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MEASUREMENT SCIENCE DEVELOPMENT FOR OPEN SYSTEMS

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Abstract: In the report present results of investigation methods of an estimation of measurements in nonlinear dynamic systems. It is shown that a main feature that must be taken into account at the analysis of measuring results consists in the condition of the stability and the stationary of the open dynamic system.

Keywords: uncertainty, measurement, dynamic open systems, linear and nonlinear systems.

1. INTRODUCTION

Open dynamic systems in various areas of a science, engineering and environment are studied already long time [1-5]. Investigation of these systems properties are presented in numerous publications, reports and books. During too time of a technique of performance of measurements and the subsequent analysis of results of these measurements exist only for simple linear and for stable steady dynamic systems.

As behavior of nonlinear dynamic open systems can be very difficult, the analysis of measurement results in these systems demands theoretical justification created on the basis of mathematical model of studied nonlinear dynamic system [6].

In the present report questions of formation of conditions for the analysis of results of measurements in nonlinear open dynamic system are discussed.

Experimental investigations of climatic, ecological and other global systems based on measurement results of systems parameters. In nonlinear dynamic open systems measurement results provide adequate description of investigated system conditions, under condition to take into account of character of behavior of dynamic system.

The existing measurement science is applicable for analysis of measurement results of parameters stationary and stable systems. Carrying out the analysis of results of measurements in dissipative open dynamic systems, the existing measurement science can give the deformed answer. Interest for this question is connected with short-term and long-term forecasts of climatic changes.

Influence conditions of dynamic system on results of measurements are well investigated. The most typical situations was studied - transients and steady periodic solutions. However, such model of behavior of dynamic systems for description of climatic system is rather approximate.

Situation when measurements are carried out in dynamic system which is in a steady condition is considered, but this steady condition represents strange attractor or a condition close to steady strange attractor.

In the present thesis, a theoretical base of measurement science development for the analysis of measurement results in real dynamical systems, for example, climatic and ecological is described. Similar systems, despite that they open systems, represent steady and dissipative systems.

2. THEORETICAL BASES

Influences of environment always are the objective reality of which in the course of measurements it is impossible to get rid. In the assumption that external influence factors are absent, measured value should characterise a unique value of the investigated parameter.

If the isolation from environment of object which investigated carried out, the fluctuations of environment do not influence on results of measurements.

Existing methods of the analysis of results of measurements were elaborated and developed on the theoretical bases of the systems which are in stationary and stable conditions.

Other basic condition underlies the analysis of results of measurements - the condition of uniqueness of value of the measured physical size. This condition together with the theorem of local existence and uniqueness of the decision of system of the ordinary differential equations [7] represent a fundamental basis measurement science and Guide of uncertainty.

What in phase space are described as steady, drawing points or steady, drawing trajectories (simple attractors [7] concern steady conditions of dynamic system). Mathematically steady conditions in phase space are described as knot or the focus which quantitative estimation is connected with negative real parts of roots of the characteristic equation [7].

Results of measurement in dynamic system make sense only in the conditions of simple attractor, i.e. in the field of a trajectory attraction in phase space. In this case uncertainty of result of measurements will be defined as speed of returning of system in a steady condition, and influence of internal and external fluctuations. As the entry condition of dynamic system is set by some likelihood distribution uncertainty of knowledge of entry conditions is established by these fluctuations.

The analysis of measurement results in such system demands the correct account of irregular character of behaviour of dynamic system. Character of dynamic, casual behaviour of system, and its objective parameters define algorithm of processing of results of measurements.

In work the qualitative theory of the differential equations is used for description of behaviour nonlinear dynamic system. It is shown that at the heart of methods of an estimation of uncertainty of measurements lie such physical processes, as fluctuations of components and chaotic behaviour of dynamic system.

3. IN THE FIELD OF SIMPLE ATTRACTOR

In this part we present shot results development of measurement science in the conditions of simple attractor.

Considering dynamic system as the purpose of a measuring problem, it is necessary to use as equation of measurements the decision mathematical model of dynamic system. The dynamic system, which parameters is planned to measure, usually described system from n differential equations (1)

$$\frac{\partial}{\partial \mathrm{t}} \vec{X} = \vec{\mathrm{f}}(\vec{\mathrm{X}}) \;, \tag{1}$$

Where: $\vec{\mathrm{X}} = (\mathrm{X}_1, \mathrm{X}_2 ... \mathrm{X}_n)$ set of the dynamic variables characterizing a condition of dynamic system; $\vec{f} = (f_1, f_2 ... f_n)$ - vector function, usually it smooth [8] and defined regarding phase space.

