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APPARENT POWER ESTIMATION BY INTERPOLATION OF THE PRODUCT OF THE DFT COEFFICIENTS

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Abstract: This paper describes the possibility of estimating the apparent power of a signal component using the interpolation of the products of amplitude discrete Fourier transform (DFT) coefficients. As with energy-based approach for the amplitude square estimation of the one-channel signal, the apparent power in the two-channel case can be estimated with the multiplied amplitude DFT coefficients around the component peak and a suitable interpolation algorithm. The use of the Hann window with well known frequency spectrum and the largest local amplitude DFT coefficients give lower systematic errors than the energy-based approach, although the frequency has to be estimated first.

Keywords: non-coherent sampling, apparent power estimation, leakage effect, interpolated DFT, Hann window.

1. INTRODUCTION

The measurements performed on electrical power systems frequently concern the disturbances existing in the supply voltage, or in the current absorbed by an electrical plant, when the ideal conditions are not complied with. The widespread use of non-linear solid-state devices in electrical and electronic appliances is the main cause of harmonic and interharmonic distortions in voltages and currents. In the last decade, considerable research has been carried out on the analysis of efficient methods capable of accurately estimating the power parameters of the frequency components of interest [1-3]. The adopted estimation procedures can be classified as either time-domain (parametric) [4] or frequency-domain (nonparametric) methods [4-5]. Parametric procedures are model-based and require computationally intensive algorithms to determine the coefficients of the model that fits the available data. On the other hand, the model order issue does not apply when using nonparametric techniques, which estimate the parameters of interest by first evaluating the DFT of the digitized signal and then the suitable parameter of each spectral tone. The main drawback of the frequency-domain methods is the well-known leakage effect due to non-coherent sampling [4]. Assuming non-coherency in the sampling process, spectral granularity and leakage may adversely affect the accuracy of the estimation process. To cope with such issues, various methods have been devised for the accurate estimation of tone amplitudes and their squares, and among them two main methods are frequently used: the energy-based method (EBM) estimates the power in each spectral component by evaluating the total energy falling inside a band including the window main lobe [6], and the interpolated DFT estimates the amplitude of the spectral lines of interest using two or more neighboring DFT coefficients, starting from those centered in each spectrum's local maximum [7-8]. While the energy-based method is a more intuitive technique and only needs generic window specifications, the interpolated DFT requires more calculations and a thorough knowledge of spectral window behavior but performs with a reduced systematic bias error as it is shown in this paper.

The interpolated DFT estimation procedure of the apparent power can be improved by interpolation – suitable summation – of the multiplied amplitude DFT coefficients around the component investigated. It is based on smoothing sampled data by windowing, prior to their numeric integration, and then averaging the amplitude DFT coefficients to reduce the leakage effects. In the numerical-based wattmeters, samples of two channels (voltage $u(k)$ and current $i(k)$) are acquired simultaneously and equally spaced. The active power is usually estimated by averaging of the instantaneous power $p(k)=u(k)\cdot i(k)$ multiplied by suitable window weights $w(k)$ to reduce the leakage effect:

$$\overline{P}=\frac{1}{W(0)}\sum_{k=0}^{n-1}w(k)\cdot u(k)\cdot i(k) \qquad W(0)=\sum_{k=0}^{n-1}w(k) \tag{1}$$

where $W(0)$ is the window gain and for the rectangular window is equal to $W(0)=n$. Quantity $\overline{P}$ represents the average active power with contributions of all $M+1$ frequency components

$$\overline{P}=U_0 I_0+\sum_{m=1}^{M}U_m I_m\cos(\varphi_{m,u}-\varphi_{m,i}) \tag{2}$$

where U_0 and I_0 represent the DC voltage and current components and $U_m=A_{m,u}\big/\sqrt{2}$ and $I_m=A_{m,i}\big/\sqrt{2}$ are the effective values of the voltage and current frequency components. For better understanding what is happen in the power system the estimation of the particular power component $U_m I_m \cos(\varphi_{m,u} - \varphi_{m,i})$ is very important. It can be noticed that three quantities have to be estimated: two amplitudes and their product, and the phase difference. The last one can be estimated without knowing the frequency if the simultaneousness of the sampling on both channels is assumed, and the measurement time of the signals is the same [9]. However, the problem remains of the apparent power estimation of the particular component $S_m = U_m I_m$.

