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AN INVESTIGATION OF SAVITZKY-GOLAY FILTERS FOR THE CALCULATION OF DERIVATIVES FOR PRIMARY SHOCK CALIBRATION

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Abstract: This manuscript reports about an investigation of two kinds of digital filters used for numerical differentiation of the displacement signal in the case of primary shock calibration of accelerometers. One is a 4th order Butterworth low-pass (4th BW) filter of infinite impulse response filters, and another is a Savitzky-Golay (S-G) filter, which applies local polynomial approximation. The computational comparison was applied to low and high shocks using known analytical excitation functions. On the optimized condition of 4th BW and S-G filters, each sum of residuals was compared. As the result, the S-G filters showed for low and high shocks a better performance as low-pass filter than the 4th BW filters. In addition, the appropriate window width for the local polynomial approximation was derived.

Keywords: shock, acceleration, infinite impulse response filter, derivative, Savitzky-Golay filter

1. INTRODUCTION

The international standardization document ISO 16063-13 [1] describes shock and complex sensitivities of accelerometers for shock calibration. For the primary shock calibration, it is important to derive an almost undistorted acceleration waveform using laser interferometer and digital signal processing. A set-up using a homodyne laser interferometer needs digital filtering with a 2nd derivative to calculate acceleration from the measured displacement. To obtain the smooth waveform of acceleration, ISO 16063-13 recommends us to use 4th BW filter and twice central differentiations. The 4th BW filter works to remove not only high frequency noise from photo detector but may also be used to suppress the resonant frequency of mechanical parts, such as the anvil [2].

A Savitzky-Golay filter [3] is applied in comparison for differentiation and smoothing to get a less distorted waveform of acceleration, compared to 4th BW filter. For the performance evaluation, numerical simulations for a homodyne set-up were carried out using known analytical excitation functions. This paper reports the characteristic about acceleration measurements using the S-G filter. For the conference, the validation of acceleration measurement using laser interferometer will be also presented.

2. BASIC INFORMATION

Figures (1) and (2) present the assumed waveforms of acceleration over time and their respective frequency domain spectra for low and high shocks. In case of low shock with a duration of 0.5 ms, the accuracy in several m/s^2 goes up to about 10 kHz. The appropriate cut-off frequency of 4th BW filter would be around 10 kHz to measure acceleration with the accuracy in several m/s^2. Thus, considering the spectrum of acceleration is significant in shock calibration.

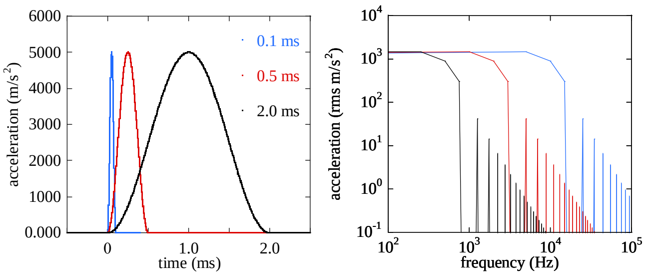

Figure 1 Low shock waveforms on computer simulation and each frequency component

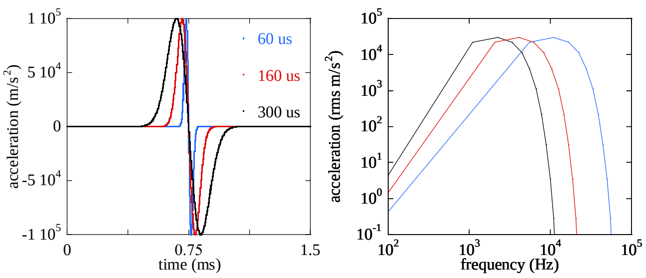

Figure 2 High shock waveforms on computer simulation and each frequency component.

3. SIMULATION PROCEDURE

For the computer simulation, equation (1) and (2) provide the definition of the mathematical function used for low and high shocks, respectively.

$$a_{low}(t) = A\sin^2\left[\frac{\pi}{\tau}(t-\tau)\right], \qquad (\tau < t < 2\tau) \qquad (1)$$

$$a_{high}(t) = A\sigma e^{0.5}\left[-\frac{(t-10\sigma)}{\sigma^2}\right]\exp\left[-\frac{(t-10\sigma)^2}{2\sigma^2}\right], \qquad (0 < t < 20\sigma) \qquad (2)$$

where the subscript *low* and *high* mean low and high shocks, respectively. The value of A is the peak of shock acceleration. The low shock waveform is given by the square of sine function, and the duration is defined as τ. In the range of $0 < t < \tau$ and $2\tau < t < 5\tau$, the waveform of acceleration vanishes. The high shock acceleration waveform is the 1st derivative of Gaussian velocity function, and the duration is defined as 4σ. The calculation range of acceleration is selected from -10σ to 10σ, so that the initial displacement sufficiently becomes small as compared with the half wavelength of Ne-He laser.

Figure 3 shows the procedure of the 1st evaluation during the computer simulation. Here, quadrature signals with 400 mV_{p-p}, white noise with 10 mV_{p-p} and 16 bit quantization in ± 16 V are a typical specification of the homodyne laser interferometer in NMIJ.

