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MEASURING MULTIPOLE FIELD ERRORS IN MAGNET APERTURES OF LARGE ASPECT RATIO BY MEANS OF AN OSCILLATING WIRE ON ELLIPTIC TRAJECTORIES

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Abstract: Errors in the transversal field of accelerator magnets are usually expressed in terms of the coefficients of a harmonic expansion on a circular trajectory in the magnet's aperture.

An elliptic trajectory more suited to apertures with large width-to-height ratios allows the field inside the domain to be reconstructed with a higher accuracy. Displacing an oscillating wire along an elliptical trajectory and acquiring the amplitude of the wire oscillation at each point yield a periodic signal to be expanded into the elliptic multipoles. The paper discusses the theoretical background, the practical implementation in the measurement system, and the accuracy achieved in the measurements of a quadrupole magnet.

Keywords: Magnetic Measurements, Accelerator Magnet, Multipole field expansions.

1. INTRODUCTION

The field quality in the aperture of accelerator magnets is suitably described by a set of Fourier coefficients known as field harmonics or multipole coefficients [1]. The computation technique finds a general solution satisfying the Laplace equation in a suitable coordinate system [2]. For accelerator magnets, the boundary values for this solution are often prescribed in circular coordinates, i.e. the radial field component is measured or calculated at a reference radius, often chosen as two-thirds of the aperture radius [3]. The classical technique for normal conducting magnets involved the mapping of the field components on the magnet's central plane, whereas the rotating search coil measurements were extensively used for the round apertures in superconducting magnets [4]. This rotating search coil method is well established even for large gains and time resolutions; it requires, however, tight-tolerance fabrication and calibration of the search coils. Moreover, the measurement is influenced by vibration and movements of the rotating coil, electrical noise on the system, and angular encoder imperfections [5].

Rotating coils are less well suited for magnet apertures with large width-to-height ratios. A field expansion on arbitrary rectangular or elliptical boundaries of domains within the magnet aperture is therefore investigated for static or quasi-static two-dimensional fields. The advantage of this approach is that the expansion is valid, convergent, and accurate in a larger domain, namely, inside an ellipse circumscribed to the reference circle of multipoles in polar coordinates.

In this paper, the analytical formulation and a preliminary experimental validation of a method for measuring multipole field errors in magnet apertures of large aspect ratio by means of an oscillating wire on elliptic trajectories are presented.

2. THE OSCILLATING WIRE TECHNIQUE

Recently, a technique based on an oscillating wire was developed [6]. The wire, fed by a sinusoidal current, is positioned step-by-step on the generators of a cylindrical domain inside the magnet aperture.

The magnetic field is mapped by measuring the amplitude of the wire displacements, which depend on the wire position and the integral strength of the magnetic flux density. This technique allows measurements when rotating coils are not applicable, for example in magnets with very small aperture (less than 10 mm) as needed for future linear accelerators.

A wire fed by an electrical current moves in a magnetic field according to the Lorentz force law, under appropriate conditions of mechanical tension, elasticity, length, etc. The wire displacement, in its linear range, is proportional to the integrated transverse field. A sinusoidal current is fed to the wire, at a frequency lower than its natural vibration frequency, in order to remain in the forced motion regime. The wire is displaced step-by-step on a trajectory in the magnet. At each position k, the corresponding wire displacements are measured with the same impressed current and mechanical tension.

The wire displacement can be represented in the Cartesian plane X-Y by two components δ_x and δ_y proportional to the magnetic field components B_y and B_x, respectively [7]. Collecting the wire displacements $d_x^k(r_0)$ and $d_y^k(r_0)$ allows the calculation of the relative multipole coefficients directly. This is shown for the circular harmonics in [8].

3. ELLIPTICAL FIELD COMPONENTS

In this paper, the idea of applying the oscillating wire method to an elliptic trajectory is presented. The method of elliptical multipole measurements was developed in [9]. This will especially be useful for magnets with high aspect ratio of their aperture, or for magnets with mounted elliptical beam vacuum chambers.

As shown in Fig. 1, a system of elliptic coordinates is defined by $x = a\, cosh\eta cos\psi$ and $y = a\, sinh\eta sin\psi$, for $0 \leq \eta < \infty$ and $-\pi \leq \psi \leq \pi$, where $a = \sqrt{a_0^2 - b_0^2}$ is the distance between the origin and the focal points, for $a_0 > b_0$.