2. INTERPOLATED DFT METHOD

The estimations of the component apparent power has two main possibilities: estimation of the component amplitude $A_{m,u}$ or $A_{m,i}$ by interpolation of the DFT [7] and after that multiplying of the results $U_m I_m$, and the second approach is by interpolation of the multiplied amplitude DFT coefficients $|U(*)| \cdot |I(*)|$. Both approaches need the estimation of the displacement first and a window with well defined spectrum.

For one channel, the sampled analog multi-frequency signal $g(t)$ can be written as follows:

$$g(k\Delta t)_N = w(k) \cdot \sum_{m=0}^{M} A_m \sin(2\pi f_m k\Delta t + \varphi_m). \qquad (3)$$

Using N samples of signal (3), the DFT at the spectral line i is given by

$$G(i) = -\frac{j}{2}\sum_{m=0}^{M} A_m \left[W(i-\theta_m)e^{j\varphi_m} - W(i+\theta_m)e^{-j\varphi_m}\right], \qquad (4)$$

where $W(*)$ is the spectrum of the adopted window and θ_m is the signal frequency divided by the frequency resolution of the time window $\Delta f = 1/(N\Delta t)$ and can be written in two parts:

$$\theta_m = \frac{f_m}{\Delta f} = i_m + \delta_m \qquad -0.5 < \delta_m \le 0.5, \qquad (5)$$

where i_m is an integer value, and the displacement term δ_m is caused by non-coherent sampling.

When the Hann window is used, for which the spectrum $|W_{\mathrm{H}}(\theta)|_{N>>1} = |\sin(\pi\theta)| / |2\pi\theta(1-\theta^2)|$ is analytically known, all three coefficients of the maximum have the same sign (the main lobe is extended to four frequency resolution intervals $4\Delta f$) and they can be expressed as:

$$|W_{\mathrm{H}}(\delta_m)| \cong \frac{\sin(\pi\delta_m)}{2\pi\delta_m(1-\delta_m^2)}, \qquad (6)$$

$$|W_{\mathrm{H}}(1+\delta_m)| \cong \frac{\sin(\pi\delta_m)}{2\pi\delta_m(1+\delta_m)(2+\delta_m)}, \qquad (7)$$

$$|W_{\mathrm{H}}(1-\delta_m)| \cong \frac{\sin(\pi\delta_m)}{2\pi\delta_m(1-\delta_m)(2-\delta_m)}. \qquad (8)$$

For the five-point estimations, it is also useful to express the next nearby amplitude DFT coefficients:

$$|W_{\mathrm{H}}(2+\delta_m)| \cong \frac{-\sin(\pi\delta_m)}{2\pi(1+\delta_m)(2+\delta_m)(3+\delta_m)} \qquad (9)$$

$$|W_{\mathrm{H}}(2-\delta_m)| \cong \frac{\sin(\pi\delta_m)}{2\pi(1-\delta_m)(2-\delta_m)(3-\delta_m)}. \qquad (10)$$

The displacement term can be expressed as a function of the quotient of the amplitude coefficients by the interpolated DFT [7]:

$$\delta_{m\mathrm{H}} \cong 2 \cdot \frac{(|G(i_m+1)| - |G(i_m-1)|)}{|G(i_m-1)| + 2|G(i_m)| + |G(i_m+1)|}. \qquad (11)$$

