

# UNCERTAINTY EVALUATION FOR THE CALIBRATION OF DIRECT ACTING INDICATING ANALOGUE ELECTRICAL MEASURING INSTRUMENTS

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**Abstract:** In the case of calibration of direct acting indicating analogue electrical measuring instruments (hereinafter called the "analogue instruments"), uncertainty budget largely depends on dispersion of readings for scales through a needle. This paper describes experimental results for three levels of voltage (10 V, 6 V, and 3 V) of analogue instruments. In the experiments, two way layout nested design was introduced for each voltage level. From the results of present experiments, it was revealed that expanded uncertainties obtained were about 1/3 for the specification that obtained from the past experience.

**Keywords:** direct acting indicating analogue measuring instruments, nested design, ANOVA

## 1. INTRODUCTION

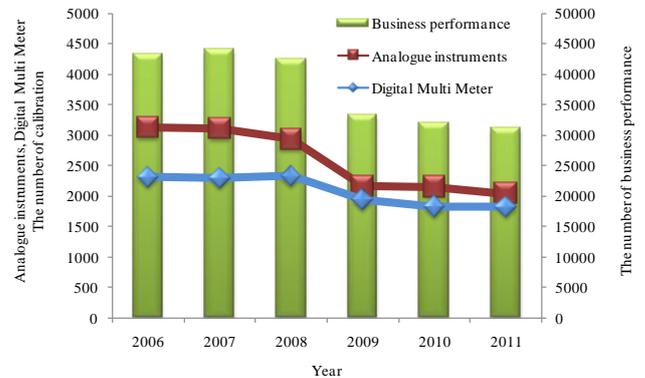
Anyone may think that analogue instruments for electrical measurement changed to digital instruments. However, trend of the number of recent calibrations at the JEMIC was nearly flat, except consolidated of brunches in 2009 (see **Figure 1**). It should be noted, that main requests of analogue instruments calibration from customers are accuracy class of 0.5 based on the JIS C 1102:2007, which depends on the IEC 60051-1 ed 5.0:1997.

The measurement for such as a low accuracy analogue instruments are required the expression of uncertainty on certificate by accredited laboratory based on the ISO/IEC 17025 increasingly now.

For these reasons, we estimate that analogue instruments are still useful and valuable for measurement in manufacture, moreover, it would still be useful at measurement until the near future. We did not have the uncertainty on the calibration certificate until about fifteen years ago.

It is necessary to evaluate the sources of measurement uncertainty for low accuracy analog instruments. For these reasons, JEMIC acquired the Japan Calibration Service System (JCSS). Under the JCSS of calibration laboratory accreditation system, calibration laboratories are assessed and accredited as Accredited Calibration Laboratories to meet the requirements of the Measurement Law in Japan, relevant regulations and ISO/IEC 17025. Then, when reporting the result of a measurement, it is expressing the uncertainty of a result of a measurement.

However, the value of uncertainty is not clarified up to now, it was given mainly based on our past experience. In this paper, we shall try to compare the uncertainty of past experience and that of our experimental result.



**Figure 1** Transition of calibrations

## 2. PROCEDURE OF CALIBRATION

The calibration process makes adjustment the output value of the standard generator in alignment with the scale and the needle, then the value of standard generator is the calibration value. In usual, we use better accuracy class standard generator for instruments under test. For example, the comparison between the accuracy of a instrument under test and the accuracy of standard as 100:1(at direct current), the calibration value is given by the following equation.

$$y = m \left( 1 + \frac{S_{\text{corr}}}{100} \right) \quad (1)$$

$y$  : Calibration value (V)

$m$  : The value of the standard generator setting (V)

$S_{\text{corr}}$  : The value of correction of the standard generator (%)  
 where,

$$S_{\text{corr}} = \left( \frac{a - n}{a} \right) \times 100 \quad (2)$$

$n$  : The value of the standard setting nearest "m" (V)

$a$  : Actual value of "n"

### 3. SOURCES OF UNCERTAINTY

We aimed at a value of combined standard uncertainty to about 0.1 %, this is from the intuitive knowledge in the past experience. Very important requirements for calibration of the analogue instruments are that the value of uncertainty and the cost should be reasonable. In any event, when calibrate low accuracy instruments for manufacturing site, one uncertainty element may become dominant on combined standard uncertainty.

