XX IMEKO World Congress
Metrology for Green Growth
September 9–14, 2012, Busan, Republic of Korea

INVESTIGATION ON THE IMPEDANCE-FREQUENCY-RESPONSE FOR A DYNAMIC BEHAVIOUR DESCRIPTION OF ELECROMAGNETIC FORCE COMPENSATED LOAD-CELLS

F.Hilbrunner[1], *H. Weis*[2], *R. Petzold*[2], *T. Fröhlich*[2], *G. Jäger*[2]

[1] Sartorius Weighing Technology GmbH, Göttingen, Germany,
[2] Institute for Process Measurement and Sensor Technology,Technische Universität Ilmenau, Ilmenau, Germany,
falko.hilbrunner@sartorius.com

Abstract: In this paper, a method for deriving significant parameters for the description of the dynamic behaviour of electromagnetic force compensated load-cells (EMC) from its impedance-frequency-response is presented. With these parameters, the dynamical behaviour of a load-cell can be depicted and classified near its main resonance frequency, or by going the inverse way the load-cell can be designed based on the favored parameters to fulfill the desired criteria concerning the dynamical behaviour. This method is based on the estimation of parameters describing the behaviour of electrodynamic speakers, called Thiele-Small-Parameters, which is state of the art for designing loudspeakers. It will be demonstrated that the controller design for the EMC load-cell can be based on the derived parameters.

Keywords: Dynamic EMC-Balance, parameter estimation, impedance-frequency-response, dynamic mass measurement, Thiele-Small-Parameters, Controller design

1. INTRODUCTION

Concerning uncertainty of measurement and achievable resolution, balances based on the principle of electromagnetic force compensation represent the state of the art.

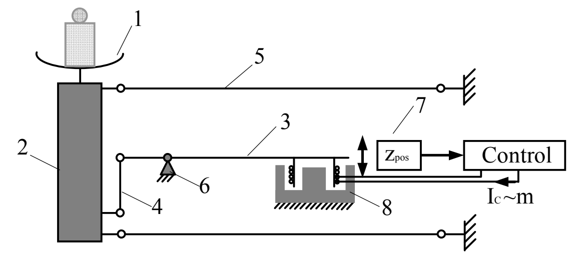


Fig. 1: Schematic setup of an EMC balance

In fig. 1, the schematic setup of an EMC-balance is depicted with 1 – Weighing pan, 2 – Load carrier, 3 – Transmission lever, 4 – Coupling member with flexure hinges, 5 – Parallel lever system with flexure hinges, 6 – Bearing of the transmission lever, 7 – Position detector, 8 – Electrodynamic actuator.

Due to many applications of these systems in industry and research, there is a strong interest of improving their performance in terms of speed and accuracy, especially for dynamic purposes. In order to simplify the design process of a load-cell to reach the desired global characteristics in the best and fastest way, a small set of parameters describing the main dynamic properties is needed. This set of parameters may also be used to characterize existing load-cells in terms of applicability for dynamic purposes and to determine controller parameters.

From the area of acoustics and sound engineering, a method of describing the dynamic behaviour of electrodynamic speakers with a set of parameters, called Thiele-Small-Parameters (TSP) is known [1, 2, 3]. These Parameters can be derived from the characteristic curve of the impedance-frequency-response of an electrodynamic speaker, giving information on the mechanical and the electrodynamic behaviour. By interpreting the TSP, the speakers can be classified in their applicability as woofer (low frequency) or tweeter (high frequency), the mechanical, electrical and total quality factors are given, and a suitable form for a loudspeaker box can be defined.

The model applied to determine the TSP reduces an electrodynamic speaker without box to a mechanical system of spring, mass and damper coupled to an electrical series circuit of ohmic resistance and inductance. We show that an EMC load-cell may be reduced to a model of the same order. Thus a set of similar parameters describing the dynamic behaviour of an EMC load-cell can be found from its impedance-frequency-response curve.

