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# SYSTEMATIC ERRORS: METHODOLOGY OF DETECTION, ELIMINATION, AND EVALUATION

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**Abstract:** The main objects and methods of systematic error theory are presented. The basic notion of systematic error is analysed, in connection with the main stages of the measurement procedure. The main problems of detection, elimination, evaluation, and estimation of systematic errors are outlined. Interrelation of the systematic and random errors is also considered.

**Keywords:** Measurement, error, systematic error, error evaluation**.**

## 1 INTRODUCTION

The problems of detecting, eliminating, evaluating, and predicting systematic errors (SEs) have always been the focus of metrology [1–6]. The nature of the problems caused by SEs is shown by two examples, which can be called typical.

Example 1. Certification tests of precision angular meter have provided a large amount of experimental data, but a priori information about possible SEs is small.

Formal processing of data as random samples yields the error estimates at accuracy level of the primary standard or even exceeding it. These results, with consideration for the measurement methods, instruments, and conditions, seem improbable. Their reliability appears to be doubtful, and, moreover, they are supposed to be significantly underestimated. At the same time, simple methods of data partitioning into homogeneous parts or application of variance analysis [7] make it possible to detect SE components, to approximately evaluate them, and, finally, to obtain estimates of the total errors looking realistic or at least matching the expected ones (at the specified measurement conditions).

Analysis of the measurement procedure and results leads to the following conclusions. Unjustified reduction of error estimates can be conditioned only by mutual compensation of error systematic components or by their unfounded randomization. Acquiring a large data volume required a long time, therefore, the measurement conditions varied during the data accumulation, thus causing SEs to change as well. SEs could be randomized during the changes owing to SE behavior (constant or variable and regular) or to the nature of variation in conditions. Two basic types of SE changes can be distinguished, for which various analysis methods are applicable.

1) With irregular quickly varying changes, SE realization in time barely differs from RE realization. Then, reliability of statistical estimates is conditioned by the homogeneity of SE+RE mixture and by SE-SE and SE-RE correlation. Since application of corrections is out of the question, we have to put up with using statistical errors, naturally obtained with consideration for the above correlations.

Then, only the problem of applying the obtained estimates remains unsettled. If the data volume acquired during the device operation is commensurable with the test data volume, the estimates remain valid, though being of merely academic interest; further they should be obtained anew for a new data array. For a much smaller array, the estimates should be recalculated with allowance for the volumes and possible correlations. Actual accuracy of practical measurements will therefore vary depending on the measurement procedure and conditions.

2) The other case is the measurement conditions remaining constant at certain intervals and varying in stepwise manner. Therefore, constant SEs appear to be piecewise-constant, however, their mutual compensation can critically reduce the resultant statistical estimate, making it unfounded and unrealistic. Then intervals of SE constancy should be detected, interval levels should be evaluated, and correction should be applied if possible.

Example 2. Ccertification of a precision marine gravimeter [8] is fundamentally different, from the methodical standpoint. In this case, we know the main physical factors which cause SEs; also known are the dependencies of the error components on these factors. This being so, we can introduce some corrections for the main effects, evaluate the residual SEs, and obtain total error.

*Processing the gravimeter data includes data filtering and applying a number of corrections, namely:*

- *Eotvos correction;*
- *correction for null point drift;*
- *total dynamic correction for gyro platform tilt, horizontal accelerations, residual orbital effect, and mixed-type effects;*
- *correction for vertical accelerations (with account for SNS antenna separation from the gravity sensor);*
- *correction for gravimeter data reduction to ellipsoid;*
- *tidal correction.*

*Additionally, Bouguer correction is applied in same cases.*

*However, corrections can be applied not to all SE components. Therefore, after applying all possible corrections in gravimeter tests, other SEs should be summed, along with residual SEs, including:*

*– calibration error;*

*– error due to platform oscillations;*

*– error due to vertical acceleration;*

*– error due to sinusoidal vibration;*

*– error due to temperature variations;*

*– error due to external magnetic field;*

*- error due to inaccurately measured ship speed and site latitude.*

These examples, along with many others, show that SE, as an object of metrology, is just as broad and intricate as RE. At the same time, the literature on RE theory is abundant, whereas only some theoretical propositions concerning SE can be found in a few authoritative publications. The lack of SE theory impedes the development of effective practical methods, and in most cases, the problems of SE are solved individually, depending on the skills of the experimenter, and thus, are not devoid of subjectivity. On the other hand, these systematic components within the measurement error and measurement instrument (MI) error are critical for the measurement objective because of the following physical and metrological reasons.