Sources of uncertainty in this calibration measurement are shown as the following:

1) Alignment of the instruments and the eyes,  $u_1$

The empirical range is about 1/5 of a scale division, and its probability distribution is assumed to be rectangular distribution from the past experience. Incidentally, there are one hundred scale marks at equal intervals on the scale of about 13 cm. The distance between any two consecutive scale marks (hereinafter called the “scale division”) is 1.3 mm, one fifth length of it equal 0.26 mm.

**Figure 3** shows the alignment of instrument and human eyes. In calibration, actual values are indicated when aligned the needle, the scale, and the eyes. It is precisely expected about the alignment of instrument and the eyes. But in fact, it could not be exactly the same during the measurement. In this case, alignment of the instruments and the eyes is equivalent to dominated to combined standard uncertainty. For the 10 V range, a scale division is 0.1 V, hence, 1/5 of becomes 0.02 V. In the calibration, we have to align twice, one is at the zero point, and the other is at the test point, then  $u_1$  is given by

$$u_1 = \sqrt{(0.02/2\sqrt{3})^2 + (0.02/2\sqrt{3})^2} \approx 0.00816 \text{ V} \quad (3)$$

2) Uncertainty of standard generator,  $u_2$

The calibration certificate gives as the expanded uncertainty with coverage factor of  $k = 2$ .

3) Stability of standard generator in a year,  $u_3$

The long-term stability was not drifting, it was random variations. Therefore, the estimated uncertainty bounds on the variability of maximum value of the change per each year.

4) Temperature coefficient of standard generator,  $u_4$

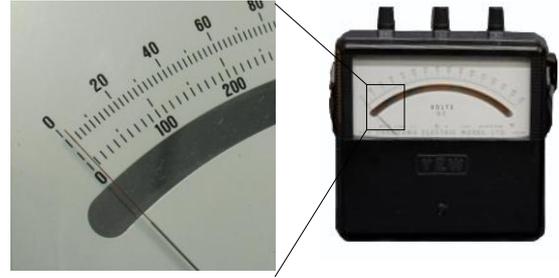
The manufacturer gives the standard generator temperature coefficient by  $23 \text{ }^\circ\text{C} \pm 2 \text{ }^\circ\text{C}$ , on the other hand, the test room temperature  $23 \text{ }^\circ\text{C} \pm 5 \text{ }^\circ\text{C}$  is allowed. For this conditions, the temperature difference is between  $\pm 5 \text{ }^\circ\text{C}$  and  $\pm 2 \text{ }^\circ\text{C}$ .

Then, combined standard uncertainty  $u_c$  is given by

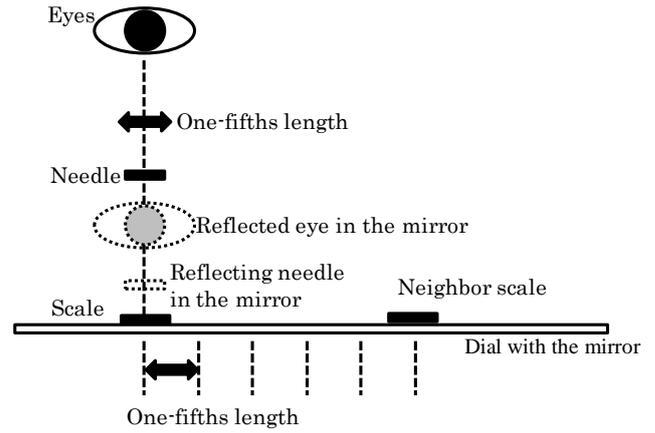
$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_4^2} \quad (4)$$

Therefore, expanded uncertainty  $U$  is given by

$$U = ku_c = 2u_c \quad (k: \text{coverage factor}) \quad (5)$$



**Figure 2** External view and close-up to indicating device



**Figure 3** Alignment of the instruments and the eyes

In addition, the effective degree of freedom is infinity, because all standard uncertainty components from  $u_1$  to  $u_4$  are infinity.