2. THIELE-SMALL-PARAMETERS

Thiele-Small-Parameters describe the dynamic behaviour of an electrodynamic speaker near its resonance frequency. In literature, the following main parameters are introduced:

Table 1: List of Thiele-Small-Parameters (see [1, 2, 3])

Symbol	unit	Explanation
Q_{ms}	1	Mechanical quality factor of the speaker at f_s
Q_{es}	1	Electrical quality factor of the speaker at f_s

Sym-bol	unit	Explanation
Q_{ts}	1	Overall quality factor of the speaker at f_s
f_s	Hz	Resonant frequency of the driver
R	Ω	DC resistance of the voice coil
L	mH	Voice coil inductance
$l{\cdot}B$	Tm N/A	Coupling factor - Product of B-field strength in air gap and length of coil wire in B-field
M_{ms}	kg	Moved mass
R_{ms}	kg/s	Mechanical resistance of speaker suspension
C_{ms}	m/N	Compliance of speaker suspension
V_{as}	cm^3	Equivalent compliance volume
S_d	cm^2	Projected area of speaker diaphragm
V_d	cm^3	Peak displacement volume
X_{max}	mm	Maximum linear peak excursion

For load-cells, the parameters V_{as}, S_d, V_d and X_{max} can be neglected, as the moved air volume is very small. The mechanical parameters f_s, M_{ms}, R_{ms} and C_{ms} and the electrical parameters R and $l{\cdot}B$ are coupled to each other trough the quality factors with the following equations (2.1) [2]:

$$Q_{es} = \frac{R}{(l \cdot B)^2}\sqrt{\frac{M_{ms}}{C_{ms}}} \qquad Q_{ms} = \frac{1}{R_{ms}}\sqrt{\frac{M_{ms}}{C_{ms}}}$$
$$Q_{ts} = \frac{Q_{ms}Q_{es}}{Q_{ms}+Q_{es}} \tag{2.1}$$

3. MODEL DESCRIPTION

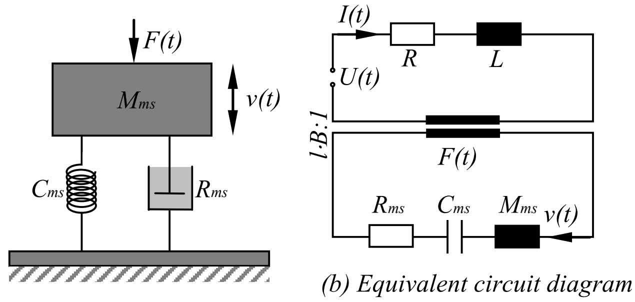
An EMC load-cell can be modelled as a system of masses, springs and dampers, as done in [4]. By reducing the model order, a simple model consisting of just one mass M_{ms}, spring C_{ms} and damper R_{ms} can be obtained [7] (fig. 2a). This model will of course provide less detail about higher order resonances of the system. By applying an electromechanical analogy [2], the system of mass, spring and damper, driven by the velocity can be transformed to an electrical circuit of inductance, capacitance and ohmic resistance. The mechanical system is bilaterally coupled to the electrodynamic actuator via the force F acting on the mechanics by applying a current I to the coil and the voltage induced U_{ind} in the coil due to the velocity v of the lever:

$$F = I \cdot l \cdot B \qquad U_{ind} = -v \cdot l \cdot B \tag{3.1}$$

The coupling factor $l{\cdot}B$ is a represents the strength of the electrodynamic actuator. The bilateral coupling of mechanical and electrical system can be reduced to this coupling factor in terms of an electromechanical analogy as an ideal transformer with turns ratio equal to $l{\cdot}B$. The electrodynamic actuator can be reduced to a series circuit of ohmic resistance and inductance [5] (see fig. 2b).

In this model, the acoustical parameters which appear for speakers were completely cancelled, as the influence of air on the dynamic behaviour of an EMC load-cell is negligible. This conclusion was drawn from a series of experiments under different pressures and vacuum where no change in the dynamic characteristics of the balances was observable.

Most EMC load-cells are set up with a lever (see fig. 1).



