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# DETERMINATION OF MODEL PARAMETERS FOR A DYNAMIC TORQUE CALIBRATION DEVICE

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**Abstract:** For the dynamic calibration of torque transducers, a calibration device has been developed. This paper describes the model of the measuring device and methods for the determination of its model parameters. The modelling of the calibration set-up is required for the identification of the corresponding model parameters of the torque transducer under test. These parameters describe the transducer's dynamic behaviour. Measurement methods and devices for the determination of mass moment of inertia and torsional stiffness are explained. This research is part of EMRP JRP IND09 - “Traceable Dynamic Measurement of Mechanical Quantities”.

**Keywords:** Dynamic torque calibration, mass moment of inertia, torsional stiffness, sinusoidal excitation, modelling of mechanical systems.

## 1. INTRODUCTION

Dynamic torque applications exist in many fields of industry, e.g. engine test stands or power torque tools. Efficiency measurement of combustion engines and electric drives are carried out by means of rotational speed and dynamic torque measurements in engine test stands. From experience with force transducers it is known that transducers may have a dynamic behaviour.

## 2. MEASURING DEVICE

At present, static torque calibrations can be carried out in a wide range and with high precision, but there are no facilities and standards to determine the dynamic behaviour of torque transducers. For this reason, a method for the primary dynamic torque calibration was developed and a proof of principle calibration device was manufactured. This device enables dynamic torque excitations in a frequency range of up to 1 kHz with up to 20 N·m sinusoidal torque. The measurement principle is in analogy to periodic force excitation based on the Newton's second law [1].

$$M(t) = \ddot{\varphi}(t) \cdot J \qquad (1)$$

Figure 1 shows the components of the calibration device and the corresponding mechanical spring-mass-damper model. The transducer under test is mounted via coupling elements to a rotational exciter. The coupling elements are composed of an upper and lower half which are connected by a steel diaphragm to reduce parasitic bending moments acting on the transducer. At the same time, they offer a high torsional stiffness. Interchangeable collets enable the mounting of transducers with shaft ends of various diameters in the measuring device. At the top, the transducer carries a known mass moment of inertia (MMOI) $J$, which is composed of components for an angular acceleration measurement set-up and a radial air bearing. The primary measurement of the angular acceleration $\ddot{\varphi}(t)$ is carried out by means of a radial grating disk and a laser-Doppler interferometer [1].

## 3. MODEL

The measurement principle of a vast majority of torque transducers is based on strain gauges. This principle is used for force transducers as well, so the implementation of a model for torque transducers was realised basing on the experience with force transducers [2].

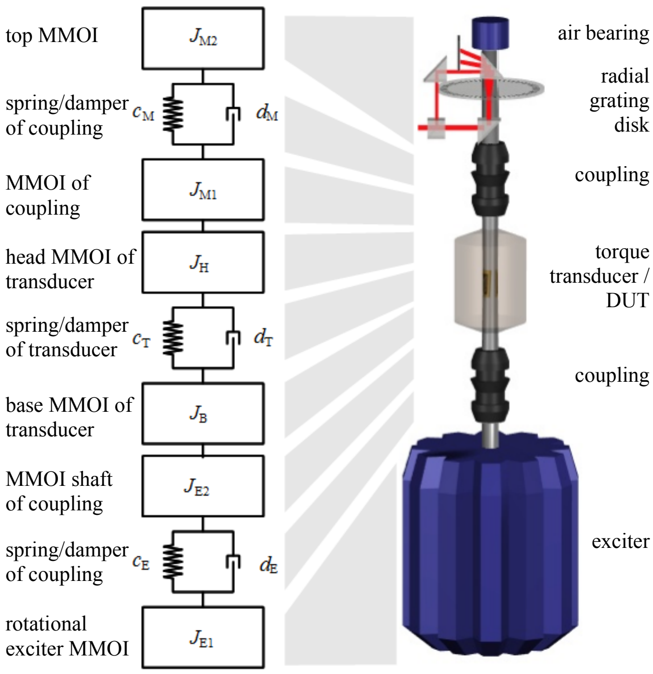


Fig. 1: Measuring device and its model.

The transducer is modelled as a linear mass-spring-damper system (see Figure 1). It consists of two mass moment of inertia elements ($J_H$, $J_B$) coupled by a torsional spring ($c_T$) and a damper ($d_T$).

