

# TAKING RIGHT FINAL CONCLUSIONS AND DECISIONS WITH THE USE OF THE MOST APPROPRIATE KEY COMPARISON REFERENCE VALUE (KCRV)

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**Abstract:** The best way that a National Metrology Institute (NMI) has to demonstrate confidence in its measurement results and, consequently, in its measurement capabilities is the the participation in key comparisons. Usually, it is not so obvious to choose the key comparison reference value (KCRV). The main purpose of this paper is to present the considerations done about the measurement results contained in some final reports of some different key comparisons, chosen as case studies, aiming to establish the KCRV.

**Keywords:** key comparison reference value (KCRV), measurement capability, degrees of equivalence, statistical consistence.

## 1. INTRODUCTION

The CIPM Mutual Recognition Arrangement (CIPM MRA) [2] aims to provide international recognition of the realisation of national standards and confidence in the measurement capabilities of participating laboratories for all users, including the regulatory and accreditation communities, thus reducing technical barriers to trade arising from lack of traceability and equivalence.

The MRA demands for demonstration that national measurement standards are traceable to the SI and to each other, as well as for providing a convenient way of presenting the results of key comparisons.

One of the important aims of the MRA is to demonstrate traceability to the SI of the measurement standards maintained by the national metrology institutes (NMIs) while at the same time providing a quantitative measure of the difference between these standards. The method chosen in the MRA is to use key comparisons to evaluate the degree of equivalence, defined as the difference from the KCRV with its uncertainty and the degree of equivalence between pairs of NMIs.

The evaluation of key comparison data is a relevant task in the metrology world, due to its importance to the global trade, as well as such comparisons are intended to test the principal techniques in a metrology field.

The degree of equivalence of each national measurement standard is expressed quantitatively by its deviation from the KCRV, as well as its uncertainty at the 95% level of confidence.

The degree of equivalence between pairs of national measurement standards is expressed quantitatively by the difference of their deviations from the KCRV, as well as the uncertainty of this difference at the 95% level of confidence.

National measurement standard here means the measurement result of a travelling standard, presented by a participating laboratory in a key comparison.

The selection of the key comparison analysis method is not always a simple task, due to the key comparisons importance. Depending on the chosen method the final conclusions and decisions can vary, and a particular laboratory, which would have compatible results with the KCRV by a method, wouldn't have by another, evidencing a technical problem and possibly generating a "political" one.

Depending on the metrology area, the peculiarities of it, the information at disposal (data set) and so on, the better analysis method is chosen and agreed between participants to take final conclusions and decisions.

## 2. MAIN DATA ANALYSIS METHODS

- Arithmetic mean
- Weighted mean
- Weighted mean with a maximum weight
- Median
- Value of an international standard
- Global value know *a priori*

- Individual values known *a priori*
- No reference value

Note: The comments that follows, related to these 11 data analysis methods, where based on the description done by Claudine Thomas in [6].

### ***2.1. Arithmetic mean***

The arithmetic mean is the most used method to represent the KCRV.

Such an approach would only be justified if all the participants have reported uncertainties of the same order of magnitude.

The arithmetic mean doesn't take into account the uncertainty of the individual results contributing to the reference value. For a relative small number of participants results with large deviations, but still not to be considered as outliers, can strongly influence the mean.

The standard uncertainty of the arithmetic mean can either be determined by application of the error propagation law, i.e. taking into account the uncertainties of the individual results, or by the spread of the results, i. e. by the standard deviation divided by the square root of the number of results contributing to the mean.

The difference between each laboratory individual measurement and the KCRV, as well as the expanded uncertainty of this difference, are used to compute the degrees of equivalence.

The correlation between the uncertainty of the arithmetic mean and the individual uncertainties of the laboratories should be taken into account.

The difference between the individual measurements of two laboratories and its expanded uncertainty is used to compute the pair-wise degree of equivalence, which doesn't depend upon the value and the uncertainty of the reference value.

### ***2.2. Weighted mean***

After the arithmetic mean, the weighted mean is the second most used method to represent the KCRV.

It is calculated by the mean of all measurement values weighted by the inverse square of the standard uncertainties associated with the measurements.

The weighted mean method requires the individual uncertainties from the laboratories be estimated according to a common approach. If this is not the case, a single "wrong" value with a strongly underestimated uncertainty could strongly influence or even fully determine the weighted

mean. On the other hand, a high quality measurement with overestimated uncertainty would contribute to the reference value only to a small extent.

The standard uncertainty of the weighted mean is calculated either by appropriately combining the individual uncertainties, or by the spread of the results.

The degrees of equivalence are generally computed as for the arithmetic mean.

### ***2.3. Weighted mean with a maximum weight***

This method is used when some individual uncertainties are identified as smaller than the state-of-art in a metrology area.

On this case, the individual uncertainties are turned to the minimal uncertainty normally achievable, which gives to the corresponding measurements a maximum weight.

This approach should be taken into account to compute the combined standard uncertainty of the KCRV and promote the need to be use a dedicated formula to compute the expanded uncertainties included in the calculation of the degrees of equivalence.

### ***2.4. Median***

The median is generally used when a robust statistic is needed and its combined standard uncertainty is its standard deviation.

### ***2.5. Value of an international standard***

The values of international standards maintained at the BIPM are considered as KCRVs in some metrology areas.

### ***2.6. Global value known a priori***

The measurand is a ratio that should be equal to 1 with an uncertainty equal to zero. The uncertainties included in the degrees of equivalence involve the individual uncertainties and their correlations.

### ***2.7. Global value Individual value known a priori***

Samples are distributed to the participants and previously measured by the pilot laboratory using a high-order technique. Consequently, each participant is attributed a different reference value.

### ***2.8. No reference value***

It is the case where no KCRV could safely be estimated.

The pair-wise degrees of equivalence are the only parameters which can be calculated.

Note: In the reference document [6], several key comparisons are cited by the author to illustrate the explanations given to the main data analysis methods. The final reports of those key comparisons can be obtained in the BIPM Key Comparison Database (KCDB).

### 3. CASE STUDIES

Some case studies will be presented on the final paper, showing the application of some of these mentioned methods, as consequence of the data sets of different comparisons.

Before calculating the key comparison reference values, outliers or erroneous results, which could significantly bias the reference value, should be excluded. The application of two methods to check the statistical consistency of the data will be discussed. One of them is the Birge ratio test " $R_B$ " and the other one is the Chi-square test.

### 4. CONCLUSION

A critical analysis of the measurement results should be done before deciding for the best representative of the KCRV.

Possible outliers can be detected by using some specific methods and, consequently, they should be excluded from the final data analysis.

It should be clear that, independently of the type of the key comparison travelling standard, which had been circulating, not necessarily, it should be always applied the same statistic method (for example, weighted mean, simple arithmetic mean, median and so on) to determine the KCRV. This will depend of several aspects, like, for example, the laboratories declared measurement uncertainties.

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