XX IMEKO World Congress
Metrology for Green Growth
September 9–14, 2012, Busan, Republic of Korea

UNCERTAINTY EVALUATION FOR THE COMPOSITE ERROR OF ENERGY METER AND INSTRUMENT TRANSFORMER

H. Kato, H. Imai

Japan Electric Meters Inspection Corporation,
4-15-7 Minato-ku, Tokyo, Japan,
E-mail : h-kato@jemic.go.jp

Abstract: Japan Electric Meters Inspection Corporation (JEMIC) is a Designated Institute of power and energy standards in Japan. All the energy meters for electricity trading in Japan are traceable to the National Standard in JEMIC. In industrial world, power is supplied by various voltage and current. However, the energy standard has been calibrated at 110 V and 100 V, for 5 A. Therefore, it is often used in combine with the instrument transformer. This paper describes the effect of composite error of instrument transformers to uncertainty evaluation of energy meters.

Keywords: AC Power, AC Energy, Energy Meter, Instrument Transformer

1. INTRODUCTION

Instrument transformer errors are evaluated by two components (Ratio error and Phase displacement). In addition, an energy meter is evaluated by instrumental error. In this paper, the composite error of the instrument transformer (instrument voltage transformer: VT, current transformer: CT) is calculated from JIS C 1736-1(2009) Annex-A, in combination with the instrumental error of energy meter. Therefore, we examined the amount of contribution of the uncertainty and error of instrument transformer for energy meters.

2. CALIBRATION METHOD

The calibration of energy meter is carried out by using the following equipment.

1) Standard energy meter
2) Power Source
3) Frequency counter
4) Voltage transformer
(This equipment is used only when the voltage is different from the standard energy meter.)
5) Current transformer
(This equipment is used only when the current is different from the standard energy meter.)

Figure 1 is a calibration circuit and **Figure 2** is a view of calibration system. The power is supplied on standard and device under test through an instrument transformer, then the output pulse can be measured for calibration using the frequency counter.

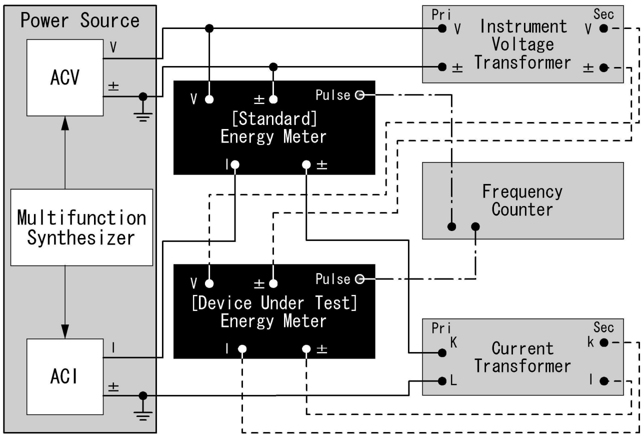

Figure 1 The block diagram of calibration circuit

Figure 2 The view of calibration system

3. ABOUT ERROR CALCULATION EXPRESSION (FUNCTION MODEL)

Described following is an example of error calculation formula of a single-phase two-wire.

1) Energy meter error calculation formula:

$$\varepsilon_x = f(p_x, \ \Delta p_f, \ p_s, \ \varepsilon_s, \ \varepsilon_{tr})$$

$$= \frac{\dfrac{p_x + \Delta p_f}{k_{wx}} - \dfrac{p_s + \Delta p_f}{k_{ws}}}{\dfrac{p_s + \Delta p_f}{k_{ws}}} \times 100 + \varepsilon_s + \varepsilon_{tr} \qquad (1)$$

where,

ε_x : Energy meter error of device under test (%)

P_x : Average of the output pulse of device under test

k_{wx} : Instrument constant of device under test (pulse/kWs)

P_s : Average of the output pulse of standard

k_{ws}: Instrument constant of standard (pulse/kWs)

Δp_f : Correction value of frequency counter

ε_s : Energy meter error of standard (%)

ε_{tr} : Composite error of instrument transformer (%)

2) Composite error calculation formula:

$$\varepsilon_{tr} = \left[\frac{1}{\cos\phi} \left[\left(1 + \frac{\varepsilon_v}{100}\right)\left(1 + \frac{\varepsilon_c}{100}\right) \cos(\phi + \theta_v - \theta_c) \right] - 1 \right] \times 100$$

$$\approx \varepsilon_v + \varepsilon_c + 0.0291(\theta_c - \theta_v)\tan\phi \qquad (2)$$

where,

ε_{tr} : Composite error of instrument transformer (%)

ε_v : Ratio error of VT (%)

ε_c : Ratio error of CT (%)

θ_v : Phase displacement of VT (min.)