(a) mechanical model *(b) Equivalent circuit diagram representing a balance*

Fig. 2: reduced model for EMC load-cell

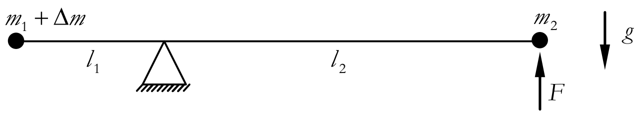


Fig. 3: Lever ratio of EMC load-cell

On one side the weighing pan (m_1) is fixed to the lever, and to the other side the voice coil (m_2) (fig. 3). Masses applied to the weighing pan can be considered as Δm. In order to condense the distributed masses to a single mass point as in [7], the geometry of the balance has to be taken into account. As the dynamic properties of the balance should be retained after the transformation of the masses, the moment of inertia has to be considered:

$$J_1 = (m_1 + \Delta m) \cdot l_1^2 \qquad J_2 = m_2 \cdot l_2^2$$
$$\rightarrow \; J = J_1 + J_2 = (m_1 + \Delta m) \cdot l_1^2 + m_2 \cdot l_2^2 \tag{3.2}$$

As only the weighing pan is accessible to the user and the applied mass Δm should not be subject of transformations to avoid confusion, the condensed mass M_{ms} should be concentrated at the weighing pan. The condensed mass considering the lever ratio $h = l_2/l_1$ then becomes:

$$J = M_{ms} \cdot l_1^2 \quad \rightarrow \quad M_{ms} = m_1 + \Delta m + m_2 \cdot h^2 \tag{3.3}$$

With (3.3), the moved mass of the EMC-balance is modelled to be acting concentrated at the position of the weighing pan.

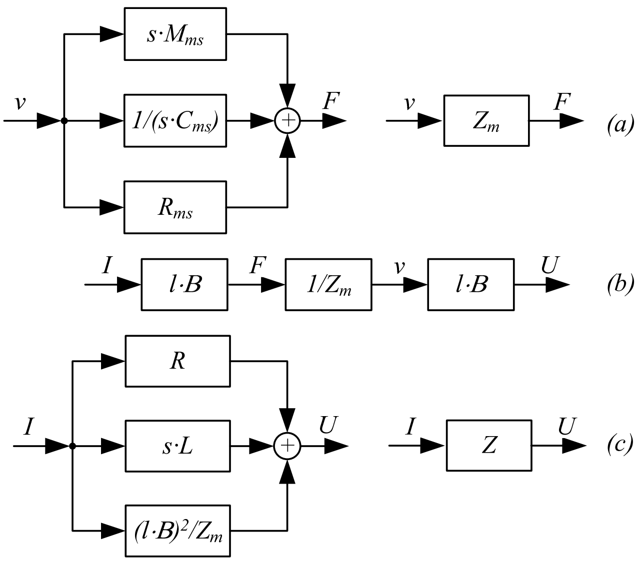


Fig. 4: System theoretical model of an EMC load-cell

The representation of the impedance of the balance in fig. 2 can be transferred to a block diagram according to system theory (fig. 4). The mechanical impedance behaviour of the System (fig. 4a) is modelled with three blocks for mass, spring and damper with the velocity v as input and a force F as output.

$$G_{v,F} = \frac{F}{v} = Z_m = s \cdot M_{ms} + R_{ms} + \frac{1}{s \cdot C_{ms}} \qquad (3.4)$$

By inverting the mechanical impedance Z_m, the force F becomes the input and the system responds with a movement (velocity) to its excitation. With (3.1), the input can be transformed to a current I, and the output to a voltage U (see fig. 4b). This way, the time derivative of the mechanical compliance is transformed to an electromechanical resistance. The transfer function of the equivalent circuit diagram in fig. 2b with input current I and output voltage U then can be reduced to a sum of ohmic, inductive and mechanical resistance (fig. 4c).

