subtraction of "background", *i.e.* data representative of constant echoes, acquired when no moving objects are present in the observed area – does not work. Therefore, more advanced methods seem to be necessary for the extraction of signals from data acquired by means of UWB radar sensors.

A simple extension of the above-mentioned method for signal extraction, described by Stormo [5], consists in subtracting a dynamic "background", obtained by taking the arithmetic average of several last data sequences, instead of the static one, measured prior to monitoring. A more complex method, based on the singular value decomposition (SVD) has been used by Yinan *et al.* [2] to extract signals from the data sequences acquired by means of a Novelda R2A sensor, and has provided promising results. Verma *et al.* [10] have compared a similar SVD-based method with several other signal extraction methods using data from a through-wall-imaging system based on an UWB radar sensor; they have claimed the superiority of another method, based on the independent component analysis.

3. COMPARED METHODS

The methods of signal extraction, studied in this paper, consist in elimination of selected components of the singular value decomposition of the following matrix, whose rows contain the data sequences acquired at the time moments $t_i, t_{i-1}, ..., t_{i-J+1}$:

$$\mathbf{X}_i = \begin{bmatrix} x_1(t_{i-J+1}) & \cdots & x_N(t_{i-J+1}) \\ \vdots & \ddots & \vdots \\ x_1(t_i) & \cdots & x_N(t_i) \end{bmatrix} \tag{3}$$

where J is a parameter. The SVD of the matrix $\mathbf{X}_i\big|_{J\times N}$ is its factorisation of the form:

$$\mathbf{X}_i\big|_{J\times N} = \mathbf{U}\big|_{J\times J}\,\mathbf{\Sigma}\big|_{J\times N}\,\mathbf{V}^{\mathrm{T}}\big|_{N\times N} \tag{4}$$

where:

$$\mathbf{\Sigma} = \left[\mathrm{diag}\{\sigma_1, \ldots, \sigma_J\} \;\; \mathbf{0}\big|_{J\times(N-J)}\right] \tag{5}$$

$$\mathbf{U}^{\mathrm{T}}\mathbf{U} = \mathbf{I} \;\text{ and }\; \mathbf{V}^{\mathrm{T}}\mathbf{V} = \mathbf{I} \tag{6}$$

with $\mathbf{I}$ being the identity matrix, and $\sigma_1 \geq \ldots \geq \sigma_J > 0$ being the so-called singular values of the matrix $\mathbf{X}_i$. The algorithms of SVD are implemented in numerous software packages for data processing, in particular in MATLAB; one of them is described in [11, Section 11.1].

In the two already mentioned papers [2, 10], different assumptions about the relationship between the clutter and the components of the SVD of the matrix $\mathbf{X}_i$ imply two different SVD-based methods of signal extraction, involving two different ways of dealing with the matrices $\mathbf{U}, \mathbf{\Sigma}$ and $\mathbf{V}$ in order to obtain an estimate of the clutter-free data, $\{\hat{x}_{s,1}(t_i), ..., \hat{x}_{s,N}(t_i)\}$. In this paper, four methods, resulting from some modifications of those two methods, are compared.

The method described in [2] is based on the assumption that the clutter is represented by the components of the SVD corresponding to the first two singular values, and – consequently – that the useful signal can be extracted by discarding those components:

$$\hat{x}_{s,n}(t_i) = \sum_{j=3}^{J} u_{J,j}\sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{7}$$

where $u_{J,j}$ is the j-th element of the J-th row of the matrix $\mathbf{U}$ and $v_{n,j}$ is the j-th element of the n-th row of the matrix $\mathbf{V}$. The same result may be generated by taking the last row of the matrix $\mathbf{U\Sigma'V}^{\mathrm{T}}$, where $\mathbf{\Sigma'}$ is the matrix obtained by zeroing the first two diagonal elements of the matrix $\mathbf{\Sigma}$. On the other hand, the method described in [10] is based on the assumption that only the first singular value corresponds to the clutter:

$$\hat{x}_{s,n}(t_i) = \sum_{j=2}^{J} u_{J,j}\sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{8}$$