θ_c : Phase displacement of CT (min.)

ϕ : Load power factor angle (deg.)

4. SOURCES OF UNCERTAINTY

The sources of uncertainty for the calibration of energy meter with instrument transformer are described as follows:

1) Uncertainty contributes to the mean value of the output pulse count of device under test.

A. Repeated measurement : $u(p_{x1})$

B. Resolution of digital display : $u(p_{x2})$

C. Difference of input voltage : $u(p_{x3})$

2) Uncertainty contributes to the correction value of the frequency counter.

A. Uncertainty of calibration using an upper standard : $u(\Delta p_{f1})$

B. Long-term stability : $u(\Delta p_{f2})$

3) Uncertainty contributes to the mean value of the output pulse count of standard.

A. Repeated measurement : $u(p_{s1})$

B. Resolution of digital display : $u(p_{s2})$

4) Uncertainty contributes to the Energy meter error of standard.

A. Uncertainty of calibration using standard : $u(\varepsilon_{s1})$

B. Stability : $u(\varepsilon_{s2})$

C. Linearity : $u(\varepsilon_{s3})$

D. Temperature dependence : $u(\varepsilon_{s4})$

E. Due to correlation : $u(\varepsilon_{s5})$
(Except for single-phase two-wire)

5) Uncertainty contributes to the composite error of instrument transformer

A. Uncertainty of resultant error : $u(\varepsilon_{tr1})$

B. Long-term stability (VT) : $u(\varepsilon_{tr2})$

C. Long-term stability (CT) : $u(\varepsilon_{tr3})$

5. CALCULATION OF COMBINED STANDARD UNCERTAINTY

Combined variance [$u_c^2(\varepsilon_x)$] of function model [ε_x]is estimated by the following equation.

$$u_c^2(\varepsilon_x) = c_{px}^2 u^2(p_x) + c_{\Delta pf}^2 u^2(\Delta p_f) + c_{ps}^2 u^2(p_s) + c_{\varepsilon s}^2 u^2(\varepsilon_s) + c_{\varepsilon tr}^2 u^2(\varepsilon_{tr})$$

Sensitivity coefficient (partial derivatives : c_i) is as follows.

$$c_{px} = \frac{\partial f}{\partial p_x} = \frac{100}{k_{wx} \times p_s}$$

$$c_{\Delta pf} = \frac{\partial f}{\partial \Delta p_f} = \frac{100}{k_{wx} \times p_s}$$

$$c_{Whm} = \frac{\partial f}{\partial p_s} = -\frac{100(p_x + \Delta p_f)}{k_{ws} \times p_s^2}$$

$$c_{\varepsilon s} = \frac{\partial f}{\partial \varepsilon_s} = 1$$

$$c_{\varepsilon tr} = \frac{\partial f}{\partial \varepsilon_{tr}} = 1$$

Therefore, the combined variance [$u_c^2(\varepsilon_x)$] will be the following equation.

$$u_c^2(\varepsilon_x) = \left(\frac{100}{k_{wx} \times p_s}\right)^2 u^2(p_x) + \left(\frac{100}{k_{wx} \times p_s}\right)^2 u^2(\Delta p_s) + \left[-\frac{100(p_x + \Delta p_f)}{k_{ws} \times p_s^2}\right]^2 u^2(p_s) + u^2(\varepsilon_s) + u^2(\varepsilon_{tr})$$

In addition, the combined standard uncertainty is composed of the following equation.

$$u_c(\varepsilon_x) = \sqrt{\left(\frac{100}{k_{wx} \times p_s}\right)^2 u^2(p_x) + \left(\frac{100}{k_{wx} \times p_s}\right)^2 u^2(\Delta p_f) + \left[-\frac{100(p_x + \Delta p_s)}{k_{ws} \times p_s^2}\right]^2 u^2(p_s) + u^2(\varepsilon_s) + u^2(\varepsilon_{tr})} \qquad (3)$$

The effective degree of freedom [$\nu_{eff}(\varepsilon_x)$] is determined by the following equation.

$$\nu_{eff}(\varepsilon_x) = \frac{u_c^4(\varepsilon_x)}{\dfrac{c_{px}^4 u^4(p_x)}{\nu_{eff}(p_x)} + \dfrac{c_{ps}^4 u^4(p_s)}{\nu_{eff}(p_s)}} \qquad (4)$$

6. DETERMINATION OF COMPOSITE ERROR DUE TO INSTRUMENT TRANSFORMER

Using the formula for calculating the composite error function model, we evaluated the uncertainty due to composite error. In the calculation, we used the expansion expression.