As in the EBM approach, the amplitude product can be estimated with the multiplied amplitude DFT coefficients and the use of a suitable interpolation algorithm. For interpolation we need at least two amplitude DFT coefficients (the largest and the second largest local DFT coefficients):

$$|U(i_m)| = \frac{A_{m,u}}{2}|W(\delta_m)| \pm |\Delta_u(i_m)|, \qquad (12)$$

$$|I(i_m)| = \frac{A_{m,i}}{2}|W(\delta_m)| \pm |\Delta_i(i_m)|, \qquad (13)$$

$$|U(i_m+s)| = \frac{A_{m,u}}{2}|W(1-\delta_m)| \mp |\Delta_u(i_m+s)|, \qquad (14)$$

$$|I(i_m+s)| = \frac{A_{m,i}}{2}|W(1-\delta_m)| \mp |\Delta_i(i_m+s)|, \qquad (15)$$

where $|\Delta_{u,i}(*)|$ are the leakage errors and s is the sign of displacement $\delta_m \ge 0 \rightarrow s = +1$, $\delta_m < 0 \rightarrow s = -1$.

Multiplying and summing the coefficients give the following expressions:

$$|U(i_m)| \cdot |I(i_m)| = \frac{A_{m,u}A_{m,i}}{4}|W(\delta_m)|^2 \pm \frac{|W(\delta_m)|}{2}\left(A_{m,u}|\Delta_i(i_m)| + A_{m,i}|\Delta_u(i_m)|\right) + |\Delta_i(i_m)| \cdot |\Delta_u(i_m)| \qquad (16)$$

$$|U(i_m+s)| \cdot |I(i_m+s)| = \frac{A_{m,u}A_{m,i}}{4}|W(1-\delta_m)|^2 \mp \frac{|W(1-\delta_m)|}{2}\left(A_{m,u}|\Delta_i(i_m+s)| + A_{m,i}|\Delta_u(i_m+s)|\right) + |\Delta_i(i_m+s)| \cdot |\Delta_u(i_m+s)| \qquad (17)$$

$$\left|U(i_m)\right|\cdot\left|I(i_m)\right|+\left|U(i_m+s)\right|\cdot\left|I(i_m+s)\right|=$$
$$=\frac{A_{m,u}A_{m,i}}{4}\left(\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2\right)$$
$$\pm\frac{A_{m,u}}{2}\left(\left|W(\delta_m)\right|\left|\Delta_i(i_m)\right|-\left|W(1-\delta_m)\right|\left|\Delta_i(i_m+s)\right|\right)$$
$$\pm\frac{A_{m,i}}{2}\left(\left|W(\delta_m)\right|\left|\Delta_u(i_m)\right|-\left|W(1-\delta_m)\right|\left|\Delta_u(i_m+s)\right|\right)$$
$$\left|\Delta_i(i_m)\right|\cdot\left|\Delta_u(i_m)\right|+\left|\Delta_i(i_m+s)\right|\cdot\left|\Delta_u(i_m+s)\right| \quad (18)$$

Since the long-range leakage contributions are negligible $\left|\Delta_i(*)\right|\cdot\left|\Delta_u(*)\right| << A_{m,u}A_{m,i}/4\cdot\left(\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2\right)$ and the difference in the second part of (18) is also very small $\left(\left|W(\delta_m)\right|\left|\Delta(i_m)\right|-\left|W(1-\delta_m)\right|\left|\Delta(i_m+s)\right|\right)<<1$, the component apparent power can be estimated by:

$$\frac{A_{m,u}A_{m,i}}{2}=U_mI_m=2\frac{\left|U(i_m)\right|\cdot\left|I(i_m)\right|+\left|U(i_m+s)\right|\cdot\left|I(i_m+s)\right|}{\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2}$$

$$U_mI_m=\frac{2}{\left|W(\delta_m)\right|^2}\frac{\left|U(i_m)\right|\cdot\left|I(i_m)\right|+\left|U(i_m+s)\right|\cdot\left|I(i_m+s)\right|}{1+\left(\dfrac{\left|W(1-\delta_m)\right|}{\left|W(\delta_m)\right|}\right)^2} \quad (19)$$