It can be shown that the smallest singular values are the most affected by noise [12]. Therefore, further improvement of the two above-mentioned methods can be obtained by discarding not only the components of the SVD corresponding to the largest singular values, but also those corresponding to the K smallest singular values, where K is a parameter whose value can be optimised empirically. This leads to more general definitions of two methods for signal extraction, which will be hereafter labelled with the acronyms M1 and M2:

$$\text{(M1)} \quad \hat{x}_{s,n}(t_i) = \sum_{j=3}^{J-K} u_{J,j}\sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{9}$$

$$\text{(M2)} \quad \hat{x}_{s,n}(t_i) = \sum_{j=2}^{J-K} u_{J,j}\sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{10}$$

The use of $K = 0$ in (9) and (10) leads to the methods described in [2] and [10], respectively, while the use of $K = J - 3$ in (9) or $K = J - 2$ in (10) is equivalent to adopting the assumption that the useful signal is sufficiently represented by a component of the SVD corresponding to a single singular value.

The value $u_{J,j}$ can be thought of as the weight with which the component of the decomposition corresponding to the j-th singular value contributes to the signal measured at the current time (t_i). Thus, by omitting the values $u_{J,j}$ in (9) and (10), one can obtain an estimate of the clutter-free data which is, in a sense, averaged over the J signals measured between t_{i-J+1} and t_i. This implies the definition of two more methods for signal extraction, which will be hereafter labelled with the acronyms M3 and M4:

$$\text{(M3)} \quad \hat{x}_{s,n}(t_i) = \sum_{j=3}^{J-K} \sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{11}$$

$$\text{(M4)} \quad \hat{x}_{s,n}(t_i) = \sum_{j=2}^{J-K} \sigma_j v_{n,j} \quad \text{for} \quad n = 1, ..., N \tag{12}$$

The assumption that underlies these methods, resulting from this reasoning, is that the averaging effect might allow for obtaining smoothed data, more useful for estimating the trajectories of the positions and magnitudes of the echoes.

It can be shown that if $K = J - 3$ is used in (11) or $K = J - 2$ in (12), the resulting method is equivalent to taking the second or first principal component of the matrix $\mathbf{X}_i$, although those principal components are calculated without centering the data [13] (which is part of the standard version of the Principal Component Analysis). Methods of signal extraction related to the Principal Component Analysis have also been considered in [10].

4. METHODOLOGY OF EXPERIMENTATION

The SVD-based methods of signal extraction, defined in the previous section, have been applied to the data acquired by means of a Novelda NVA6201 sensor located in front of a person walking around a room. Several objects generating clutter, such as a metal sink, were also present in the room during the experiment. 2000 data sequences, corresponding to consecutive time moments, each consisting of 1280 samples, covering the range of 1–6 m from the sensor, have been recorded using a special MATLAB application. An analogous data set has been acquired with no moving objects present in the observed area, in order to estimate the "background" consisting of echoes reflected from static objects.

The uncertainty of the estimates of echo parameters, obtained using the modified CLEAN algorithm [14], applied to the estimates of clutter-free data, has been used as the criterion of comparison. In order to obtain a template of the echo, required by that algorithm, a sequence of data has been acquired using the same sensor placed in front of a metal plate, other conditions unchanged.

The estimates of the echo position and magnitude trajectories have been approximated in the least-squares sense using a 5-th degree polynomial. The sum of squares of relative residues of this approximation has been used as an indicator of uncertainty of the echo parameters estimates. The percentage of missed echoes and the average number of false detections have also been used as criteria of comparison. The first of these parameters has been defined as the number of data sequences in which no echoes have been detected, divided by the total number of data sequences. The second of these parameters has been defined in the following way:

$$n_{\mathrm{FD}} = \left(\frac{1}{I_{\mathrm{D}}} \sum_{i=1}^{I} e_i \right) - 1 \tag{13}$$

where I_{D} is the number of data sequences in which at least one echo has been detected and e_i is the number of echoes detected in the i-th data sequence.

5. RESULTS OF EXPERIMENTS

The results of signal extraction by means of the four SVD-based methods, obtained for $J = 15$ and $K = 12$, and by means of the reference method, based on the "background" subtraction, are illustrated in Fig. 1 using an exemplary data sequence.