The distribution of the head and base MMOI of the transducer under test ($J_H$ and $J_B$) is dependent on the mechanical design of the transducer.

Because of the fact that torque transducers are always coupled to the mechanical environment at both ends, different coupled mass moments of inertia of different stiffness and damping may have influence on the frequency response of the system, which affects the frequency-dependent output signal of the transducer. To be able to identify the model parameters of the transducer, it is necessary to include the entire measurement set-up in the model.

The measuring device is coupled to both ends of the transducer, as is its model. It is assumed to be linear and time invariant (LTI), as is the transducer's model.

As the coupling elements cannot be assumed to be totally rigid to torsional loads, these are represented in the model of the measuring device by two torsional spring and damper elements which are coupled to mass moments of inertia. Rigid connections are divided into properties of the transducer and properties of the measuring device (e.g. $J_{E2}$ and $J_B$ in Figure 1) in order to be able to determine solely the model parameters of the transducer.

The mass moment of inertia $J_{M2}$ includes all components of the measuring device on top of the upper half of the coupling. $J_{M1}$ includes the lower part of the coupling, as well as adapters for mounting the transducer, the same applies to $J_{E2}$ but in opposite direction.

Only if the dynamic properties of the measuring device are known, the determination of the transducer's model parameters from measurement data would be possible. Measurement techniques for the determination of these parameters have to be developed.

## 4. MASS MOMENT OF INERTIA

The mass moment of inertia that generates the dynamic torque acting on the transducer, when applied to an angular acceleration, needs to be determined with high precision.

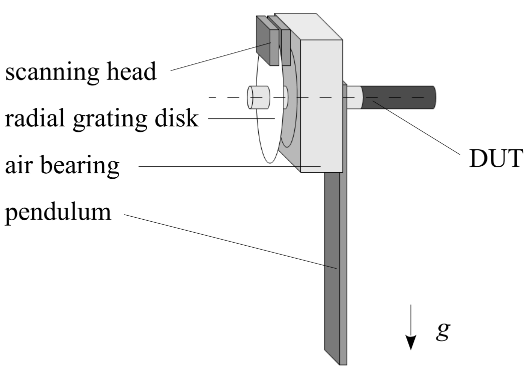


Fig. 2: Pendulum for the measurement of the mass moment of inertia.

For this purpose, a dedicated measuring device for mass moment of inertia has been developed (see Figure 2). Its measurement method is based on the principle of a physical pendulum. For small angles of pendulum excitation, the non-linearity of the pendulum can be neglected. Assuming harmonic oscillations, the frequency of the pendulum's swing depends on its mass moment of inertia.

$$J = \frac{m \cdot g \cdot l}{\omega^2} \tag{2}$$

To minimise frictional losses, the pendulum set-up uses an air bearing [3]. The device under test (DUT) is mounted on the axis of rotation. The mass moment of inertia of the DUT is derived from the change of the pendulum's swing frequency. For the measurement of the angular excitation, an optical angle encoder is used. Its grating disk has an angular pitch of 0.04°. The angular resolution can be additionally increased by interpolation methods.

As the pendulum's mass moment of inertia is unknown, it has to be determined first. This is to be carried out by adding known mass bodies to the pendulum at different distances from the axis of rotation, i.e. adding well defined mass moments of inertia given by the Huygens-Steiner theorem.

The auxiliary mass bodies of the pendulum have well-known dimensions and weight. With this information, the mass moment of inertia for rotation around the centre of gravity can be calculated for each of them. Based on the measured values of the pendulum's frequency with additional MMOI (i.e. mass bodies mounted on the pendulum's lever at different distances from the axis of rotation) the unknown mass moment of inertia of the unloaded pendulum structure can be extrapolated [1].

The air bearing provides nearly undamped pendulum oscillations. To determine the damping parameter of the measuring device the decay of the pendulum's amplitude was measured.