First, SEs characterize the principal imperfection of the measurement procedure and, therefore, of MI design, thus actually limit from below the obtainable measurement (MI) error (which can't be further reduced through data processing). The latter can appear an insurmountable obstacle to required accuracy.

During the development and use of measurement procedures and instruments, the ratio between systematic and random components is asymmetrical. Random components should be turned to systematic (preferably constant) by stabilizing both the components and the influencing factors. Once stability is achieved, the size of systematic components and/or factors should be reduced. Residual instabilities represented by RE are processed (filtered) to make them less influential.

Second, SE activities at any stage of a procedure/instrument life cycle are a physical study, i.e., are associated with the objective contents of the measurement problem and relies on physical phenomena and laws. Generally, SE components conditioned by physical factors – temperature, vibration, pressure, magnetic field strength, humidity, linear or angular acceleration – are detected. It makes a principal distinction between SEs and REs, which are studied using statistical models mostly. The latter always result from conjecturing the base probabilistic models, which usually remain non-verified [9, 10]. Therefore, the ground for RE studies is vague and subjective.

The above-said seems to weightily justify the truism "Metrology is an art of studying the systematic errors". Unfortunately, the most important and informative aspects of studying the errors are beyond the scope of the metrological handbook for the last 30 years – Guide [11], along with hundreds (or even thousands) of its predecessors. The only relevant clause (3.2.4.) reads "It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects." From the objective probability standpoint, Guide [11] merely intensifies the tendency of statistical conjectures.

Traditionally, SE analysis and evaluation is associated with the study of fundamental physical laws whereon the measurement principle, method, and MI design are based [1-3]. SE original classification, principles and methods used to detect and eliminate constant SE are given in the classical monograph by M. Malikov [1]. These principles were developed by Rabinovich and summarized in [2], which focuses on practical heuristic methods for SE evaluation and summation with RE. Methodical aspects of error component analysis are detailed in [4]; we should also mention the methodologically important comprehensive SE analysis as applied to a certain type of measurement [3].

The aim of this study is to develop the systematically initial concepts for the development of SE theory through the analysis of main SE activities. To achieve this aim, the following problems should be solved:

- structuring the SE initial notion;
- defining the subject and method of SE theory;
- formulating the basic theoretical problems and approaches to solve them.

This study addresses the stages of measurement method preceding the data processing, that is, aims to clear up the clause 3.2.4 in Guide quoted above.

## 2 NOTION OF SYSTEMATIC ERROR

The notion of SE is defined in the theory of errors [1, 2] phenomenologically; therefore, its interpretation is unambiguous only for constant SE. However there are several other important groups of SEs, including regular (deterministic) SEs of various types and irregular SEs of different structures. First of all, we speak about piecewise-constant SE (see above) caused by some factor that remains constant at certain time intervals, varying from one interval to another. Further, we may refer to increasing SE conditioned by factors with accumulated effects; these trends are typical of measurements in many applications, such as navigational instruments. In addition, there exists a type of periodical SEs, often physically associated with realization of a piecewise-constant SE in the cyclic process, a harmonic SE being an important special case of this component.

The phenomenological nature of SE (as well as RE) is manifested in possible RE-SE and SE-RE transition. Thus, RE of a MI calibration procedure shows as a SE of resulting calibration dependency, and the constant SE of MI specific sample behaves like a RE at a set of samples of one-type MI.

There may be two approaches to SE modelling: SE model is either included in a general stochastic error model (using complex models of random processes, including non-stationary and non-ergodic processes) or treated separately using other non-stochastic models introduced for it. In this case, SE detection must conform to the general objectives – evaluation and decrease of measurement errors. For example, including SE in a random process complicates the processing statistical methods, making the results less reliable. An alternative approach deals with the choice of non-stochastic models and difficulties in integration of SE estimates obtained on their basis with statistical RE estimates.

SE detection and evaluation vary considerably depending on their variation in time. Thus, strictly constant SEs cannot be detected within this measurement procedure. The only way possible in this case is to study their physical sources using additional methods and measurement instruments. Particularly, if information on SE source is available, we can try to affect the source by regular methods and record the predicted change in the error. Thus, the expected temperature SE can be determined by controlled temperature variations, which lead to identical shift of all data. Accelerometer SE can be determined by displacing its measurement axis from initial position through two preset angles, and then the unknown initial angle and the sought SEs are found from two resultant equations.