$$\varepsilon_{tr} = f(\varepsilon_v, \ \varepsilon_c, \ \theta_c, \ \theta_v, \ \phi)$$
$$= \varepsilon_v + \varepsilon_c + 0.0291(\theta_c - \theta_v)\tan\phi$$

Combined variance [u_c^2 (ε_{tr})] of function model [ε_{tr}] is estimated by the following equation.

$$u_c^2(\varepsilon_{tr}) = c_{\varepsilon v}^2 u^2(\varepsilon_v) + c_{\varepsilon c}^2 u^2(\varepsilon_c) + c_{\theta c}^2 u^2(\theta_c) + c_{\theta v}^2 u^2(\theta_v) + c_\phi^2 u^2(\phi)$$

Sensitivity coefficients (partial derivatives : c_i) are as follows.

$$c_{\varepsilon v} = \frac{\partial f}{\partial \varepsilon_v} = 1$$
$$c_{\varepsilon c} = \frac{\partial f}{\partial \varepsilon_c} = 1$$
$$c_{\theta c} = \frac{\partial f}{\partial \theta_c} = 0.0291\tan\phi$$
$$c_{\theta v} = \frac{\partial f}{\partial \theta_v} = -0.0291\tan\phi$$
$$c_\phi = \frac{\partial f}{\partial \phi} = \frac{0.0291(\theta_c - \theta_v)}{\cos^2\phi}$$

Therefore, the combined variance [u_c^2 (ε_{tr})] will be described by the following equation.

$$u_c^2(\varepsilon_{tr}) = u^2(\varepsilon_v) + u^2(\varepsilon_c) + (0.0291\tan\phi)^2 u^2(\theta_c) + \left[-(0.0291\tan\phi)^2 u^2(\theta_v)\right] + \left[\frac{0.0291(\theta_c - \theta_v)}{\cos^2\phi}\right]^2 u^2(\phi)$$

In addition, the combined standard uncertainty [u_c (ε_{tr})] is composed of the following equation.

$$u_c(\varepsilon_{tr}) = \sqrt{u^2(\varepsilon_v) + u^2(\varepsilon_c) + (0.0291\tan\phi)^2 u^2(\theta_c) + \left[-(0.0291\tan\phi)^2 u^2(\theta_v)\right] + \left[\frac{0.0291(\theta_c - \theta_v)}{\cos^2\phi}\right]^2 u^2(\phi)} \quad (5)$$

We examined the influence of uncertainty of composite error by three types of transformers. It should be noted that the uncertainty of the following is a coverage factor $k = 2$.

1) Uncertainty of calibration using a high accuracy transformer .
  A. Inductive voltage divider (IVD).
    Uncertainty of ratio error : 0.2 (ppm)
    Uncertainty of phase displacement : 3.7 (ppm)
  B. Current transformer.
    Uncertainty of ratio error : 20 (ppm)
    Uncertainty of phase displacement : 29 (ppm)
2) Uncertainty of calibration using a general transformer.
  A. Instrument voltage transformer.
    Uncertainty of ratio error : 50 (ppm)
    Uncertainty of phase displacement :0.3 (min.) ≈ 87 (ppm)
  B. Current transformer.
    Uncertainty of ratio error : 60 (ppm)
    Uncertainty of phase displacement :0.2 (min.) ≈ 58 (ppm)
3) Uncertainty of calibration using a high voltage and high current transformer.
  A. Instrument voltage transformer.
    Uncertainty of ratio error : 200 (ppm)
    Uncertainty of phase displacement:0.6 (min.) ≈ 175 (ppm)
  B. Current transformer.
    Uncertainty of ratio error : 70 (ppm)
    Uncertainty of phase displacement:0.3 (min.) ≈ 87 (ppm)

Uncertainty of composite error due to instrument transformer is determined using composite error calculation formula by the following expanded formula (5). The result is as follows **Table 2**.

Table 1 Uncertainty of transformers (ppm)

Classification		(1)	(2)	(3)
IVD or VT	ε_v	0.2	50	200
	θ_v	3.7	87	175
CT	ε_c	20	60	70
	θ_c	29	58	87

Table 2 Uncertainty based on composite error (ppm)

	Composite error		
Power factor	(1)	(2)	(3)
PF 1	20.00	78.10	211.90
PF 0.5 Lagging	20.29	78.36	212.40

Note: PF 0.5 Lagging is 60° angle of the load power factor.