Using the Hann window (equations (6) and (8)) the quotient of the used coefficients gives

$$\frac{\left|W_{\mathrm{H}}(1-\delta_m)\right|}{\left|W_{\mathrm{H}}(\delta_m)\right|}=\frac{1+\delta_m}{2-\delta_m} \quad (20)$$

and from here:

$$U_mI_m=\frac{2}{\left|W(\delta_m)\right|^2}\frac{\left|U(i_m)\right|\cdot\left|I(i_m)\right|+\left|U(i_m+s)\right|\cdot\left|I(i_m+s)\right|}{1+\left(\dfrac{1+\delta_m}{2-\delta_m}\right)^2}. \quad (21)$$

Increasing the number of the amplitude DFT coefficients used improves the apparent power estimation. For the three-point estimation, the three largest local amplitude DFT coefficients should be used:

$$\left|U(i_m-1)\right|\cdot\left|I(i_m-1)\right|=\frac{A_{m,u}A_{m,i}}{4}\left|W(1+\delta_m)\right|^2$$
$$\mp\frac{\left|W(1+\delta_m)\right|}{2}\left(A_{m,u}\left|\Delta_i(i_m-1)\right|+A_{m,i}\left|\Delta_u(i_m-1)\right|\right)$$
$$+\left|\Delta_i(i_m-1)\right|\cdot\left|\Delta_u(i_m-1)\right| \quad (22)$$

$$\left|U(i_m+1)\right|\cdot\left|I(i_m+1)\right|=\frac{A_{m,u}A_{m,i}}{4}\left|W(1-\delta_m)\right|^2$$
$$\mp\frac{\left|W(1-\delta_m)\right|}{2}\left(A_{m,u}\left|\Delta_i(i_m+1)\right|+A_{m,i}\left|\Delta_u(i_m+1)\right|\right)$$
$$+\left|\Delta_i(i_m+1)\right|\cdot\left|\Delta_u(i_m+1)\right| \quad (23)$$

Summation of the multiplied coefficients at the same spectral bins

$$\left|U(i_m-1)\right|\cdot\left|I(i_m-1)\right|+\left|U(i_m)\right|\cdot\left|I(i_m)\right|+\left|U(i_m+1)\right|\cdot\left|I(i_m+1)\right|=$$
$$=\frac{A_{m,u}A_{m,i}}{4}\left(\left|W(1+\delta_m)\right|^2+\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2\right)$$
$$\pm\frac{A_{m,u}}{2}\left(-\left|W(1+\delta_m)\right|\left|\Delta_i(i_m-1)\right|+\left|W(\delta_m)\right|\left|\Delta_i(i_m)\right|-\left|W(1-\delta_m)\right|\left|\Delta_i(i_m+1)\right|\right)$$
$$\pm\frac{A_{m,i}}{2}\left(-\left|W(1+\delta_m)\right|\left|\Delta_u(i_m-1)\right|+\left|W(\delta_m)\right|\left|\Delta_u(i_m)\right|-\left|W(1-\delta_m)\right|\left|\Delta_u(i_m+1)\right|\right)$$
$$+\left|\Delta_i(i_m-1)\right|\cdot\left|\Delta_u(i_m-1)\right|+\left|\Delta_i(i_m)\right|\cdot\left|\Delta_u(i_m)\right|+\left|\Delta_i(i_m+1)\right|\cdot\left|\Delta_u(i_m+1)\right| \quad (24)$$

gives the possibility to estimate the apparent power if the leakage tails are neglected $\left|\Delta_i(*)\right|\cdot\left|\Delta_u(*)\right|<<1$ and considering that the second part in (24) is close to zero $\left[-\left|W(1+\delta_m)\right|\cdot\left|\Delta(i_m-1)\right|+\left|W(\delta_m)\right|\cdot\left|\Delta(i_m)\right|-\left|W(1-\delta_m)\right|\cdot\left|\Delta(i_m+1)\right|\right]\approx 0$.