From fig. 4c a transfer function can be derived which corresponds to the impedance of the balance (3.5)

$$Z = \frac{U}{I} = R + s \cdot L + \frac{(l \cdot B)^2}{s \cdot M_{ms} + R_{ms} + \frac{1}{s \cdot C_{ms}}} \qquad (3.5)$$

4. DERIVATION OF PARAMETERS

In literature [1, 2, 3], the derivation of the TSP is introduced as fit procedure of the impedance-frequency-response. The measurement of the impedance is performed twice: The first time for the basic state, the second time with a well known additional mass on the speaker. By adding a mass, for speakers the resonance frequency f_s changes and the parameters can be determined non-ambiguously. For load-cells with high lever ratios and heavy damping, the variation of the resonance frequency becomes weak or even invisible. In order to overcome these shortcomings and improve the quality of the parameter estimation, a slightly different method of deriving the parameters from the impedance-frequency-response is presented in this paper: Instead of performing two measurements – one in the initial state and one with an additional well known mass – just one impedance-frequency-response is taken into account. The parameters from (3.5) are fitted to the measurement in the frequency range via nonlinear optimization. To retrieve unique parameters from this fit, one parameter of the mechanical part of the model has to be determined and fixed to its value in advance [5].

For balances it is possible to robustly determine the coupling factor $l{\cdot}B$ from static measurements: By applying known masses on the weighing pan and measuring the current trough the coil to return the lever to zero position, a very good estimate for $\widetilde{l{\cdot}B}$ can be derived, taking the lever ratio into account.

$$\Delta m{\cdot}g{\cdot}l_1 = l_2{\cdot}F_{Magn} = l_2{\cdot}I{\cdot}\widetilde{l{\cdot}B} \quad \rightarrow \quad \widetilde{l{\cdot}B} = \frac{\Delta m{\cdot}g}{h{\cdot}I} \qquad (4.1)$$

When the balance is treated as a black box with a pure translational mechanical part where all forces are acting on the side of the weighing pan, as depicted in fig. 2a, the unknown lever ration h has to be included into the coupling factor $l{\cdot}B$. This value then results to:

$$\Delta m{\cdot}g = F_{Magn} = I{\cdot}l{\cdot}B \quad \rightarrow \quad l{\cdot}B = \frac{\Delta m{\cdot}g}{I} \qquad (4.2)$$
$$\rightarrow \quad l{\cdot}B = h{\cdot}\widetilde{l{\cdot}B}$$

With this procedure, the model (3.5) was fitted to the impedance-frequency-response of an analytical balance BP211D provided by Sartorius [8] (see fig. 5). The model fits the impedance-frequency-response from the measurements very well. From the fit procedure, the following parameters could be derived:

Table 2: Identified TS-Parameters

Symb	[unit]	Value
Q_{ms}	[1]	0.097
Q_{es}	[1]	0.074
Q_{ts}	[1]	0.0420
f_s	[Hz]	3.4
R	[Ω]	250.67
L	[mH]	75.3
$l{\cdot}B$	[Tm] or [N/A]	198.2
M_{ms}	[kg]	0.541
R_{ms}	[kg/s]	119.0
C_{ms}	[m/N]	$4.0{\cdot}10^{-4}$

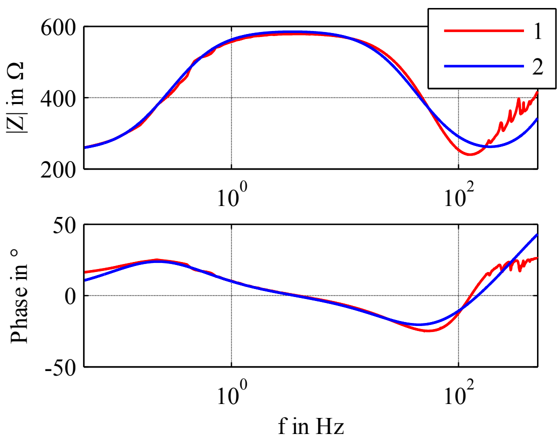


Fig. 5: measurement and fit of impedance frequency response of an EMC load-cell, 1 – measurement, 2 – simulation

5. INVESTIGATION ON THE CONTROLLABILITY OF AN EMC-BALANCE BASED ON TSP

In order to counterbalance the deflection of the lever when a mass piece is applied to the weighing pan, the force acting on the lever provoked by the current trough the coil has to be controlled. The controller of a balance receives as input the actual position x of the lever and puts out a signal to the coil which could be a voltage U or a current I. State of the art for a control loop at many balance manufacturers is to set up a voltage driving controller. The transfer function of the balance with voltage input and position output is depicted in fig. 6.