Finally, uncertainty budget is shown as in **Table 1**

**Table 1** Uncertainty budget for 10 V

Standard uncertainty component	Source of uncertainty	Value of standard uncertainty	Type of distribution
$u_1$	Alignment for the instrument and the eyes	8.16 mV	Rectangular
$u_2$	Uncertainty of standard generator	0.30 mV	Normal
$u_3$	Stability of standard generator in a year	0.58 mV	Rectangular
$u_4$	Thermal coefficient of standard generator	0.10 mV	Rectangular
$u_c$ : Combined standard uncertainty		8.19 mV	0.0819%
$k$ : Coverage factor		2	
$U$ : Expanded uncertainty		16.4 mV	

#### 4. DESIGN OF EXPERIMENT

The significant source of uncertainty is alignment of the instrument and the eyes, therefore, experimental evaluation is very important to confirm the reliability. In this time, we tried an experiment, then we compared the relation between empirical uncertainty from the past experience and experimental results calculated by Analysis of Variance (ANOVA). In the present experiment, two way layout nested design was introduced. The experimental components introduced are as follows, and **Figure 4** shows the layout.

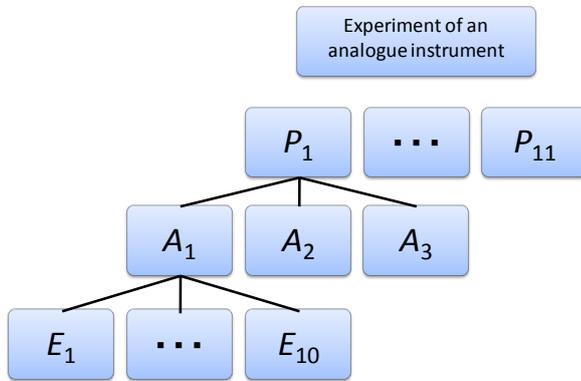
*P*: 11 persons (operators)

*A*: 3 calibration points (10 V, 6 V and 3 V in 10 V Range )

*E*: Measurement for 3 sets and 10 times of repetition is carried out at each calibration point.

Mathematical model is given by,

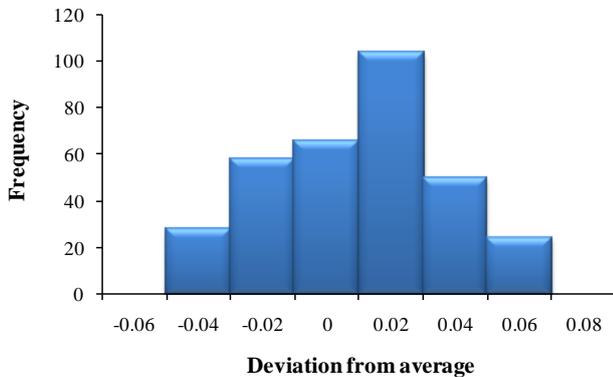
$$V_{ijk} = \mu + P_i + A_j + E_{ijk} \quad (4)$$



**Figure 4** Tree diagram of the experiment

#### 5. ANALYSIS OF VARIANCE

We compared between the empirical uncertainty and the results obtained from ANOVA in the present experiments. The ANOVA program was exposed by the NMIJ/AIST. Nested design was chosen from ANOVA program, because this experiment was not randomized. The experimental data and histogram of 10V shown by **Table 2** and **Figure 5**. The other points histogram are similar to histogram of 10V. And now, summary of ANOVA are shown by **Table 3**, **Table 4** and **Table 5**.