The estimates of echo parameters, obtained by means of the modified CLEAN algorithm, applied to data resulting from the subtraction of a previously acquired map of constant echoes, and to the estimates of $\tilde{x}_{s,n}(t_i)$, obtained using the SVD-based methods for signal extraction, together with the results of their approximation, are provided in Fig. 2 and Fig. 3. A subset of data, corresponding to a fragment of the measurement period when a person was present in the observed area and moving away from the sensor, has been selected for the presentation of all results in this section.

An echo reflected from a static object, located at a point represented by the 700th sample, is clearly visible if the SVD-based method has not been applied. It has been completely removed in the results obtained by means of the SVD-based methods.

The dependence of the indicators of uncertainty of the echo parameters estimation on the value of K for a fixed value of J is shown in Figs. 4–7, and on the value of J for a fixed value of K – in Figs. 8–11.

The computational complexity of the procedure for signal extraction increases with J since larger data matrices have to be processed. At the same time, it decreases with K because fast algorithms are available for calculating the components of the SVD corresponding to a limited number of the largest singular values [15].

6. CONCLUSIONS

The obtained results confirm the hypothesis that the subtraction of static "background" is not sufficient for signal extraction in case of the Novelda NVA6201 radar sensor. Those results also justify the recommendation of the proposed method M4 for signal extraction with a small buffer size J and a large value of the parameter K, which is slightly different than the methods proposed in [2] and [10]. The method M4 with $J = 15$ and $K = 13$ has been implemented in a simple measuring system based on an impulse-radar sensor, allowing for the estimation of the position of a moving object *ca.* 20 times per second.

The joint optimisation of the parameters of the described SVD-based methods for signal extraction and of the algorithm for the estimation of the echo parameters remains an issue to be considered in future research.

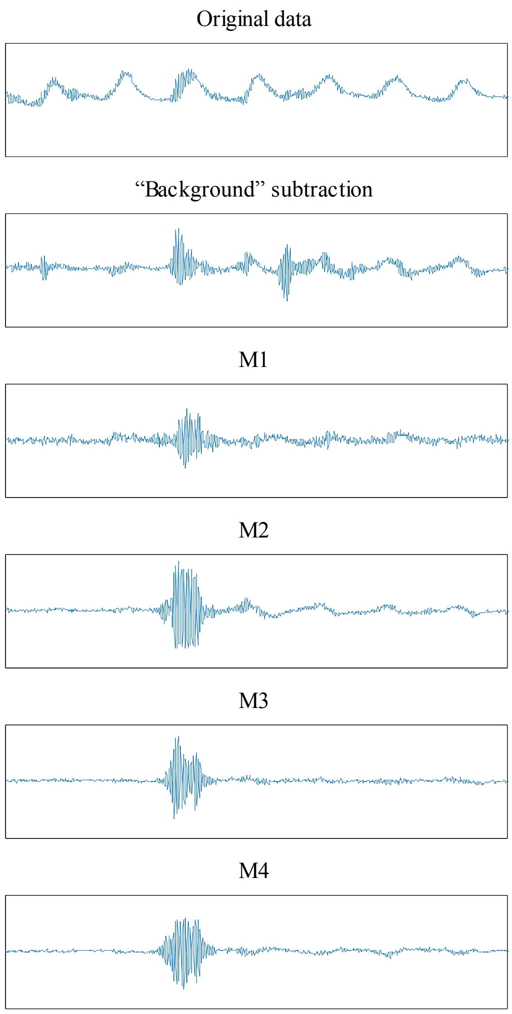

Fig. 1. The results of signal extraction by means of different methods for an exemplary data sequence.