The decision $\vec{\mathrm{X}}(\mathrm{t}) = \mathrm{F}^{\mathrm{t}}(\vec{X}(0))$ represents a curve which in each point is a tangent to this vector field. Owing to uniqueness of the decision for each entry condition and any smooth vector field $\vec{\mathrm{X}}(0)$ there is a unique phase trajectory $\vec{f}$. In this case, the analytical decision is used as the equation of measurements at the analysis of results of dynamic measurements.

The measuring problem, in dynamic system, makes sense, only in a case when the system is in stationary, steady and drawing conditions (mathematically it corresponds to steady points in phase space). The description of a condition of dynamic system near to special points, is carried out by means of Lyapunov's [8] method which allows to define on a sign on real parts of roots of the characteristic equation presence of drawing properties at special points of dynamic system.

In report discussion one simple example - equation describing dynamic system near to a steady point in phase space. The linear equation of Langevin is

$$\frac{\partial}{\partial \mathrm{t}} \eta = -\lambda \eta + \varphi(\mathrm{t}) \tag{2}$$

with entry conditions $\eta(0) = A_0$ and $\varphi(0) = \Phi_0$. In the conditions of absence of noise size $\eta(\mathrm{t})$ change is described by expression $\eta(\mathrm{t}) = A_0 \exp(-\lambda t)$. The equation describing behaviour of dynamic system looks like $\eta(\mathrm{t}) = A_0 \exp(-\lambda t)$. In time $t = 1/\lambda$ the system from the indignant condition comes back in a steady condition $\mathrm{X} = \mathrm{X}^{(0)}$.

Fluctuations of size η under the influence of casual forces $\varphi(\mathrm{t})$ are described by expression

$$\eta(\mathrm{t}) = \int_0^t d\tau \varphi(\tau) \exp\{-\lambda(t-\tau)\} . \tag{3}$$

Well - known time realization and, hence, statistical characteristics of $\varphi(\mathrm{t})$, it is possible to find statistical characteristics of process $\eta(\mathrm{t})$. It is clear that character of time behaviour of process $\eta(\mathrm{t})$ will be defined also by speed of returning in a steady condition, i.e. value and sign of parameter λ .

Uncertainty of measurement of size defined from statistical characteristics fluctuation forces and parameters of the determined system.

$$u(\eta) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{m} (\eta_i - \overline{\eta})^2} \; ;$$
$$\eta_i = \eta(\mathrm{t_i}) = \int_0^{t_i} d\tau \varphi(\tau) \exp\{-\lambda(t_i - \tau)\} \tag{4}$$

4. IN THE FIELD OF STRANGE ATTRACTOR

In this, second, part we present results development of measurement science in the conditions of strange attractor.

Statement of a problem of measurement of a variable of the dynamic system which is in a condition of determined chaos has a number of features. First, by results of preliminary researches (obligatory) existence of a mode strange attractor [9-13] is established. Secondly, all fixed values of the measured size correspond to a real condition of system. Thirdly, uncertainty of result of measurement is understood as the size of drawing set attractor in phase space.

Experimental researches of chaotic modes of dynamic system are carried out by processing of time series of results of the supervision registered during time many than characteristic time of strange attractor. Methods of the analysis of the time series, used for studying of chaotic movement [14], allow defining such characteristics as fractal dimension, Lyapunov's indicators, entropy and dimension of an investment [15].

A time series of results of supervision represents sequence of the values $\mathrm{x}(\mathrm{t})$ received through a time interval τ $\mathrm{t_i} = \mathrm{t_0} + (\mathrm{i}-1)\tau$, $\mathrm{x_i} = \mathrm{x}(\mathrm{t_i})$, $\mathrm{i} = 1, \ldots, \mathrm{N}$. For research of structure strange attractor on time numbers of supervision of F. Takens [16] has mathematically developed and has proved the device on the basis of application of vectors of a delay. This method uses expression for correlation integral

$$C_m(\varepsilon)=\lim_{N\to\infty}\sum_{i,j=1}^{N}\frac{1}{N^2}\Theta(\varepsilon-\left|y_i^{(m)}-y_j^{(m)}\right|), \quad (5)$$

where: Θ - function of Heaviside; ε - the set distance between points of sets on аттракторе; $\left|y_i^{(m)}-y_j^{(m)}\right|$ - the distance module between two points i and j the set generated by a principle

$$\left|y_i^{(m)}-y_j^{(m)}\right|=\sqrt{\sum_{n=0}^{m-1}(x_{i+n}-x_{j+n})^2}$$

The correlation integral is used for quantitative and quality standard chaotic behaviour of dynamic system or external white noise.