$$S_m=\frac{A_{m,u}A_{m,i}}{2}\doteq 2\frac{\left|U_{i_m-1}\right|\cdot\left|I_{i_m-1}\right|+\left|U_{i_m}\right|\cdot\left|I_{i_m}\right|+\left|U_{i_m+1}\right|\cdot\left|I_{i_m+1}\right|}{\left|W(1+\delta_m)\right|^2+\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2} \quad (25)$$

The expression of the apparent power estimation is close to the expression for the energy-based estimation of the power of the sine-wave amplitude A_m^2 [6]:

$$\frac{A_m^2}{2}=2\frac{\sum_{i=i_m-r}^{i_m+r}\left|G(i)\right|^2}{NNPG},\ r_{\mathrm{optimal}}=p, \quad (26a)$$

where $NNPG$ is the window Normalized Noise Power Gain [10], defined as

$$NNPG=\frac{1}{N}\sum_{k=0}^{N-1}w^2(k),\ NNPG_{\mathrm{Hann}}=0{,}375. \quad (26b)$$

In the interpolated DFT approach, the denominator is a suitable summation of the largest window coefficients instead of *NNPG* and the numerator is sum of the multiplied coefficients at the same spectral bins $\left|U_i\right|\cdot\left|I_i\right|$:

$$S_m=\frac{A_{m,u}A_{m,i}}{2}=2\frac{\left|U_{i_m-1}\right|\cdot\left|I_{i_m-1}\right|+\left|U_{i_m}\right|\cdot\left|I_{i_m}\right|+\left|U_{i_m+1}\right|\cdot\left|I_{i_m+1}\right|}{\left|W(1+\delta_m)\right|^2+\left|W(\delta_m)\right|^2+\left|W(1-\delta_m)\right|^2} \quad (27)$$

Using the Hann window, the quotient of the used coefficients $\left|W_{\mathrm{H}}(1+\delta_m)\right|/\left|W_{\mathrm{H}}(\delta_m)\right|$ (equations (6) and (7)) can be expressed as:

$$\frac{\left|W_{\mathrm{H}}(1+\delta_m)\right|}{\left|W_{\mathrm{H}}(\delta_m)\right|}=\frac{1-\delta_m}{2+\delta_m} \quad (28)$$

and the apparent power can be estimated without the value of the displacement sign:

$$\frac{A_{m,u}A_{m,i}}{2}=\frac{2}{\left|W(\delta_m)\right|^2}\cdot\frac{\left|U_{i_m-1}\right|\cdot\left|I_{i_m-1}\right|+\left|U_{i_m}\right|\cdot\left|I_{i_m}\right|+\left|U_{i_m+1}\right|\cdot\left|I_{i_m+1}\right|}{\left[1+\left(\dfrac{1+\delta_m}{2-\delta_m}\right)^2+\left(\dfrac{1-\delta_m}{2+\delta_m}\right)^2\right]} \quad (29)$$

The five-point estimation of the apparent power can follow the same procedure using two more DFT coefficients ((9) and (10)):

$$S_m = \frac{A_{m,u}A_{m,i}}{2} = \frac{2}{\left|W(\delta_m)\right|^2} \cdot$$

$$\cdot \frac{|U_{i_m-2}|\cdot|I_{i_m-2}| + |U_{i_m-1}|\cdot|I_{i_m-1}| + |U_{i_m}|\cdot|I_{i_m}| + |U_{i_m+1}|\cdot|I_{i_m+1}| + |U_{i_m+2}|\cdot|I_{i_m+2}|}{\left[1 + \left(\frac{1+\delta_m}{2-\delta_m}\right)^2 + \left(\frac{1-\delta_m}{2+\delta_m}\right)^2 + \left(\frac{\delta_m(1+\delta_m)}{(2-\delta_m)(3-\delta_m)}\right)^2 + \left(\frac{\delta_m(1-\delta_m)}{(2+\delta_m)(3+\delta_m)}\right)^2\right]} \qquad (30)$$