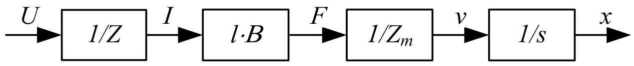


Fig. 6: voltage driven transfer function of an EMC-balance

The transfer function of these blocks results to (5.1).

$$G_{U,x} = \frac{x}{U} = \frac{1}{Z}\cdot\frac{l\cdot B}{s\cdot Z_m} = \frac{1}{Z}\cdot\frac{l\cdot B}{s^2\cdot M_{ms} + s\cdot R_{ms} + \frac{1}{C_{ms}}} \quad (5.1)$$

When instead a current driving control loop is set up, the first block of fig. 6 can be cancelled out, and the transfer function reduces to (5.2)

$$G_{I,x} = \frac{x}{I} = \frac{l\cdot B}{s\cdot Z_m} = \frac{l\cdot B}{s^2\cdot M_{ms} + s\cdot R_{ms} + \frac{1}{C_{ms}}} \quad (5.2)$$

With (5.1) and (5.2) it can be shown that for ideal voltage or current sources, a current driven control becomes faster than a voltage driven one, as the influence of the resistance Z is cancelled out. Hence for (5.2) the current acts directly on the coil, as a current response to a voltage step on a coil (5.1) is always delayed towards the voltage.

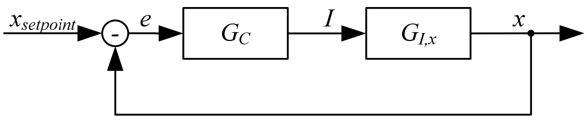


Fig. 7: setup of the control loop of an EMC-balance

For a control loop as depicted in fig. 7, the command transfer function computes to (5.3) [6]:

$$G_{R(x_{setp.},x)} = \frac{G_C\cdot G_{I,x}}{1 + G_C\cdot G_{I,x}} \quad (5.3)$$

The disturbance transfer function for disturbances at the input of the balance computes to (5.4)

$$G_{R(I,x)} = \frac{G_{I,x}}{1 + G_C\cdot G_{I,x}} \quad (5.4)$$

In most commercially available EMC-balances, a PID-controller is applied. Its transfer function computes to (5.5)

$$G_{C(PID)} = K_R\cdot\left(1 + \frac{1}{sT_n} + sT_v\right) \quad (5.5)$$

With such a controller, the command transfer function results in equation (5.6)

$$G_{R(x_{setp.},x)} = \frac{l\cdot B\cdot C_{ms}\cdot K_R\left[T_n\cdot T_v\cdot s^2 + T_n\cdot s + 1\right]}{\left[\begin{array}{l}\left(C_{ms}\cdot T_n\cdot M_{ms}\right)\cdot s^3 \\ +\left(C_{ms}\cdot R_{ms}\cdot T_n + l\cdot B\cdot C_{ms}\cdot K_R\cdot T_n\cdot T_v\right)\cdot s^2 \\ +\left(T_n + l\cdot B\cdot C_{ms}\cdot K_R\cdot T_n\right)\cdot s + l\cdot B\cdot C_{ms}\cdot K_R\end{array}\right]} \quad (5.6)$$

Consequently, the disturbance transfer function introduced in (5.4) computes for the closed loop depicted in fig. 7 to (5.7)

$$G_{R(I,x)} = \frac{\left(l\cdot B\cdot C_{ms}\cdot T_n\right)\cdot s}{\left[\begin{array}{l}\left(C_{ms}\cdot T_n\cdot M_{ms}\right)\cdot s^3 \\ +\left(C_{ms}\cdot R_{ms}\cdot T_n + l\cdot B\cdot C_{ms}\cdot K_R\cdot T_n\cdot T_v\right)\cdot s^2 \\ +\left(T_n + l\cdot B\cdot C_{ms}\cdot K_R\cdot T_n\right)\cdot s + l\cdot B\cdot C_{ms}\cdot K_R\end{array}\right]} \quad (5.7)$$

6. DESIGN OF A CONTROLLER BASED ON TSP

As already mentioned in section 5, the closed control loop for an EMC-balance consists of the controller, and the transfer function (5.2). So in order to design a controller, first of all equation (5.2) has to be determined. One possibility would be to measure it separately, in this paper we propose to calculate it from the parameters identified in section 4. To validate this approach, the transfer functions of the model and a measured one are compared to each other. For the low frequency range, there could be obtained a very good agreement of measurement and model (see fig. 8).