For SEs of known functional types, the procedure is similar. For instance, to calibrate a gravimeter, it is tilted through a number of preset angles, and then the deviations of gravimeter readings from the reference values are determined [8]. Then the data are processed to obtain the SE dependence in a given range.

In practice, of primary interest is the situation, when random noise is superimposed on regular or irregular physical processes. Additional information is required then, firstly, on the functional form of trends and correlation functions of noise. In particular, the length of the realizations needed to detect the trends significantly depends on their smoothness and correlation properties of noise.

To conclude this section we note that another interpretation of SE notion can be possible in formal sentence calculus [12]. In this case, the contents of ***measurement result*** as a sentence includes a number of components with various logical values. In particular, these are:

***true value of measurand*** - logical value "**true**";
***systematic error*** – logical value "**false**";
***random error*** - logical value "**indeterminate**".

Additional ***irregular*** elements may be present, described by other logical values (different from above). Anyhow, the logical content of the ***measurement result*** should be described by multi-valued logic (at least three-valued) rather than traditional two-valued logic.

## 3 SUBJECT MATTER AND METHOD OF SYSTEMATIC ERROR THEORY

Metrology is a practically oriented science, so it widely uses an operational approach. It seems expedient to extend it to SE theory. Therefore, in determining the subject matter and methods of SE theory it is advisable to rely on modern concepts of the measurement structure, including the interrelation between measurement elements [13], and the structure of measurement procedure [14].

SE activities start **at the first stages** of the measurement procedure, first of all, with detecting the physical factors conditioning SE components. The primary physical sources can be actually defined as the measurement equation is constructed at the problem statement stage.

As a simple example, we take indirect measurement of the solid body density:

$$\rho = m/V$$

where m is the weight of a homogeneous solid body; V is the body volume.

Initial requirements for the measurement error are also formulated at the problem statement stage. Since SE restricts the possible increase in accuracy, and (unlike RE) cannot be further reduced by increasing the amount of data and their processing, these requirements mainly apply to SE.

Major SE activities are performed **at the planning stage** of measurement experiment [13]. At this point, measurement methods and instruments are selected, and measurement conditions are specified. Then the instrumental errors, including SE and RE components, are detected. The measurement conditions determine the main influencing factors, and therefore, define whether the corrections can be applied and residual SEs (RSEs) can be evaluated.

One of the most important tasks at the planning stage is the selection of measurement method, where SP analysis is of crucial importance. All the classical measurement methods (including the differential methods, substitution and opposition, etc.) were actually intended to eliminate, compensate, or reduce a certain type of SE. A classic example is given by precision weighting methods using laboratory symmetrical balance [1, 2], where SE due to different lengths of arms ( $l_1 \neq l_2$ ) is one of the most significant For example, in substitution method the measured weight $x$ is balanced by packing $T$, and then substituted by a set of scale weights of weight M. Then $x = M$, that is, the result is free from asymmetrical balance SE.

Thus, the classical theory of measurement methods, in a sense, can be regarded as a special branch of SE theory. This is because the selection of measurement method first of all implies SE activities. Once the method is developed, SE activities are used in the method implementation.

Finally, at the planning stage the errors are a priori estimated and experimental parameters are selected, first of all, number of observations. SE activities again play a major role here. SE components, including methodical and instrumental ones are estimated, possibility and feasibility of applying corrections are revealed, and predicted RSEs are estimated.

The resulting SEs are compared with the expected REs to select the number of observations. It is done using the empirical rules of combining the components and verifying their ratio, which are summarized below in Section 5.

**At the measurement experiment stage**, there seems to be no direct SE activities. However, every effort should be made to ensure further effective SE activities during data processing, namely, to organize the conditions required by the procedure and perform the necessary measurements of influencing factors (accurately enough to apply the corrections). In addition, online data checking can be performed during the experiment to detect the outliers, large errors or failures, and non-homogeneous data.

SE activities at the data processing stage start with application of possible corrections (provided at the planning stage), and then SE-RE relationship is established. This can be done using a priori data of the expected SE components, and analysis of the measurement data sets.

To identify SE significance, the limits of RSEs are estimated and the resultant estimates are combined. Further, if RE remains significant, resultant SE and RE estimates are combined. However, SE and RE estimates are often indicated separately in high-grade precision measurements.