From this result, the effect of uncertainty due to the load power factor angle is changed by the combination of the transformer, it can be seen that the amount of that affect is very different.
In this calculation, the number we used were converted to ppm. Therefore, it can also be seen by the exerted influence of significant digits.

7. UNCERTAINTY BUDGET

Described in **Table 3 and Table 4** are examples of uncertainty budget. Energy meter setting is single-phase two-wire, 100 V, 5 A, and combination of the general transformer.
Note: Usually, when the calibration is made by 100 V and 5 A, an instrument transformer is not used, but the calculation is done by assuming it.

8. CONCLUSION

The uncertainty due to the composite error of instrument transformer contributes to the energy meter greatly because of the uncertainty of itself. It is necessary to be careful that the difference between ratio errors and phase displacement lead to change of uncertainty by the change of power factor angle. In addition, it cannot ignore the impact on the calculation results of significant digits.

Table 3 An example of uncertainty budget for PF 1

Standard uncertainty $u(x_i)$	Sources of uncertainty	Value of standard uncertainty $u(x_i)$	Type of distribution	$c_i \equiv \partial f / \partial x_i$	$u_i(\Delta\varepsilon_x) \equiv \lvert c_i \rvert u(x_i)$	Degree of freedom ν_i	Note
	Single-Phase two-wire (100 V 5 A PF 1)						
$u(Px)$	Uncertainty contributes to the average value of the output pulse count of device under test	1.04E-02 Hz		0.02	2.08E-04 %	1.06E+01	
$u(Px_1)$	Repeated measurement	1.00E-02 Hz	Normal			9	Calibration data
$u(Px_2)$	Resolution of digital display	2.89E-03 Hz	Rectangular			∞	0.01 Hz
$u(Px_3)$	Difference of input voltage	-				-	Negligible
$u(\Delta P_f)$	Uncertainty contributes to the correction value of the frequency counter	5.77E-03 Hz		0.02	1.15E-04 %	∞	
$u(\Delta P_{f1})$	Uncertainty of calibration using an upper standard	5.00E-06 Hz	Normal			∞	Calibration certificate
$u(\Delta P_{f2})$	Long-term stability	5.77E-03 Hz	Rectangular			∞	Record data
$u(P_s)$	Uncertainty contributes to the average value of the output pulse count of standard	1.04E-02 Hz		-0.02	2.08E-04 %	1.06E+01	
$u(P_{s1})$	Repeated measurement	1.00E-02 Hz	Normal			9	Calibration data
$u(P_{s2})$	Resolution of digital display	2.89E-03 Hz	Rectangular			∞	0.01 Hz
$u(\varepsilon_s)$	Uncertainty contributes to the Energy meter error of standard	2.77E-03 %		1	2.77E-03 %	∞	
$u(\varepsilon_{s1})$	Uncertainty of calibration using standard	2.50E-03 %	Normal			∞	Calibration certificate
$u(\varepsilon_{s2})$	Long-term stability	1.15E-03 %	Rectangular			∞	Record data
$u(\varepsilon_{s3})$	Linearity	1.15E-04 %	Rectangular			∞	Experimental data
$u(\varepsilon_{s4})$	Temperature dependence	2.89E-04 %	Rectangular			∞	Experimental data
$u(\varepsilon_{s5})$	Due to correlation	-				—	Not affected
$u(\varepsilon_{tr})$	Uncertainty contributes to the resultant error of instrument transformer	3.90E-03 %		1	3.90E-03 %	∞	
$u(\varepsilon_{tr1})$	Uncertainty of resultant error	3.90E-03 %	Normal			∞	Calibration certificate
$u(\varepsilon_{tr2})$	Long-term stability (VT)	5.77E-05 %	Rectangular			∞	Record data
$u(\varepsilon_{tr3})$	Long-term stability (CT)	5.77E-05 %	Rectangular			∞	Record data
			Combined variance	$u_c^2(\Delta\varepsilon_x) = \Sigma u_i^2(\Delta\varepsilon_x) =$		2.30E-05 %^{2}	
			Combined standard uncertainty	$u_c(\Delta\varepsilon_x) =$		4.79E-03 %	
			Effective degree of freedom	$\nu_{eff}(\Delta\varepsilon_x) =$		1484357.0	
			Coverage factor	$k =$		2	
			Expanded uncertainty	$U = k \times u_c(\Delta\varepsilon_x) =$		0.0096 %	
			Calibration uncertainty	$U =$		0.010 %	