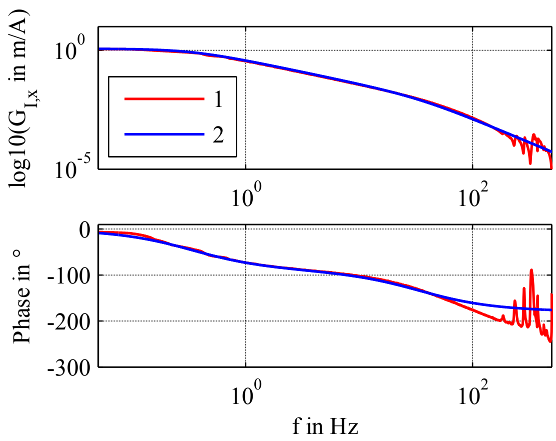


Fig. 8: Measurement of amplitude response and model calculated from parameters with: 1 – measurement, 2 – simulation

With the knowledge of this result, the controller design of a PID-controller can be based on a few parameters retrieved from the impedance-frequency-response: The time constants of the transfer function $G_{I,x}$ can be determined from (5.2), and compute to:

$$T_{1,2} = \frac{1}{\left|-\frac{R_{ms}}{2\cdot M_{ms}} \mp \sqrt{\frac{R_{ms}{}^2\cdot C_{ms} - 4M_{ms}}{4M_{ms}{}^2\cdot C_{ms}}}\right|} \quad (6.1)$$

For the investigated balance, the time constants compute to T_1 = 476.71 ms and T_2 = 4.59 ms. The open loop transfer function $G_c\cdot G_{I,x}$ results to (6.2)

$$G_C\cdot G_{I,x} = \frac{l\cdot B\cdot K_R\cdot C_{ms}}{T_n}\cdot\frac{1}{s}\cdot\frac{\left(s^2\cdot T_n\cdot T_v + s\cdot T_n + 1\right)}{\left(s^2\cdot C_{ms}\cdot M_{ms} + s\cdot C_{ms}\cdot R_{ms} + 1\right)} \quad (6.2)$$

When the controller time constants are chosen to compensate the time constants T_1 and T_2, the open loop transfer function equals to an ideal integrator:

$$T_n = \left(T_1 + T_2\right) = C_{ms}\cdot R_{ms} = 481.3\text{ ms}$$

$$T_v = \frac{T_1\cdot T_2}{\left(T_1 + T_2\right)} = \frac{M_{ms}}{R_{ms}} = 4.5\text{ ms} \quad (6.3)$$

$$\rightarrow\quad G_C\cdot G_{I,x} = \frac{l\cdot B\cdot K_R}{R_{ms}}\cdot\frac{1}{s}$$

The last free parameter to adjust is the gain K_R, with which the crossover frequency of the system can be determined. The crossover frequency of the open loop transfer function $G_c{\cdot}G_{I,x}$ can be determined from $\left|G_C{\cdot}G_{I,x}\right|=1$, and results to

$$\omega_c=\frac{l{\cdot}B{\cdot}K_R}{R_{ms}}\quad\rightarrow\quad K_R=\omega_c{\cdot}\frac{R_{ms}}{l{\cdot}B} \tag{6.4}$$

By adjusting the gain to K_R = 30.194, the crossover frequency is moved to 8Hz (see fig. 9). Hence for the closed loop command transfer function, the step response has reached its final value after 125ms.