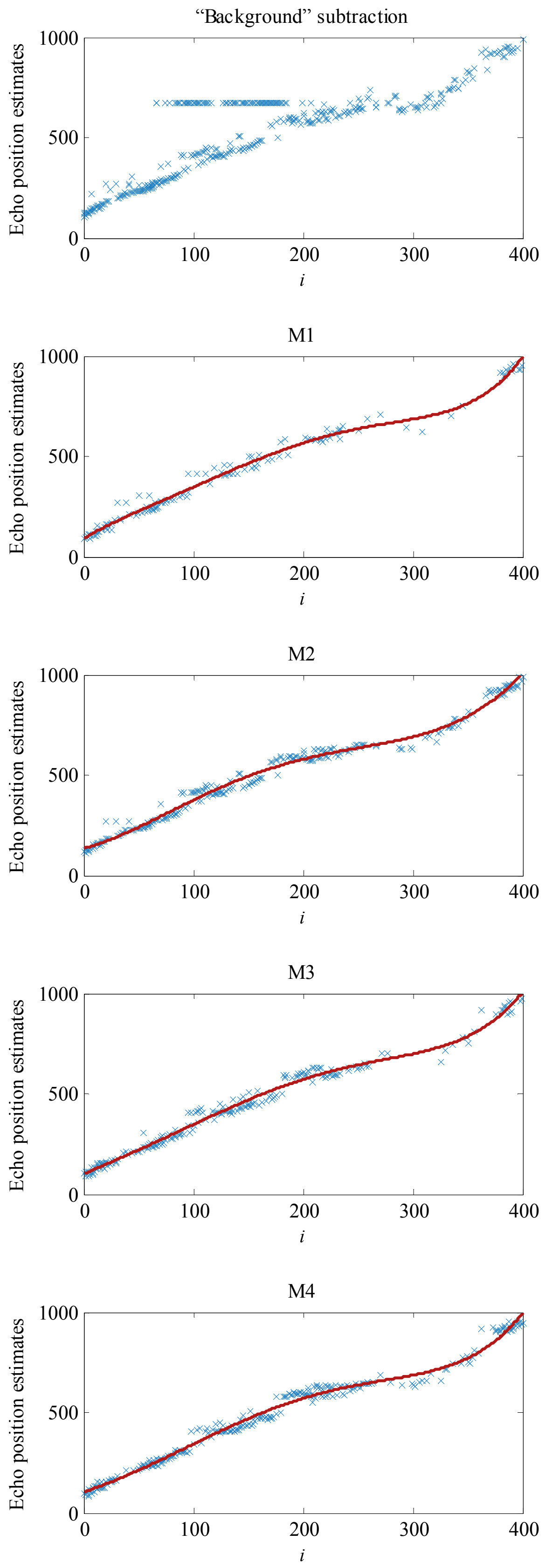

Fig. 2. The estimates of the positions of echoes obtained using the sequences $\hat{x}_{s,n}(t_i)$ computed by means of the compared methods of signal extraction (the results of approximation indicated with a grey line).