The effects of summation of the multiplied coefficients at the same spectral bins can be deduced from Fig. 1, in which the normalized deviation from the true apparent power $|e| = |S/S^* - 1|$ (S^* is the true value of the coherent sampling) has been plotted assuming the Hann window and $N = 1024$. In simulations with a single component, the angle between u ($A_u = 1$) and i ($A_1 = 1$) has been changed from -1.553 to $+1.553$ with step $\Delta\varphi = \pi/180$ and the maximal values of errors have been searched at each frequency. It can be deduced from Fig. 1 that when a number of voltage and current periods is great enough, the proposed algorithms give better results than estimations of voltage and current amplitudes by IDFT (31) first [7] and then multiplication (Fig. 1d).

$$A_m = \frac{\pi\delta_m}{\sin(\pi\delta_m)} \frac{(1-\delta_m^2)(4-\delta_m^2)}{3} \cdot \left[|G(i_m - 1)| + 2|G(i_m)| + |G(i_m + 1)|\right] \quad (31)$$

The reduction the systematic errors using (19), (29), and (30) is evident especially in the regions where the displacement is $0 < \delta_m < 0.5$.

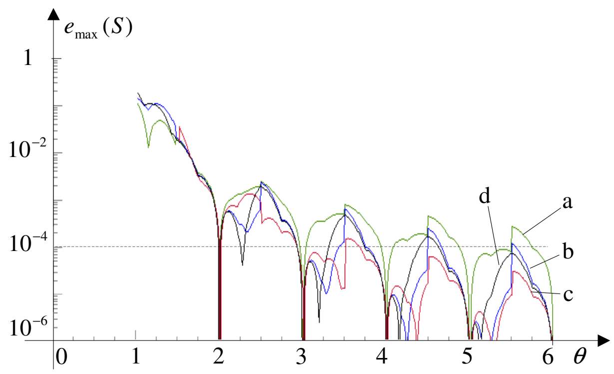

Figure 1. Maximal relative values of errors of the apparent power estimation with the multi-point DFT interpolations and the Hann window (δ is obtained with the three-point interpolation (11): a – the two-point est. (19), b – the three-point est. (29), c – the five-point est. (30), d – the three-point est. of amplitudes by (31)

The effectiveness is even further visible if we compare the proposed estimations with the five-point EBM (Fig. 2). It can be deduced that the proposed algorithms give better results than estimation with modified EB method using *NNPG* in the denominator of (30) as in (26a) (Fig. 2d).

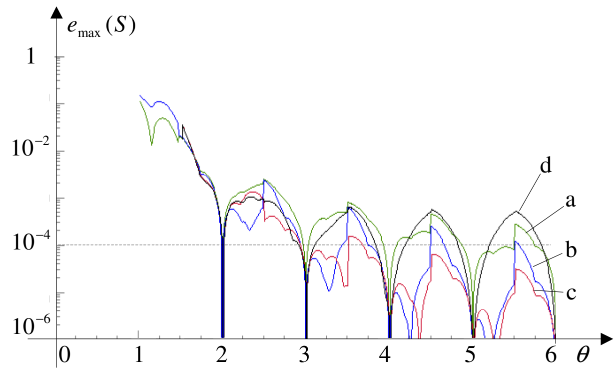

Figure 2. Maximal relative values of errors of the apparent power estimation with the multi-point DFT interpolations and the Hann window (δ is obtained with the three-point interpolation (11): a – the two-point est. (19), b – the three-point est. (29), c – the five-point est. (30), d – the five-point EBM est. using modified (30)