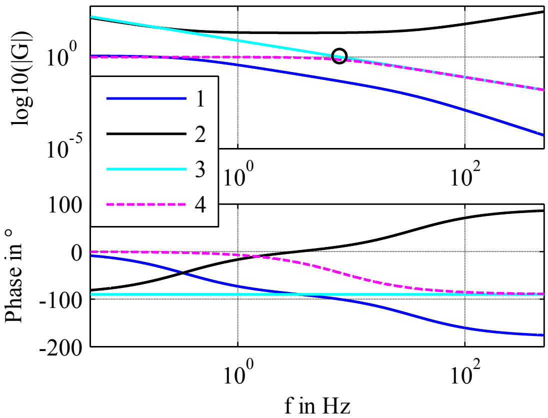


Fig. 9: Controller design based on TSP with: 1 – transfer function G_{Ix}, 2 – PID-controller, 3 – open loop of balance and controller, 4 – closed loop $G_{R(xsetp,x)}$, o – crossover frequency

In order to validate the controller design, a load step on the weighing pan was simulated and measured (see fig. 10). Both for the movement of the lever and the measurement signal calculated from the current trough the coil, a very good agreement of measurement and simulation was obtained.

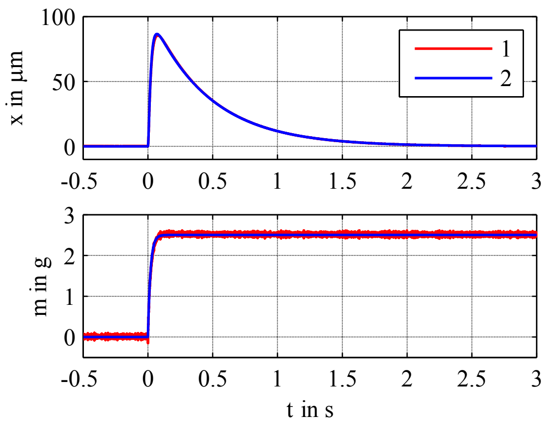


Fig. 10: step response of lever position and measurement signal (unfiltered) to a load step on the weighing pan with: 1 – measurement, 2 – simulation

7. CONCLUSION

A novel method of describing the dynamic behaviour of EMC load-cells is presented. This method involves Thiele-Small-parameters which give the opportunity to validate the applicability of a load-cell in terms of dynamic purposes. The Thiele-Small-parameters are easy to derive from two simple measurements of the coupling factor $l{\cdot}B$ and the impedance-frequency-response. From these parameters, both mechanical and electrical characteristics of the investigated system can be derived, without any manipulation.

We showed in this paper that based on the Thiele-Small-parameters a controller can be obtained fully automated from a simple algorithm. Additionally, the optimization of the design process of dynamic EMC load-cells may be simplified by fulfilling the optimal parameters derived for a defined application.

8. REFERENCES

[1] A. N. Thiele: “Loudspeakers in vented Boxes”, *I.R.E. Radio and Electronic Engineering Convention*, Sydney, 1961

[2] J. D’Appolito: “Lautsprecher-Messtechnik, PC-gestützte Analyse analoger Systeme”, Elektor-Verlag, Aachen, 2005

[3] J. Eargle: “Loudspeaker Handbook”, 2nd Edition, Kluwer Academic Publishers, Boston, 2003

[4] H. Weis, et. al., “Parametric mechatronic model of a load-cell with electromagnetic force compensation”, *IMEKO TC3 & TC5 & TC22 International Conference (2010)*, S. 29-32

[5] R. Petzold: “Analogieuntersuchungen an elektrodynamischen Lautsprechern und Waagen nach dem Prinzip der elektromagnetischen Kraftkompensation – Ermittlung von Modellparametern zur Beschreibung des dynamischen Verhaltens von EMK-Waagen”, Master-Thesis, TU Ilmenau, Ilmenau 2011

[6] J. Lunze, „Regelungstechnik 1“, 4th Edition, Springer Verlag, 2004

[7] L. Krause, „Vergleich von Kompensations- und Ausschlagprinzip am Beispiel der Wägetechnik“, tm Technisches Messen 71 (2004) 12, pp. 643-650, Oldenbourg Verlag, 2004

[8] Sartorius AG: „Sartorius BasicPlus – Electronic Semi-micro, Analytical and Precision Balances“, publication number WBP6014-e00053, 2000