**Figure 5** Histogram of 10 V

**Table 2** The value of the standard generator setting for 10 V

		$E_1$	$E_2$	$E_3$	...	$E_{10}$
$P_1$	$A_1$	9.975	9.975	9.975	...	9.975
	$A_2$	9.974	9.974	9.973	...	9.974
	$A_3$	9.976	9.974	9.974	...	9.974
$P_2$	$A_1$	9.976	9.975	9.976	...	9.974
	$A_2$	9.977	9.976	9.977	...	9.976
	$A_3$	9.976	9.975	9.976	...	9.974
$P_3$	$A_1$	9.977	9.976	9.975	...	9.976
	$A_2$	9.977	9.978	9.978	...	9.976
	$A_3$	9.977	9.976	9.975	...	9.976
.	.	...	...	...	...	...
	.	...	...	...	...	...
	.	...	...	...	...	...
$P_{11}$	$A_1$	9.975	9.976	9.976	...	9.976
	$A_2$	9.977	9.977	9.977	...	9.977
	$A_3$	9.976	9.976	9.976	...	9.976

**Table 3** Summary of ANOVA for 10 V

Component	Sum of squares $S$ (mV)	Degrees of freedom $f$	Variance $V$ (mV)	Expectation $E(V)$
$P$	0.1404	10	0.01404	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2 + 30\hat{\sigma}_p^2$
$A$	0.0925	22	0.00421	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2$
$E$	0.1376	297	0.00046	$\hat{\sigma}_e^2$
Sum	0.3705	329		

**Table 4** Summary of ANOVA for 6 V

Component	Sum of squares $S$ (mV)	Degrees of freedom $f$	Variance $V$ (mV)	Expectation $E(V)$
$P$	0.3063	10	0.03063	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2 + 30\hat{\sigma}_p^2$
$A$	0.1997	22	0.00908	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2$
$E$	0.1105	297	0.00037	$\hat{\sigma}_e^2$
Sum	0.6165	329		

**Table 5** Summary of ANOVA for 3 V

Component	Sum of squares $S$ (mV)	Degrees of freedom $f$	Variance $V$ (mV)	Expectation $E(V)$
$P$	0.3418	10	0.03418	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2 + 30\hat{\sigma}_p^2$
$A$	0.1358	22	0.00617	$\hat{\sigma}_e^2 + 10\hat{\sigma}_a^2$
$E$	0.0835	297	0.00028	$\hat{\sigma}_e^2$
Sum	0.5611	329		

## 6. EXPERIMENTAL RESULTS

The component calculated from ANOVA results are shown as the following,

1) Uncertainty due to observer ( $P$ ) for 10 V is given by,

$$\hat{\sigma}_P = \sqrt{\frac{\hat{E}(V_P) - \hat{\sigma}_A^2}{30}} = \sqrt{\frac{0.00001404 - 0.00000421}{30}} \approx 0.57 \text{ mV}$$

2) Uncertainty of reproducibility ( $A$ ) for 10 V is given by,

$$\hat{\sigma}_A = \sqrt{\frac{\hat{E}(V_A) - \hat{\sigma}_E^2}{10}} = \sqrt{\frac{0.00000421 - 0.00000046}{10}} \approx 0.61 \text{ mV}$$