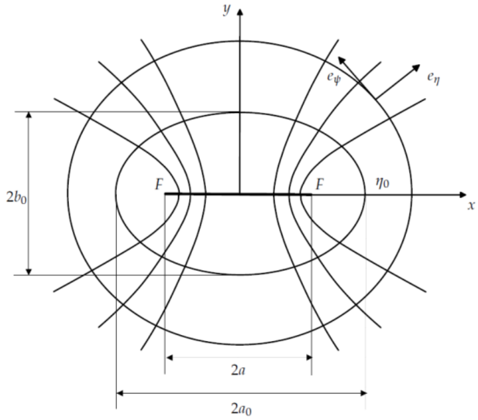

Fig.1. Elliptic coordinates.

A 2π periodic signal $\tilde{B}_\eta$ can be obtained from the Cartesian field components by:

$$\tilde{B}_\eta(\eta,\psi) = \left(a\; sinh\eta\; cos\psi\; B_x + \; a\; cosh\eta\; sin\psi\; B_y\right) \quad (1)$$

At a given reference ellipse, this signal $\tilde{B}_\eta(\eta_0,\psi)$ can be expressed as the Fourier series:

$$\tilde{B}_\eta(\eta_0,\psi) = \sum_{n=1}^{\infty}\left(\tilde{B}_n(\eta_0)\sin(n\psi) + \tilde{A}_n(\eta_0)\cos(n\psi)\right) \quad (2)$$

$\tilde{B}_n(\eta_0)$ and $\tilde{A}_n(\eta_0)$ are comparable with the coefficients calculated on a circular trajectory, but do not contain the space metric of the curvilinear coordinate system. This is the reason for the tilde notation in Eq. (1).

The magnetic field components in the entire elliptic aperture domain can then be expressed as:

$$B_\eta(\eta,\psi) = \\ = \frac{1}{h}\sum_{n=1}^{\infty}\left(\tilde{B}_n(\eta_0)\frac{\cosh(n\eta)}{\cosh(n\eta_0)}\sin(n\psi) + \tilde{A}_n(\eta_0)\frac{\sinh(n\eta)}{\sinh(n\eta_0)}\cos(n\psi)\right)$$

(3)

$$B_\psi(\eta,\psi) = \\ = \frac{1}{h}\sum_{n=1}^{\infty}\left(\tilde{B}_n(\eta_0)\frac{\sinh(n\eta)}{\cosh(n\eta_0)}\cos(n\psi) - \tilde{A}_n(\eta_0)\frac{\cosh(n\eta)}{\sinh(n\eta_0)}\cos(n\psi)\right)$$

where $h = a\sqrt{cosh^2\eta - cos^2\psi}$ are the metric coefficients of the elliptic coordinate system. The Cartesian components $B_x(\eta,\psi)$ and $B_y(\eta,\psi)$ are calculated from these by:

$$B_x(\eta,\psi) = H\left(sinh\eta\;\; cos\psi B_\eta(\eta,\psi) - cosh\,\eta\; sin\psi\; B_\psi(\eta,\psi)\right)$$

$$B_y(\eta,\psi) = H\left(cosh\eta\;\; sin\psi B_\eta + sinh\,\eta\; cos\psi\; B_\psi(\eta,\psi)\right) \quad (4)$$

where H is:

$$H = \frac{\sqrt{sinh^2\eta + sin^2\psi}}{cosh^2\eta\; sin^2\psi + cos^2\psi\; sinh^2\eta} \quad (5)$$

Using this method, the field in the entire elliptical domain can be reconstituted from the measurement of the field components along the reference ellipse.

4. EXPERIMENTAL RESULTS

The experimental validation of the proposed method was carried out in a large-aperture quadrupole in order to compare the accuracy with the inscribed and circumscribed circular trajectories. In any case, the method will work also in cases where an elliptical beam-screen restricts the access to the field domain.

In Figs. 2, the two Cartesian components of the wire oscillations, acquired by two perpendicular optocouplers mounted at the stages of the system and measuring the deflection of the oscillating wire, are shown.

