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ASPECTS OF MEASUREMENT POLICY WITH UNCERTAIN MEASUREMENTS

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Abstract: This paper describes how to use uncertain measurements in decision making. How to verify measurement policy with uncertainty involved. Simulation of process behavior with different uncertainties and cost structures are tackled. An example with four different decision options is shown and process of simulation is explained and visualized.

Keywords: Measurement strategy, uncertainty, measurement process

1. INTRODUCTION

Measurements are an integral part of process and quality management. Measurements are uncertain, thus there always is uncertainty about the quality and decisions are to be made under uncertainty.

Simulation of measurement process behaviour is an important tool to visualize and understood the measurement policy and the risks during the decision making.

This paper concerns quality pipe problem introduced in papers published at earlier IMEKO conferences [1, 2, 3]. Measurement scheduling, strategy and optimization is used in various fields – for example, aerospace [4, 5], linear quadratic Gaussian control problems [6].

1.1. Quality pipe

It is common industrial practice that every quality variable has its own quality specifications, acceptance limits – “quality pipe”. This can also be understood as a tool for monitoring and controlling the process. Different quality pipes can be formed by forming information channels in which measurements, *a priori* information and information based on covariance matrix can be combined [7].

This kind of quality pipe can serve as basis for process monitoring where statistical process control (SPC) tool can be applied or this can be turned around - regular statistical process control problem can be understood as a solution to control problem, in which there is an action that returns the system back to its origin, but has a cost associated to it.

Measurement strategy leading to policy should be such that enough information for controlling the process (that is decision making) is acquired by doing measurements. On the other hand the costs of making these measurements should be acceptable level. This sets an optimization problem that is very process dependent; the costs of measurements vary from process to process as well as the difficulty level of decisions.

In this paper we research an optimal action - measurement policy with certain cost structure of one quality parameter using simulation. Examples of simulation of optimal decision policy are presented with four decision options – do nothing, control action, and two uncertain measurements.

This paper is organized as follows: First introduction to the topic and quality pipe problem. Then definition of our system under inspection and third chapter deals with simulation and system evolution. Paper ends with conclusions with discussion and future work suggestions.

2. SYSTEM

In this chapter let us define out system, which is used simulation. Our system has four possible actions – first “no action” leading to linear dynamics:

$$\begin{aligned} x_{n+1} &= a x_n + \varepsilon_{n+1} \\ \varepsilon_{n+1} &\sim N(0,\sigma_\varepsilon^2) \end{aligned} \quad (1)$$

Second - “make an action” that returns the system to zero

$$\begin{aligned} x_{n+1} &= \varepsilon_{n+1} \\ \varepsilon_{n+1} &\sim N(0,\sigma_\varepsilon^2) \end{aligned} \quad (2)$$

There is cost $c(x)=\alpha x^2$ associated with state and cost c_A associated with action. Now our quality pipe is at figure 1, marked as blue an as shown when uncertainty increases our safe zone narrows. Interesting part of the quality pipe is near the edges where measurements are usually made. If our process is certainly out of the pipe then no measurements are needed but we have to make control actions to get the process inside the pipe. Also if process is certainly inside the pipe then nothing has to done. But when process is between then more information from the measurements are needed.

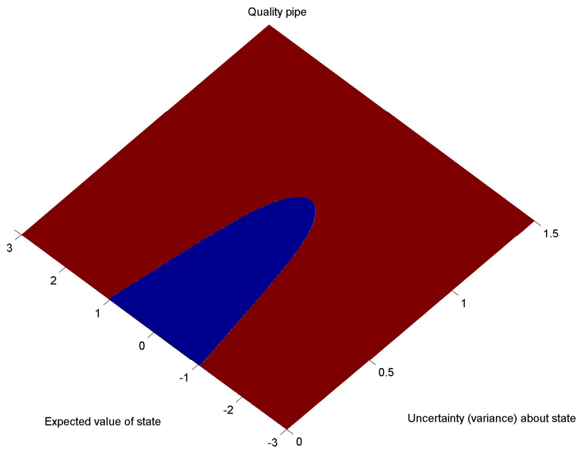

Figure 1. Quality pipe visualization

Third and fourth - two imperfect (uncertain) measurements with parameters - cost of measurement - c_M and c_{M2} and uncertainty - σ^2_m and σ^2_{m2}.

$$\begin{aligned} y_{n+1} &= x_{n+1} + v_{n+1} \\ v_{n+1} &\sim N(0,\sigma_m^2) \end{aligned} \quad (3)$$

$$\begin{aligned} y_{n+1} &= x_{n+1} + w_{n+1} \\ w_{n+1} &\sim N(0,\sigma_{m2}^2) \end{aligned} \quad (4)$$

Figures 2 shows an optimal decision policy when calculation horizon is 2, for parameters and calculations, see reference 1.

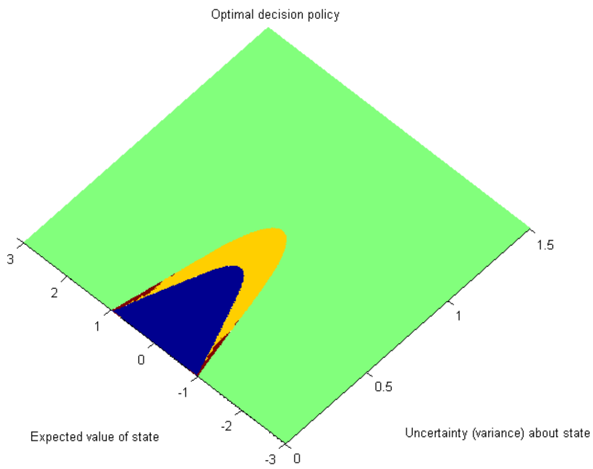

Figure 2. Plane of expected value and uncertainty, with horizon (N) 2. Blue denotes the no measurement/no action, red denotes measurement 1/no action, yellow denotes measurement 2/no action and green no measurement/action.