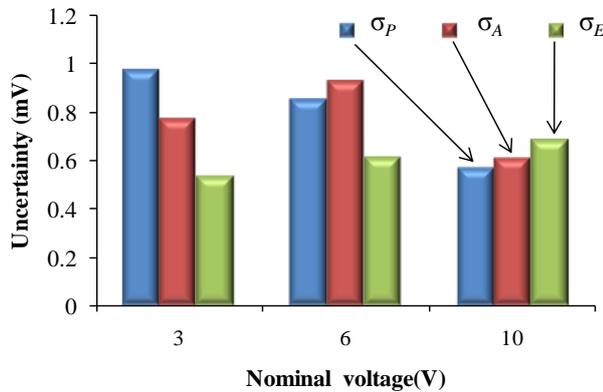
3) Uncertainty of repeatability ( $E$ ) for 10 V is given by,

$$\hat{\sigma}_E = \sqrt{\hat{\sigma}_E^2} = \sqrt{0.00000046} \approx 0.68 \text{ mV}$$

Results of calculation are shown as **Table 6** and **Figure 6**:

**Table 6 Results of ANOVA**

Component	Test point (mV)		
	3 V	6 V	10 V
$\sigma_P$	0.97	0.85	0.57
$\sigma_A$	0.77	0.93	0.61
$\sigma_E$	0.53	0.61	0.68



**Figure 6 Comparison of components of ANOVA**

Usually, our calibration is done by only one measurement. Therefore, uncertainty of alignment of the instrument and the eyes for 10 V is given as follows:

$$u_1 = \sqrt{\frac{\hat{\sigma}_P^2}{1} + \frac{\hat{\sigma}_A^2}{1} + \frac{\hat{\sigma}_E^2}{1}} = \sqrt{0.00057^2 + 0.00061^2 + 0.00068^2} \approx 1.1 \text{ mV}$$

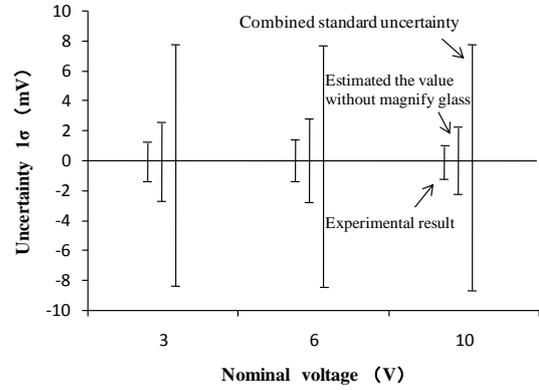
Results of other calculations are shown as follows:

$$u_1 \approx 1.4 \text{ mV for } 6 \text{ V,}$$

$$u_1 \approx 1.3 \text{ mV for } 3 \text{ V}$$

From the above results, it is confirmed that the uncertainty

does not depend on the position of scale. In addition, uncertainties obtained from the results of ANOVA (1.1 mV, 1.4 mV and 1.3 mV), correspond to about 1/6 compared with the uncertainty of empirical value (8.16 mV). However, we used a magnify glass that can be expanded to about double in the experiment. Therefore, uncertainties are estimated to be from 2.2 mV to 2.8 mV (see **Figure 7**).



**Figure 7 Comparison of sizes of uncertainty**

## 7. CONCLUSION

We are using the value of the uncertainty obtained by the past experience usefully, but it was not corroborated up to now. This time, we tried to make a comparison of the past experienced value and the ANOVA of the experimental value.

1. The source of uncertainty dominant is alignment of the instrument and the eyes, it is about 1/5 of scale division based on experience.

2. In the present experiment, two way layout nested design was effectively introduced.

3. The experimental three components are observers, reproducibility and repetition.

4. It can be considered that the result of ANOVA become 1/6 of the past experience, and become 1/3 of it without magnify glass.

5. Therefore, the experimental value may increase the value of experience when calibrating a lot of instruments on our business.

In summary, ANOVA is very useful, and at same time, the experimental results can be used for the evaluation of the uncertainty of past experience. Therefore, we concluded that uncertainty of past experience uncertainty is reasonable and useful for our usual calibration.

## 8. REFERENCES

- [1] JIS C 1102:2007 Direct acting indicating analogue electrical measuring instruments and their accessories
- [2] IEC 60051-1 ed5.0:1997 Direct acting indicating analogue electrical measuring instruments and their accessories