Lastly, SE analysis should be given special attention at the final stage, i.e., interpretation of measurement results. This is especially true if SE behaviour should be predicted when using the measurement results.

Thus, the following SE activities are performed **at different stages** of measurement procedure:

- structuring (identifying SE components);
- planning (organizing) the measures to prevent or eliminate SE;
- approximate calculation of SE components (evaluation);
- estimating the limits of RSE after the correction application (estimation);
- combining the estimates of SE components;
- combining SE and RE estimates.

These operations are performed using different methods, firstly, physical and mathematical modeling. Naturally, most widely used are the statistical models and quasi-statistical models close to them. Quasi-deterministic models based on fuzzy sets and interval analysis can also made use of.

Within these methods, the relevant data analysis and estimation techniques are employed depending on the stages and objectives of modeling and analysis.

Various methods and criteria can be helpful, including:

- heuristic methods for combining SE and RE with subsequent analysis of significance of each component in the total estimate;
- statistical analysis of experimental data, including analysis of homogeneity of observation groups, verifying the hypotheses on the significance of differences between the group means, and estimating the intergroup differences;
- statistical methods for parameter estimation, including classic sampling methods (based on strict statistical models) and nonparametric and robust methods (based on less stringent requirements for the samples, i.e., expanded statistical models);
- quasi-deterministic processing techniques based on fuzzy sets and interval analysis.

## 4 PROBLEMS OF SYSTEMATIC ERROR THEORY

The main groups of SE operations (see Section 3 above) determine the main problems of SE theory.

First of all we should mention the physic-metrological problem of **identifying the major systematic factors** conditioning SE components. Further, identified factors should be formalized and SE models should be constructed.

Referring to the measurement block diagram [13-15] and analysis of the measurement procedure [14] can help solve these problems. For example, a version of this approach provides error decomposition detailed in Table 1 below.

Once the main SE components are identified, a problem of **selecting SE components** for further analysis and estimation appears. This problem has several aspects, including the internal selection of components within the SE and external selection, i.e., determining whether SE is significant against RE. Moreover, the sequence of components to be compared matters, including whether these are individual or group components. The problem is even more important as it must be solved repeatedly at each stage of SE activities.

Obviously, these problems are difficult to formalize, and their solution depends on the accepted models of components, rules of their combination, and criteria for negligible smallness. Some practical criteria are given below in Section 5.

Approximate calculation of SE is associated with the general problem of **obtaining raw data of required quality**, including the physical equations underlying the measurements. To solve the problem, the measurement principle and method should be analyzed, MI metrological characteristics should be known, measurement conditions should be studied and regulated, measurement procedure should be properly organized, and the required reliability of SE prediction should be compared with the depth (volume) of a database to be analyzed.

The resulting SE estimates are always approximate, which restricts the effectiveness of correction application and reliability of prediction. However these approximate results serve as initial data for estimating the limits of RSEs.

Estimating the limits of RSEs is associated with the problem of **achieving proper quality of the corrections**. The latter is conditioned by the adequacy of SE models used (depending on the influencing factors) and the precision of their measurement.

Thus, the problems of adequacy of SE models are critical.

Estimating the limits of total SE (using the limits of its components) causes a problem of **matching and combining the models of components**, which also may have different properties. This is due to the adequacy of the models used and the accuracy of the measurements of influencing factors.

Combining the RSE and RE estimates is a natural continuation of previous operations provided that assumption of SE significance remains valid after corrections have been applied. It creates the problem of **optimal matching for models of various-type components**. The options are SE randomization or quasi-deterministic estimation of RE limits based on non-stochastic models.

The first option is the most common approach, within which SE approximate quasi-statistical models are used and SE and RE estimates are summed using statistical methods. In the second option, fuzzy sets and interval analysis approaches are the most effective.

## 5 PRACTICAL APPROACHES TO ESTIMATION OF SYSTEMATIC ERRORS

Some practical approaches to solve the above-stated problems, along with examples, are given below. Without claiming to be a comprehensive solution, these examples show how wide is the range of practical problems to be solved, and which questions still remain unresolved.

**Structuring SEs**

Efficient structuring is the basis for SE analysis and estimation at all stages. SEs can be decomposed using the structure of measurement procedure given above ([13, 14], and Section 3).

Formation of error components using the structural features (according to [13, 14]) is shown in Table 1. The component type – SE or RE – is also indicated.