3. THE INFLUENCE OF NOISE ON ESTIMATIONS

The reduction of systematic errors increases the contribution of the noise random part of errors. The quantization error is a minimum that has to be taken into consideration in the measurement uncertainty of the final result. In simulations, the white noise with a rectangular distribution, zero mean and standard deviation $\sigma = A_{\text{noise}}/\sqrt{3}$ was added to signals in the time domain. The corresponding signal to noise ratios for both channels in the time domain were $SNR_i = I^2/\sigma_i^2$ and $SNR_u = U^2/\sigma_u^2$. At every test point of changing frequency and phase, as in Fig. 1, 30 trials of random added noise were used for the estimation of the apparent power standard deviation (Fig. 3: at every frequency altogether $179 \cdot 30 = 5370$ trials). Noise propagations in the algorithms were compared to the Cramér-Rao lower bound [5], [11] (Fig. 3e).

$$\sigma_{\text{CRB}} \cong \frac{S}{\sqrt{N}} \sqrt{\frac{1}{SNR_i} + \frac{1}{SNR_u}} \qquad (32)$$

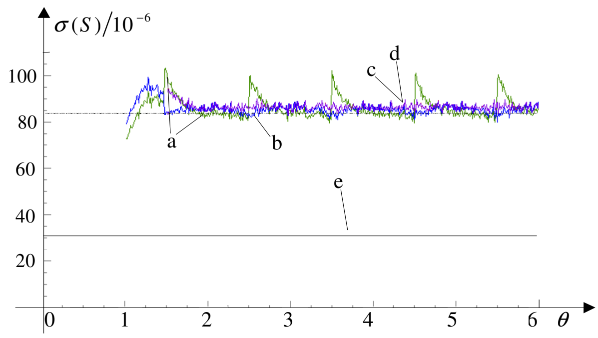

Figure 3. Standard deviations of errors of the apparent power estimation with the multi-point DFT interpolations and the Hann window (δ is obtained with the three-point interpolation (11): a – the two-point est. (19), b – the three-point est. (29), c – the five-point est. (30), d – the five-point EBM est. using modified (30), e – the CRB bound $\sigma_{\text{CRB}}(S = 0.5) = 31.25 \cdot 10^{-6}$ (32), $N = 1024$, signal: $A_u = A_i = 1$, $\sigma_u = \sigma_i = 0.001$, $SNR_u = SNR_i = 5 \cdot 10^5$.

As expected, the lowest standard deviation is with the two-point IDFT estimation (Fig. 3a). At relative frequencies larger than $\theta_m \geq 1.5$ the standard deviations are about 2.7 times larger than the Cramér-Rao lower bound (Fig. 3e).

It is also useful to analyze both contributions together (systematic and noise) searching for the maximal errors at each relative frequency (Fig. 4). In those regions where the displacement is $0 < \delta_m < 0.5$ (Fig. 4) the proposed three-point estimations using the Hann window gives better results than the energy-based method.

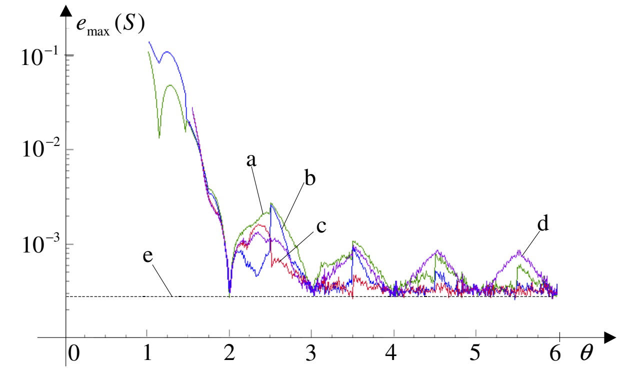

Figure 4. Maximal relative values of errors of the apparent power estimation with the multi-point DFT interpolations and the Hann window (δ is obtained with the three-point interpolation (11): a – the two-point est. (19), b – the three-point est. (29), c – the five-point est. (30), d – the five-point EBM est. using modified (30); e – the expected maximal value $\langle e_{\max} \rangle \approx 3 \cdot (2.7 \cdot \sigma_{\text{CRB}}(S)) \approx 2{,}7 \cdot 10^{-4}$.