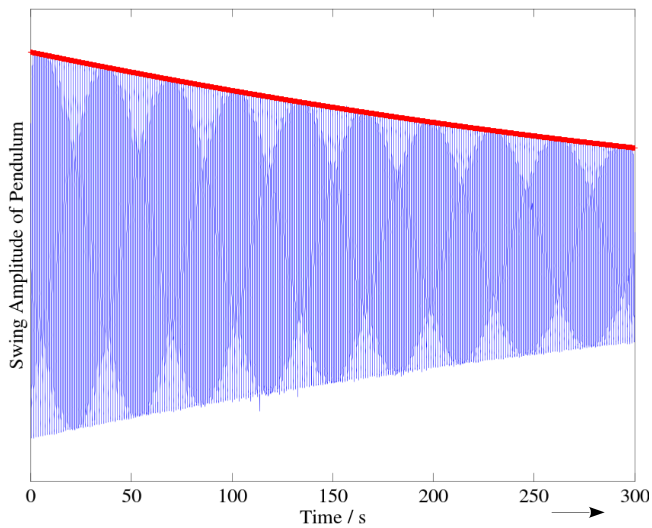


Fig. 3: Decay of the pendulum's swing amplitude.

The oscillations of the pendulum were measured by means of a laser-Doppler interferometer, as the angle measurement set-up had not yet been installed at the time of measurement. The interferometer traced the pendulum's swing at the bottom end of the pendulum.

The damping of the pendulum was determined calculating a non-linear regression approximating the measurement values with damped sine fit function.

$$y(t)=C\cdot e^{-\delta t}\cdot\sin(\omega t-\varphi) \tag{3}$$

The decay of the pendulum swing amplitude for the measurement data shown in Figure 3 is calculated and equals to satisfying values for $\delta=2.23\cdot 10^{-3}\,\mathrm{s}^{-1}$. The damping slightly influences the resonant frequency of pendulum [4]. The higher the damping, the lower is the frequency of the pendulum $\omega_1$ in comparison to the frequency of the undamped pendulum $\omega_0$. This effect has to be taken into account.

$$\omega_0=\sqrt{(\omega_1^2+\delta^2)} \tag{4}$$

For the described measurement set-up, this effect is in the range of $8\cdot 10^{-8}$.

## 5. TORSIONAL STIFFNESS

The stiffness of the components of the measuring device have great influence on the torque measurement as it affects the dynamic behaviour of the whole system. The torsional stiffness is defined as the torque-to-torsion ratio.

$$c=\frac{M}{\varphi_2-\varphi_1} \tag{5}$$

The determination of the torsional stiffness is realised by a measurement set-up (see Figure 4), in which a well-known torque is applied to the DUT while the angle of torsion is measured with high precision by means of two autocollimators.

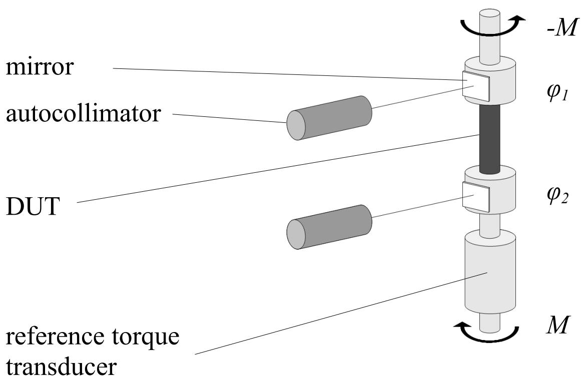


Fig. 4: Torsional stiffness measurement set-up.

Two mirrors attached with support clamps to both sides of the DUT are the sensing points of the autocollimators. The torque-dependent difference of the two angle readings of both autocollimators yields the torsional angle to be determined. The change in the angle of the mirror surface is equal to the torsion. Mirrors with a planarity of 1/10 $\lambda$ enable the determination of the angle with uncertainties of less than one arcsecond, the measurement range amounts to rotation angles of up to ± 600 arcseconds. In this measurement set-up, the statically applied torque is measured by means of a reference torque transducer.

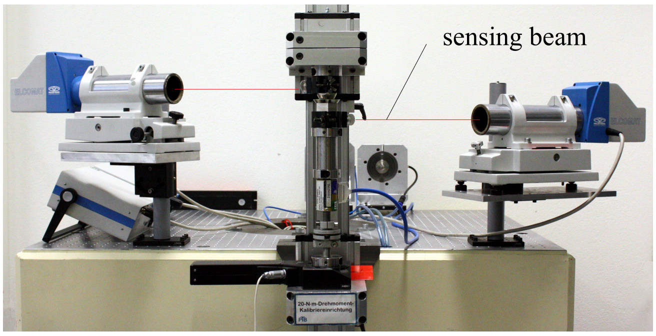


Fig. 5: Measurement set-up for torsional stiffness, indicated sensing beams of autocollimators (red).