Table 4 An example of uncertainty budget for PF 0.5 Lagging

Standard uncertainty $u(x_i)$	Sources of uncertainty	Value of standard uncertainty $u(x_i)$	Type of distribution	$c_i \equiv \partial f / \partial x_i$	$u_i(\Delta\varepsilon_x) \equiv \lvert c_i \rvert u(x_i)$	Degree of freedom ν_i	Note
	Single-Phase two-wire (100 V 5 A PF 0.5 Lagging)						
$u(Px)$	Uncertainty contributes to the average value of the output pulse count of device under test	1.04E-02 Hz		0.02	2.08E-04 %	1.06E+01	
$u(Px_1)$	Repeated measurement	1.00E-02 Hz	Normal			9	Calibration data
$u(Px_2)$	Resolution of digital display	2.89E-03 Hz	Rectangular			∞	0.01 Hz
$u(Px_3)$	Difference of input voltage	-				-	Negligible
$u(\Delta P_f)$	Uncertainty contributes to the correction value of the frequency counter	3.54E-06 Hz		0.02	7.07E-08 %	∞	
$u(\Delta P_{f1})$	Uncertainty of calibration using an upper standard	2.50E-06 Hz	Normal			∞	Calibration certificate
$u(\Delta P_{f2})$	Long-term stability	2.50E-06 Hz	Rectangular			∞	Record data
$u(P_s)$	Uncertainty contributes to the average value of the output pulse count of standard	1.04E-02 Hz		-0.02	2.08E-04 %	1.06E+01	
$u(P_{s1})$	Repeated measurement	1.00E-02 Hz	Normal			9	Calibration data
$u(P_{s2})$	Resolution of digital display	2.89E-03 Hz	Rectangular			∞	0.01 Hz
$u(\varepsilon_s)$	Uncertainty contributes to the Energy meter error of standard	3.00E-03 %		1	3.00E-03 %	∞	
$u(\varepsilon_{s1})$	Uncertainty of calibration using standard	2.70E-03 %	Normal			∞	Calibration certificate
$u(\varepsilon_{s2})$	Long-term stability	1.15E-03 %	Rectangular			∞	Record data
$u(\varepsilon_{s3})$	Linearity	2.31E-04 %	Rectangular			∞	Experimental data
$u(\varepsilon_{s4})$	Temperature dependence	5.77E-04 %	Rectangular			∞	Experimental data
$u(\varepsilon_{s5})$	Due to correlation	-				—	Not affected
$u(\varepsilon_{tr})$	Uncertainty contributes to the resultant error of instrument transformer	3.90E-03 %		1	3.90E-03 %	∞	
$u(\varepsilon_{tr1})$	Uncertainty of resultant error	3.90E-03 %	Normal			∞	Calibration certificate
$u(\varepsilon_{tr2})$	Long-term stability (VT)	5.77E-05 %	Rectangular			∞	Record data
$u(\varepsilon_{tr3})$	Long-term stability (CT)	5.77E-05 %	Rectangular			∞	Record data
			Combined variance	$u_c^2(\Delta\varepsilon_x) = \Sigma u_i^2(\Delta\varepsilon_x) =$		2.43E-05 %^{2}	
			Combined standard uncertainty	$u_c(\Delta\varepsilon_x) =$		4.93E-03 %	
			Effective degree of freedom	$\nu_{eff}(\Delta\varepsilon_x) =$		1659378.7	
			Coverage factor	$k =$		2	
			Expanded uncertainty	$U = k \times u_c(\Delta\varepsilon_x) =$		0.0099 %	
			Calibration uncertainty	$U =$		0.010 %	

9. REFERENCES

[1] JIS C 1736-1 : 2009 : Instrument transformers for metering service- Part 1: General measuring instrument

[2] H. Kato, M. Miyakoda : “Re-establishment of Power and Energy Standards Calibration System ”, JEMIC Technical Report, Vol.45, No.4 pp.71-77 (2010)

[3] H. Kato, H. Miyamura and T. Ogawa : “Re-establishment of Energy Verification Standards Calibration System” , JEMIC Technical Report, Vol.46, No.3 pp.38-49 (2011)

[4] M. Kogane, N. Yamazaki, S. Kusui : “A Very Dynamic Range Standard Watthour Meter Using a High Speed Circulating Mark Space Type Multiplier”, JEMIC Technical Report, Vol.15, No.1 pp.7-23 (1980)

[5] A. Hashimoto : “Three Phase Standard Watthour Meter using Modified Precision Self-Calibration Multiplier”, JEMIC Technical Report, Vol.34, No.2 pp.1-9 (1999)

[6] M. Oku, K. Takahashi, R. Yasuda, H. Kato : “Calibration System of Standard Watthour Meters Using VXIbus Instrument Unit”, JEMIC Technical Report, Vol.30, No.3 pp.77-81 (1995)