4. EXPERIMENTAL RESULTS

The proposed algorithms were tested also with the noisy multi-component signals of the real measurement system. For the imitation of the multi-component signal, triangular shape signals were generated by the two-channel waveform generator (HP3245A: $\hat{u}_{\text{triang}} = 1\,\text{V}$, noise - $U_{\text{noise}} \approx 10\,\mu\text{V}_{\text{RMS}}$, frequency accuracy $\pm 5 \cdot 10^{-5}$) and for data acquisition two sampling voltmeters were synchronized (Agilent34411A: $U_{\text{Range}} = 1\,\text{V}$, $U_{\text{noise}} \approx 30 \cdot 10^{-6} \cdot U_{\text{Range}} = 30\,\mu\text{V}_{\text{RMS}}$, $f_{\text{sampling}} = 50\,\text{kHz}$, $N = 1000$). The testing procedure was the same as in Figs. 1 and 2. According to specifications the effective value of the noise floor in this experiment was $U_{\text{noise}} \approx 32\,\mu\text{V}_{\text{RMS}}$ per channel and consequently *SNR* for the fundamental component of voltage or current (g) in the case of triangular signal can be expressed as:

$$SNR_{g,1.\text{comp}} = G^2 / \sigma_g^2 = \frac{\left(\left(1\,\text{V} \cdot 8/\pi^2 \right) / \sqrt{2} \right)^2}{32\,\mu\text{V}} = 2{,}92 \cdot 10^7 \quad (33)$$

and also the CR lower bound can be evaluated considering $S = A_{1,u} A_{1,i} / 3 = 0.33$:

$$\sigma_{\text{CRB}} \cong \frac{S}{\sqrt{N}} \sqrt{\frac{1}{SNR_i} + \frac{1}{SNR_u}} \approx 2{,}7 \cdot 10^{-6} \qquad (34)$$

The experiment confirms the expectations also in the case of the real multi-component signals (Fig. 5). The maximal errors of the apparent power of the fundamental component were lower using the proposed three and five point estimations and give better results than the energy-based method (Fig. 5d).

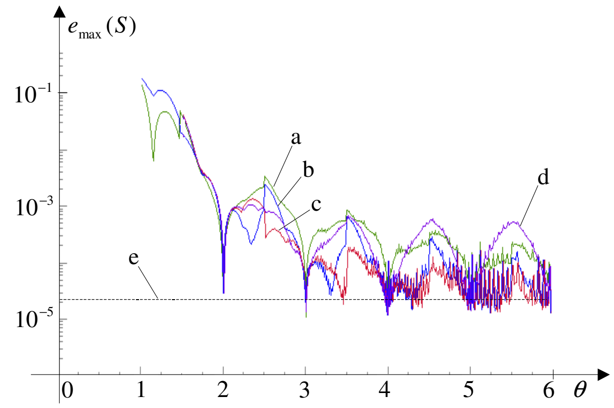

Figure 5. Maximal relative values of errors of the apparent power of the fundamental component in experiment using the triangular shape signals (δ is obtained with the three-point interpolation (11) a – the two-point est. (19), b – the three-point est. (29), c – the five-point est. (30), d – the five-point EBM est. using modified (30); e – the expected maximal value $\langle e_{\max} \rangle \approx 3 \cdot (2.7 \cdot \sigma_{\text{CRB}}(S)) \approx 2{,}2 \cdot 10^{-5}$.

4. CONCLUSION

The paper presents the possibilities of the apparent power estimation by interpolation of the products of the amplitude discrete Fourier transform (DFT) coefficients when simultaneously of sampling is anticipated. In the analyses the Hann window is used, for which the frequency spectrum is well known and has good compromise between the width of the main-lobe and the side-lobes fall-off. The interpolated multiplied DFT approach gives lower systematic error than the energy-based approach, although the frequency has to be estimated in the first step. All algorithms investigated have almost the same noise propagation.

5. REFERENCES

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