Elements x_i are results of initial supervision on which in phase space of dimension m the size is formed $y^{(m)}$ ($y_i^{(m)}=\{x_i, x_{i+1},...x_{i+m-1}\}$).

The analysis of the one-dimensional time series received as result consecutive in time of registration of values, presumably on strange attractor, with use (5) allows to define dimension of space of an investment - M, fractal dimension- D.

For strange attractors always there is such maximum value ε which excess doesn't lead to increase in value of correlation integral $C(\varepsilon)$ [17]. Use of correlation integral allows to establish existence mode strange attractor, therefore calculation of correlation integral should be the first step at performance of measurements and estimations of uncertainty.

For measuring experiment important that dynamics of behavior of investigated system is that that if during the initial moment of time uncertainty of position of a phase point of system is characterized in the size, that, through long enough time interval, the size of area in which the phase point of system can be and move, increases till the sizes strange attractors. To carry out an estimation of the size of area with reference to observable parameter it is necessary to use time series of results of supervision.

Usually uncertainty estimated on type A calculates from numbers of repeated supervision as a standard deviation [18]. Thus standard uncertainty of type A is established through function of density of the probability received from observable distribution. As a result expanded uncertainty establishes an interval about result of measurements in which limits there is a most part of values which can be attributed to the measured size.

In the field of strange attractor the casual behavior of the investigated size is caused by unstable dynamics of behavior of the system, therefore for an estimation of uncertainty of measurements it is necessary to be based on mathematical model which describes a movement trajectory. As to construct analytical model for nonlinear dynamic system, in most cases, practically, it is impossible.

Therefore for an estimation of uncertainty of results of measurements can be used, or results of numerical modeling, or results of experimental supervision. If at calculation of the expanded uncertainty the interval of probable values of the measured size in case of strange attractor the uncertainty estimation is carried out on the maximum scope of observable sizes is established.

$$u=\max_{i,j=1...N}\left|x_i-x_j\right| \quad (6)$$

Actually, the maximum diameter of one-dimensional set of results of supervision can be considered as the basic estimation of uncertainty of result of measurements.

In work conditions at which it is necessary to consider influence of behavior of dynamic system on uncertainty of results of measurements have been considered. Considering from uniform positions analytical and numerical decisions of system of the differential equations which describe steady simple or strange attractor dynamic systems, it is possible to use equations of measurements for an estimation of uncertainty of measurements.

The main feature, the received results, consists that with their help it is possible to carry out correctly the analysis of results of measurements in any dynamic systems being as in a condition simple and strange attractors. If the steady condition is described by a limiting cycle or steady focus the estimation of results is carried out taking into account drawing properties of steady conditions.

In that case when a steady condition is strange attractor the analysis of results of measuring experiment should be carried out taking into account the sizes of drawing attractor set. At measurement of one of parameters of dynamic system, uncertainty is defined by size of a projection of section of set on an axis of the investigated parameter.

Advantage of a similar estimation of results of measurements consists that the interval of values which characterizes behavior of investigated system in the field of strange attractor is established. Interest to measurements in the field of strange attractor in such systems as biological, medical and economic, it is connected with construction of the theory of an estimation of uncertainty of results of measurements, in nonlinear dynamic systems of small size.

5. CONCLUSION

In the present report the results of measurement - are estimation in nonlinear dynamic systems.

The main feature which should be considered at the analysis of results of measurement consists in a condition of stability dynamic system.

The condition of dynamic system can be steady and attractive. Now two attractive conditions well - known, one of them simple attractor, another strange attractor. The estimation of measurements in the field of simple attractor is carried out on the basis of known methods of an estimation of uncertainty of measurements.

The estimation of measurements in the field of strange attractor should be carried out taking into account irregular (chaotic) behavior of dynamic system [19-22].

Feature of carrying out of an estimation of measurements in conditions of attractor is very important, for example, at climatic measurements.

If to consider conditions of an anticyclone as simple attractor results of measurements provide an estimation of a attractive condition, in the conditions of natural fluctuations.

If the cyclone, as an attractive condition which is characterized as strange attractor results of single measurements do not allow to carry out the forecast of a condition as the behavior of dynamic system essentially deforms results of supervision over system as a whole is considered.

Realization of an estimation of quality of results of measurements at climatic researches is necessary taking into account the established mode of behavior of climatic system as a whole. It is necessary to develop the theory of measurements in nonlinear systems which represent the purpose of measuring experiment, taking into account character of behavior of these systems in time.

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