*Table 1*

*Major components of measurement error according to its elements*

| Designation | Factors conditioning the error components | Component type |
|---|---|---|
| $\xi_1$ | discrepancy between the properties of object of research (OR) and its model (model inadequacy) | SE |
| $\xi_2$ | inaccuracy of installation and MI preparation | SE + RE |
| $\xi_3$ | measurand distortion due to MI-OR interaction | SE + RE |
| $\xi_4$ | noise (interference) at MI input | RE |
| $\xi_5$ | MI imperfection, discrepancy between metrological characteristics and MI real features | SE + RE |
| $\xi_6$ | MI response to variations in influencing factors | SE + RE |
| $\xi_7$ | MI inertia, rate of change of MI input signal | MI dynamic components SE + RE |
| $\xi_8$ | inaccurate implementation of measurement method | Methodical components SE |
| $\xi_9$ | noise at MI output | SE + RE |
| $\xi_{10}$ | recording of MI signal or taking MI readings | SE + RE |
| $\xi_{11}$ | imperfection of data processing algorithm | SE + RE |
| $\xi_{12}$ | inaccurate implementation of the software processing algorithm | SE + RE |
| $\xi_{13}$ | imperfection of computing device | SE + RE |

Once all the SEs are detected, corrections are applied to some components if possible and RSEs are estimated. Therefore, non-corrected or residual SEs are given in Table 1.

Similarly, instrumental SE can be divided into components corresponding to MI design features.

**Detecting the SEs**

To detect SEs in an observation sequence $x_1, ..., x_n$, special statistics and criteria can be employed. Monotonous SEs, for example, can be detected using Allan variance and Abbe criterion [7].

Abbe criterion tests the null hypothesis of constant mean:

$$H_0: M x_k = c,$$

vs the alternative of monotonous data shift:

$$H_1: \quad M x_{k+1} = M x_k + h .$$

Abbe statistics is defined as a ratio of two variance estimates:

$$r = \sigma_a^2 / S^2 ,$$

where $\sigma_a^2 = \sum (x_{k+1} - x_k)^2 / 2(n-1)$ is the Allan variance,

$S^2 = \sum (x_k - \bar{x})^2 / (n-1)$ is the sample variance.

If the calculated ratio $r$ is less than critical: $r < r_{min}(n, \alpha)$, where $\alpha$ is the accepted significance level, systematic data shift is recognized.

Abbe criterion can be applied to detect SEs of other types, including the randomized ones.

**Summing the SEs**

The major difficulties in SE estimation are caused by the problem weak formalization. SEs are summed using heuristic methods, based on a priori data and plausible assumptions [2].

SE is represented as a sum of its components conditioned by different factors:

$$\vartheta = \sum_1^m \vartheta_i ;$$

with the estimated limits of all components: $| \theta_i | < \Theta_i$.

Summation consists in finding the common limit of total SE $\Theta$ from $\Theta_i$.

The simplest estimate of the sought limit $\Theta_a$ is the sum of component limits:

$$\Theta_a = \sum_1^m \Theta_i .$$

This arithmetical limit is the most reliable and it conforms to the interval model of SE components. However it is usually overestimated, and in practice realistic estimates are sought by using approximate empirical formulas. Arithmetical summation is suitable only for strictly constant SEs.

In precision measurements, corrections are applied to the major SE variables, so the most SREs are semi-constant, that is, varying within the specified limits over repeated measurements. Therefore they may be summed using randomization and approximate quasi-statistical methods. Then the approximate confidence limit of total SE can be found by an empirical formula [2]

$$\Theta_P = k_P \sqrt{\sum_1^m \Theta_i^2},$$

where $k_p$ is the factor depending on the confidence probability P and the number of components m, and from ratio between limits.

**Summing SEs and REs**

First of all, significance of summands should be checked, that is, if one of them is negligibly small. Qualitative criterion for comparison of SE and RE is based on ratio of their characteristics:

$$r = \Theta / S,$$

where $\Theta$ is SE limit,
S is RMS of RE.

The comparison is based on the above mentioned quasi-statistical approach and yields the following empirical rules [2]:

If $r < 0{,}8$, SE is considered negligibly small compared with RE, and the limit of total error is taken to be RE confidence limit: $\Delta = \varepsilon(P)$.

If $r > 8$, RE is considered negligibly small compared with SE, and the limit of total error is taken to be SE limit: $\Delta = \Theta$.