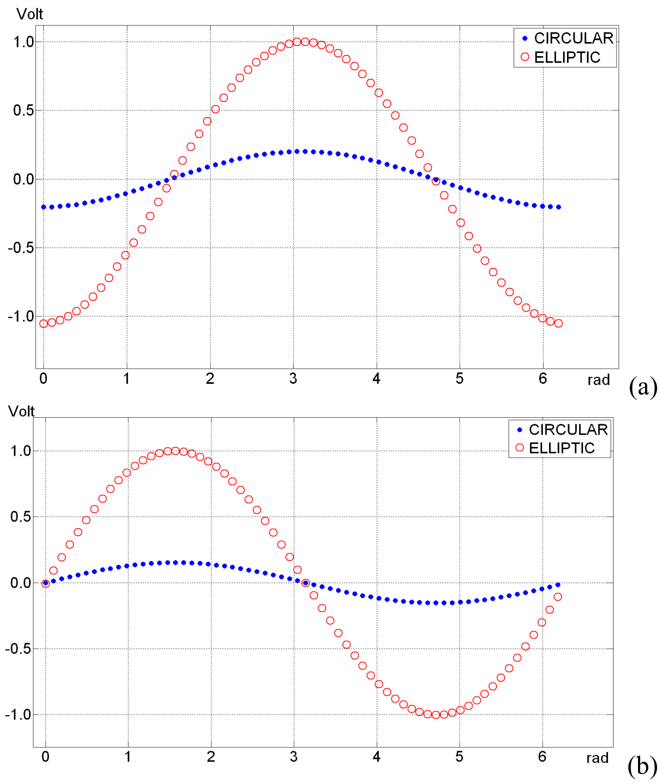

Fig. 2: 64 signals from sensor in x (a) and y (b) directions, for circular (radius 32 mm) and elliptic (a_0 = 45 mm and b_0 = 32 mm) trajectory.

It can be seen that the maximum amplitude of the oscillation is larger for the elliptical trajectory, because the field in the quadrupole increases linearly away from the centre. The voltage signals proportional to the field components are then combined into the 2π period signal (1).

The method was validated by analysing the Cartesian field component on the mid-plane of the magnet inside the reference trajectory. For the circular trajectory, in Fig. 3, it can be seen that the field components reconstructed from the Fourier coefficients diverge outside the reference trajectory. This is due to the finite sample number and the accuracy of the measurement of these coefficients.

In Fig. 4, the y-components reconstructed from the Fourier coefficients obtained from the circular (blue) and elliptic (black) trajectories are compared. Although the trajectories are restricted to $y < 32$ mm in both the cases, the elliptical trajectory yields a higher accuracy in the region of $x > 32$ mm, such as expected.

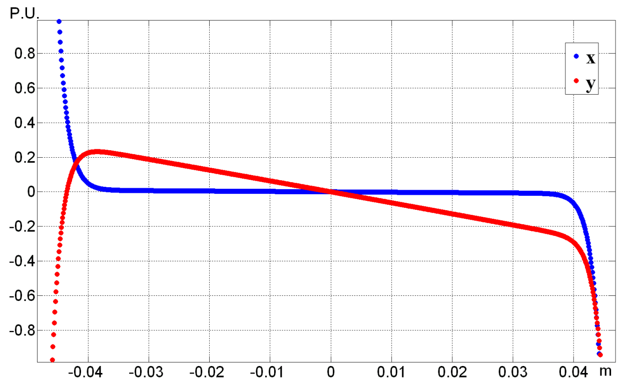

Fig. 3: relative field reconstructed for circular trajectory.

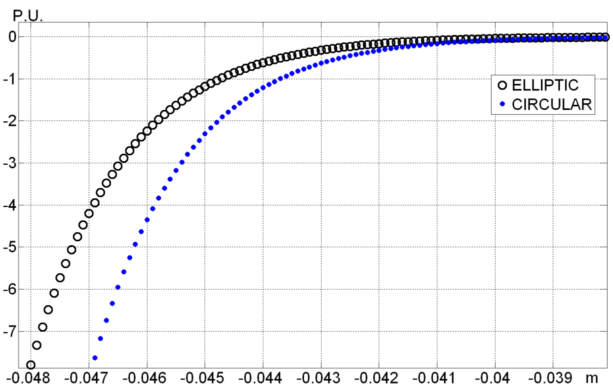

Fig. 4: relative field reconstructed outside of measured domain in both elliptic and circular trajectory.

5. CONCLUSION

The measurement system employing the oscillation wire technique has the intrinsic advantage that different wire trajectories can be traced without hardware modification. This flexibility was demonstrated with the field reconstruction from Fourier coefficients obtained from the field components measured along a reference ellipse. The results were compared to the classical analysis along a circular trajectory.

In both the cases, the expansion coefficients of the complex field can be computed from the field given along the reference curve. As the ellipse covers a larger area in the gap, the area of the convergence of the Fourier series is also larger.

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