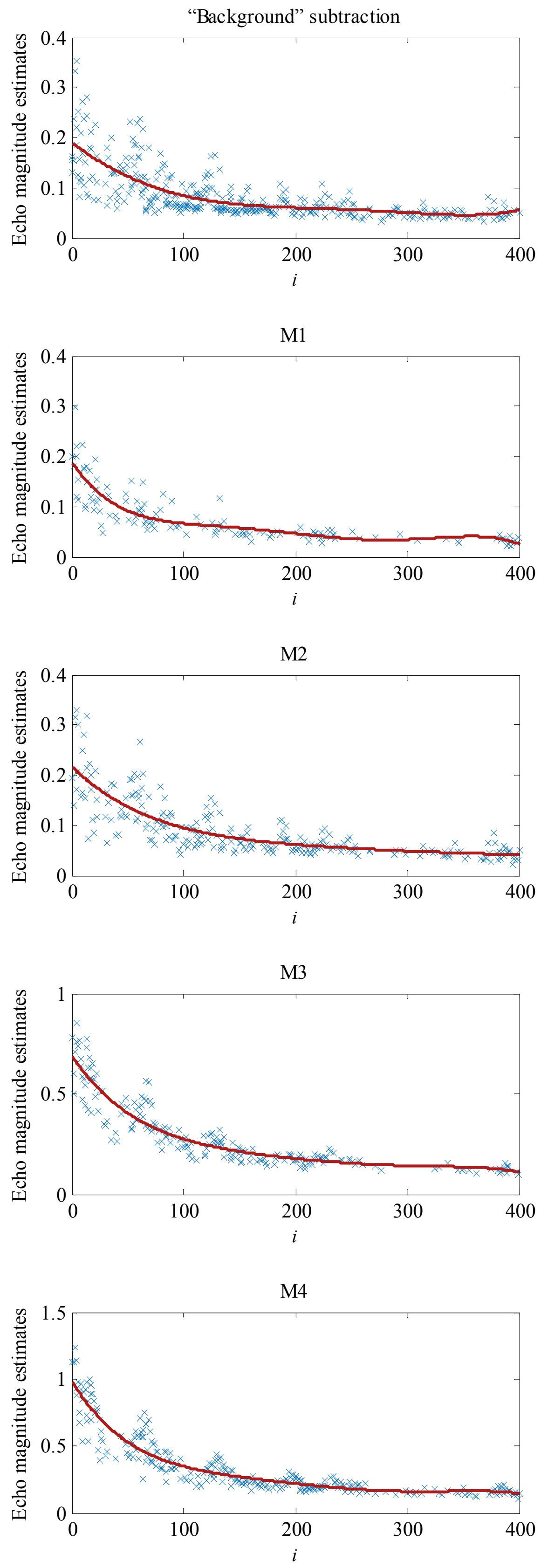

Fig. 3. The estimates of the magnitudes of echoes obtained using the sequences $\hat{x}_{s,n}(t_i)$ computed by means of the compared methods of signal extraction (the results of approximation indicated with a grey line).

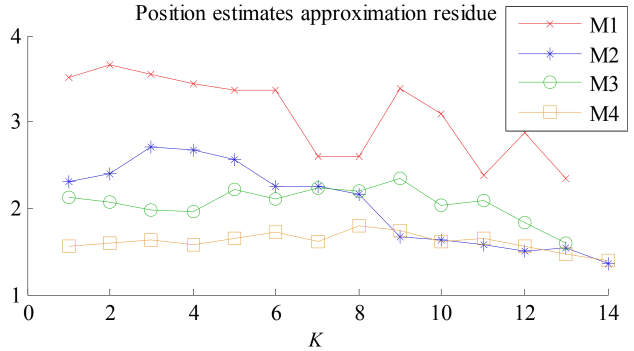

Fig. 4. Dependence of the approximation residue of the echo position estimates on the value of *K* for four SVD-based methods of signal extraction.

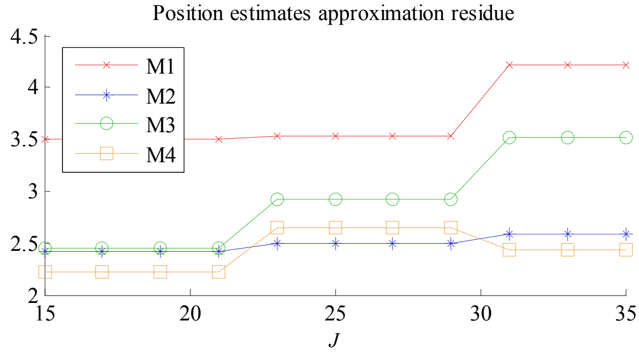

Fig. 8. Dependence of the approximation residue of the echo position estimates on the buffer size *J* for four SVD-based methods of signal extraction.

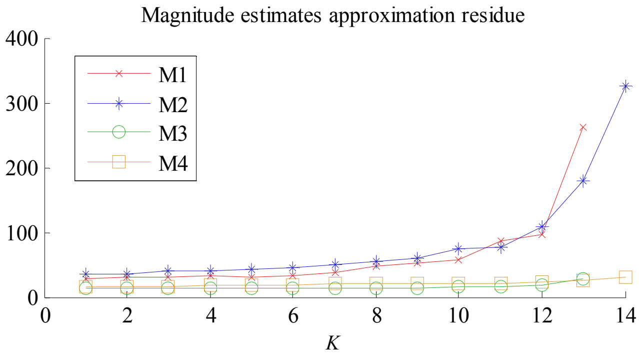

Fig. 5. Dependence of the approximation residue of the echo magnitude estimates on the value of *K* for four SVD-based methods of signal extraction.

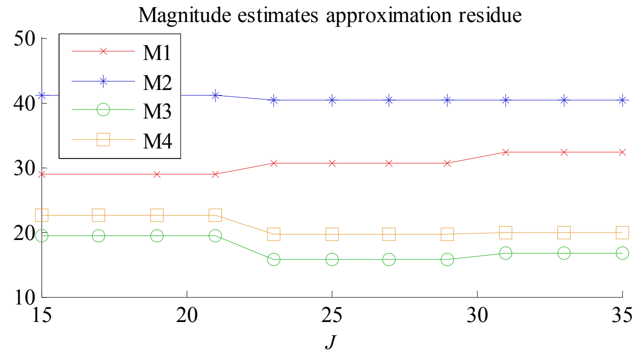

Fig. 9. Dependence of the approximation residue of the echo magnitude estimates on the buffer size *J* for four SVD-based methods of signal extraction.