3. MEASUREMENT BEHAVIOR AND QUALITY PIPE

This chapter references basics of simulation. Simulation is based on running actual process, so depending on the action system either returns to zero (eq. 2) or continue with its dynamics (eq. 1). Simulation process goes step by step and depending on the information we have through uncertain measurement process, see figure 3.

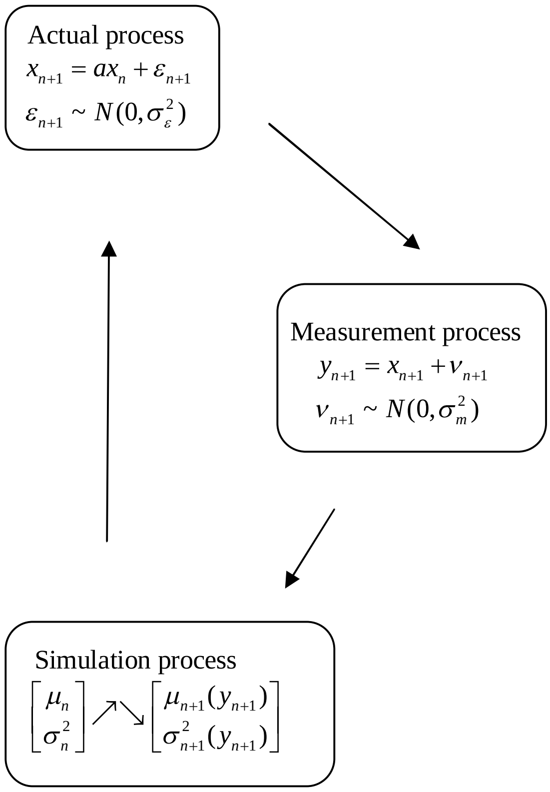

Figure 3. Simulation process with actual hidden process and measurements.

At final paper this is elaborated in detail with different initial parameters as well as different parameters for measurement process. Sensitivity of process is also discussed. Simulation uses results from paper presented IMEKO TC1, TC7 and TC13 joint symposium at Jena [1]. There result is based on dynamic programming and this result will also be referred at the full paper.

Short simulation examples are being seen at figures 4 and 5. Background image presents optimal decision policy at the horizon of 5. In both figures line shows how the simulation process has been evolving (with number showing the order).

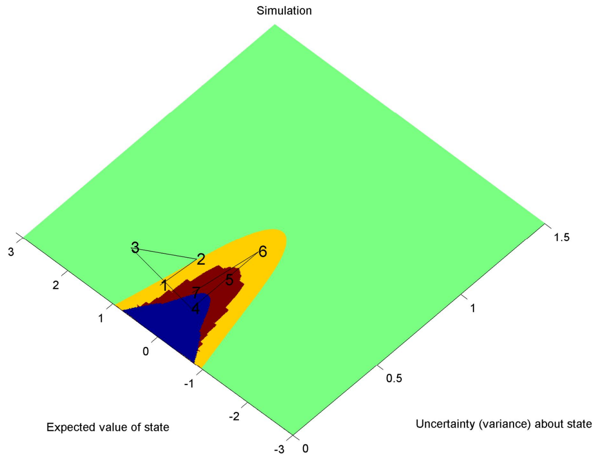

Figure 4. Short simulation example with initial process value of 0.7. Action sequence has been then 1, 4, 2,1, 1, 4 and 1.

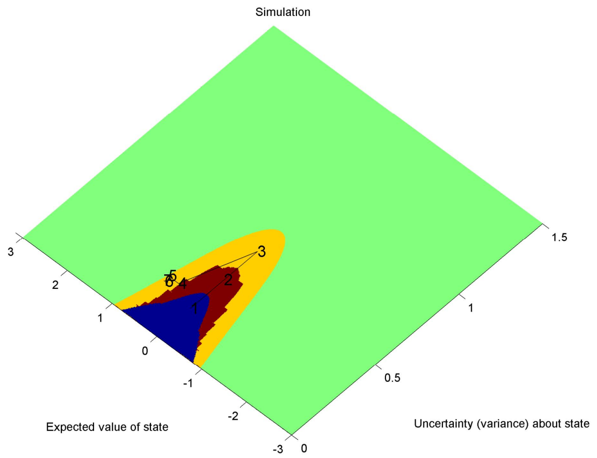

Figure 5. Short simulation example with initial process value of 0. Action sequence has been then 1, 1, 4, 3, 4, 4 and 4.

With results from simulation we can base analysis of the frequency of measurements and actions. By this we can address more information to quality pipe problem and thus make more efficient measurement policies and schedules. This is discussed in more detail in final paper.

4. CONCLUSIONS

This paper tackles quality pipe problem using simulation and analysing the results of frequency of measurements and actions. System has four possible actions (no action, action and two uncertain measurement options). Simulation process and initial result is shown and will be elaborated in full paper.

Full paper will include sensitivity analysis and discussion of uncertainty and cost parameters with more elaborate analysis of the frequency of measurements and actions.

5. REFERENCES

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