To measure the torsional stiffness, PTB's 20 N·m Torque Calibration Machine was equipped with two electronic autocollimators in order to perform measurements of the torsional angle. Figure 5 shows photograph of the used set-up. In the centre, the Torque Calibration Machine is located, on the right is the autocollimator tracing the bottom mirror, on the left, the autocollimator sensing at the top mirror.

A measurement routine was developed for the evaluation of the torsional stiffness. The applied load steps are based on the DIN 51309 standard for torque transducer calibrations. After preloading to avoid hysteresis behaviour in the torque calibration system, the applied torque (load) increases in steps of 10% to the full load. The number of load steps shown in Figure 6 was increased in comparison to the standard document. For the measurement, both clockwise and counter-clockwise torsional loading cycle was applied.

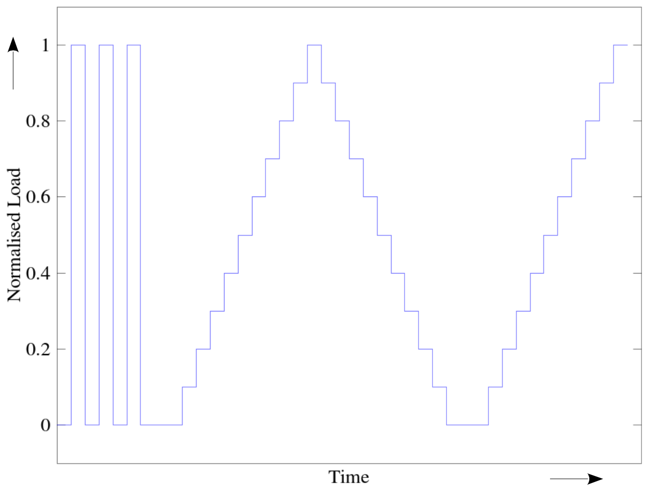


Fig. 6: Load steps of the torsional stiffness measurement routine.

The voltage output of the bridge amplifier connected to the reference torque transducer was acquired simultaneously with the angle values of the two autocollimators. Several hundred measurement values were recorded for each torque step.

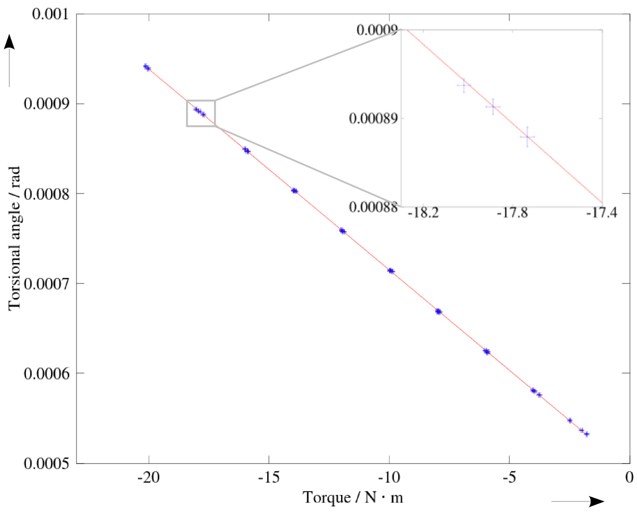


Fig. 7: Measurement result with standard deviation (blue) and regression line (red).

The first measurement results are very encouraging, as it is seen in Figure 7. The torsional angle values for the different torque steps show a linear dependency. The first order regression line excellently fits the mean values of the different torque levels. The value for torsional stiffness results from the gradient of the regression line. The angle offset is given by the initial absolute angle values of the autocollimators.

## 6. CONCLUSIONS

This paper describes the modelling and the determination of the model parameters of the dynamic torque calibration device at PTB. The modelling of the measuring device is a prerequisite to be able to determine the transducer's dynamic properties.

The described methods enable the measurement of the torsional stiffness and the mass moment of inertia. The torsional stiffness is determined by applying a well-known torque and by measuring the torsion angle on both sides of the DUT. The mass moment of inertia is determined by means of the swing frequency of a compound pendulum. It is intended to determine the model parameters of the components of the dynamic torque calibration device with this measurement methods.

For the future model based calibration of torque transducers, the parameters of the measuring device need to be known, in order to be able to identify the model parameters of the torque transducer under test from measurement data.

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