With intermediate values: $0{,}8 \le r \le 8$ both components should be considered, SE and RE. The limit of total error can be calculated using one of empirical formulas:

A) $\Delta = \Theta + \varepsilon(P)$;

B) $\Delta = K(\Theta + \varepsilon(P))$;

C) $\Delta = k_\Sigma\ S(\Sigma)$.

Formula (A) is for the interval model of components and usually provides overestimated limit.

Formulas (B-C) are for the quasi-statistical method of summing SE components. In (B), the factor K depends on confidence probability P and ratio r; K values for probabilities P = 0.95 and 0.99 can be taken K = 0.8 and 0.85. (C) uses the following estimates:

$S(\Sigma)$ – total RMS (with consideration for SE and RE);

$k_\Sigma = (\theta + \varepsilon(P)) / (S(\varepsilon) + S(\theta))$.

**Selecting the number of observations during the measurement**

Increased number of observations generally reduces only REs, so the rules for rational selection of observation number are established based on SE-RE ratio. For example, the number of observations can be selected so that one of the following conditions holds:

1) RE of the measurement result is negligibly small compared with SE, that is RE is not considered in summing [2];
2) SE of a multiple measurement result is much lower than the error of single measurement results [2];
3) total error of measurement result estimated using RE and SE exceeds its SE insignificantly (not more than by a specified part $\alpha$) [14].

Qualitative relations obtained using these empirical rules obviously depend on the accepted models of SE and RE, on estimation methods and SE-RE summing techniques.

## 6 CONCLUSUIONS

The formulated approaches and major problems can serve as a basis for developing SE theory. SE activities are required at all stages of measurement procedure from the problem statement and to the result interpretation. The main operations should be as follows:

identifying and selecting the major systematic factors by structuring them and corresponding systematic errors detecting;

obtaining raw data of required quality by physical modeling the measurement process;

achieving proper quality of the corrections by systematic errors modeling;

optimal matching various-type components by model expansion and combination.

Further research on these problems creates prospects for a consistent theory including both conceptual considerations and general methods for SE detection and estimation. In the future, its combination with RE theory may be required.

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## 7 REFERENCES

[1] Malikov M. F. Foundation of Metrology. – Moscow, 1949 (in Russian).

[2] Rabinovich S. G. Measurement errors and uncertainties. Theory and practice. – 3 edition, Springer-Verlag, New York, 2005.

[3] Cox M. G. Systematic-error modeling, with application to complex permittivity measurement. – 16th IMEKO – TC4 Symposium. – Florence, 2008.

[4] Campion P. J., Burns I. E., Williams A. A code of practice for the detailed statement of accuracy. – NPL, London, 1973.

[5] Finkelstein L. Measurement and instrumentation science. - An analytical review // Measurement, 1994, v. 14, N 1, p. 3-14.

[6] Finkelstein L., Grattan K. T. V. (Eds.) Concise Encyclopedia of Measurement and Instrumentation, Pergamon, Oxford, 1994.

[7] Brownlee K. A. Statistical theory and methodology in science and engineering – John Wiley & Sons, London, 1970.

[8] Krasnov A.A., Nesenyuk L.P., Peshekhonov V.G., Sokolov A.V., Elinson L.S. Marine Gravimeter of a New Generation. - Proceedings of International Symposium Terrestrial Gravimetry: Static and Mobile Measurements. – 2007.

[9] Tutubalin V. N. Probability theory. –Moscow State University Publ., Moscow, 1972 (in Russian)..

[10] Ventzel E. S. Probability theory. –“Nauka”, Moscow, 1973 (in Russian).

[11] Guide to the Expression of Uncertainty in Measurement (BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML) // ISO, 1993.

[12] Markov A. A. Logic of constructive mathematics. - “Znanie”, Moscow, 1972 (in Russian).

[13] Granovsky V. A., Siraya T. N. Methods for Data Processing in Measurement. - Energoatomizdat, Leningrad, 1990 (in Russian).

[14] Lyachnev V. V., Siraya T. N., Dovbeta L. I. Metrological foundations of the theory of measurement procedures. – Elmor, S.-Petersburg, 2011 (in Russian).

[15] Granovsky V. A., Siraya T. N. «Model adequacy in measurements: estimation and improvement» - Advanced mathematical & Computational Tools in Metrology VIII – Series on Advances in Mathematics for Applied Sciences, World Scientific, 2009.