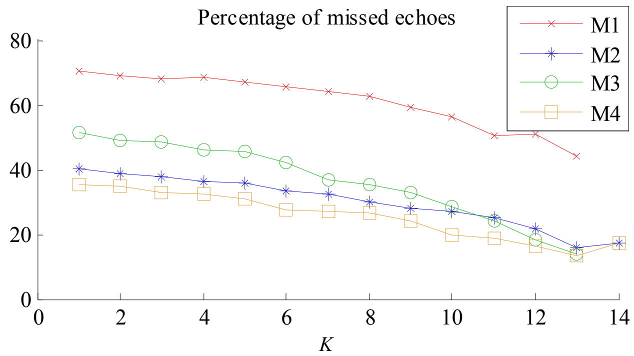

Fig. 6. Dependence of the percentage of missed echoes on the value of *K* for four SVD-based methods of signal extraction.

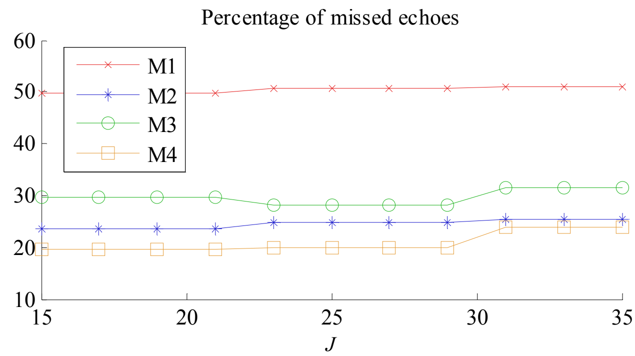

Fig. 10. Dependence of the percentage of missed on the buffer size *J* for four SVD-based methods of signal extraction.

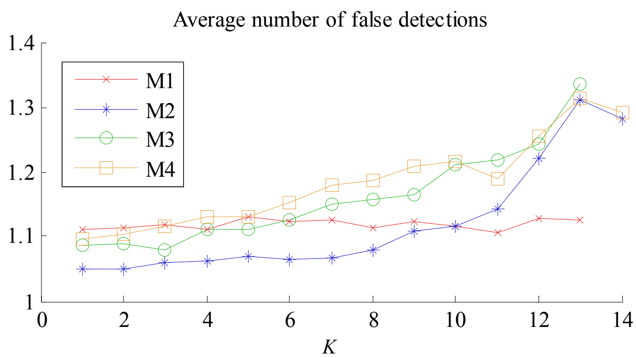

Fig. 7. Dependence of the average number of false detections on the value of *K* for four SVD-based methods of signal extraction.

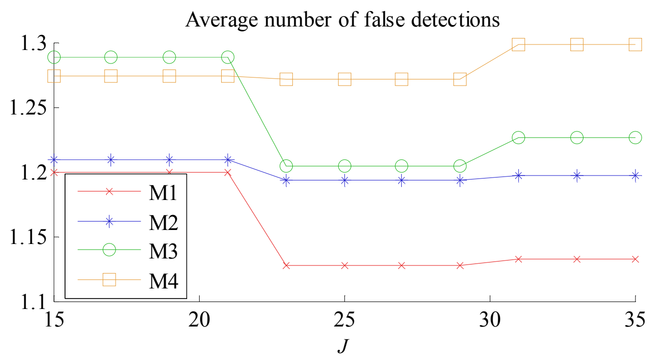

Fig. 11. Dependence of the average number of false detections on the buffer size *J* for four SVD-based methods of signal extraction.

ACKNOWLEDGEMENTS

This work has been accomplished within the project PL12-0001 financially supported by *EEA Grants – Norway Grants* (*http://eeagrants.org/project-portal/project/PL12-0001*). The authors would like to thank Dr. Andrzej Miękina for the acquisition of data used for numerical experimentation reported in this paper and Paweł Mazurek for sharing a MATLAB implementation of the CLEAN algorithm.

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