Low shock : $(\sin)^2$ with a duration of 0.1 ms to 2.0 ms
High shock : dG/dt with a duration of 60 µs to 300 µs

Quadrature signals with 400 mVp-p for homodyne interferometer

White noise with 10 mVp-p, 16 bit quantization in ±16 V

Haydemman correction, Displacement using arctan, Decimation

2th BW, differentiation, 2th BW in reverse time, differentiation

Savitzky-Golay filter (order, side points)

Comparison of each acceleration waveform

Figure 3 Procedure of computer simulation

4. 4TH ORDER BUTTERWORTH FILTER

Figure 4 presents the computed differences between 4th BW (red line) and S-G (green line) filters in the cases of low shock with a peak acceleration of 5000 m/s² and a duration of 0.5 ms. The white line stands for an assumed waveform of acceleration. On this computational result, the assumed waveform is generated with the sampling frequency of 50 MHz, and the demodulated displacement is decimated to the sampling frequency of 10 MHz. Second and third rows in figure 4 respectively are residuals and their spectrum compared with the assumed waveform. The cut-off frequency of the 4th BW filter is optimized to obtain the smallest residuals, and is 10 kHz (see figure 5). Each minimal value depends on the duration. Shock waveforms have broad frequency bandwidth as presented in figures 1 and 2. The longer the duration, the narrower is the frequency bandwidth covered by the shock waveform. The S-G filter is also optimized with 7 orders and 1300 side points (see figure 8). The second row in figure 4 implies that the distortion of the S-G filter is smaller than that of the 4th BW filter. In the third row, the S-G filter also exhibits smaller residuals in both frequency ranges below 3 kHz and beyond 10 kHz in the spectrum, compared with 4th BW filter. Consequently, such a smaller deviation of waveform would be effective to obtain more accurate results of shock and complex sensitivities. [4, 5]

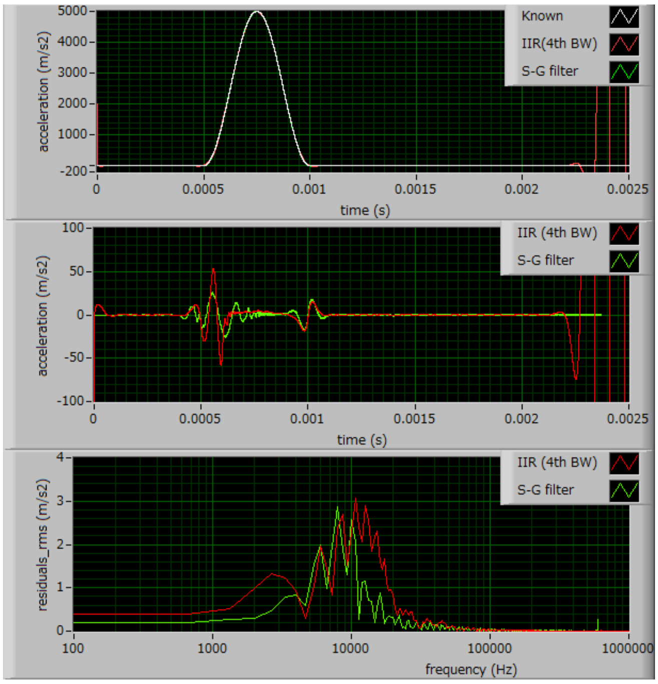

Figure 4 Comparison results of residuals between optimized 4th BW and S-G filters in low shock with a duration of 0.5 ms.

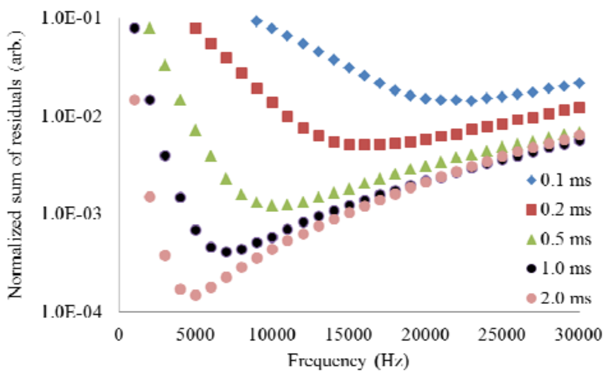

Figure 5 Dependence of normalized sum of residuals on cut-off frequency with 5 different durations in low shocks.

Figure 6 shows the deviation between the assumed and computed peak acceleration on 4th BW filter in dependence of the selected cut off frequency. This result indicates that, the longer the duration, the more accurate is the peak acceleration a laser interferometer can measure. In case of a duration of 0.5 ms, the deviation with less than 0.1 % ranges in the cut-off frequency beyond 10 kHz. As a conclusion from figures 5 and 6, the cut-off frequency of 10 kHz is suited to get both less distorted waveform and more accurate peak acceleration in case of a duration of 0.5 ms and beyond.

In order to measure more correct waveform of acceleration using a homodyne laser interferometer, the authors recommend to select durations more than 0.5 ms. However, as a remark, the appropriate duration and cut-off frequency strongly depend on the resonance of each shock calibration machine.

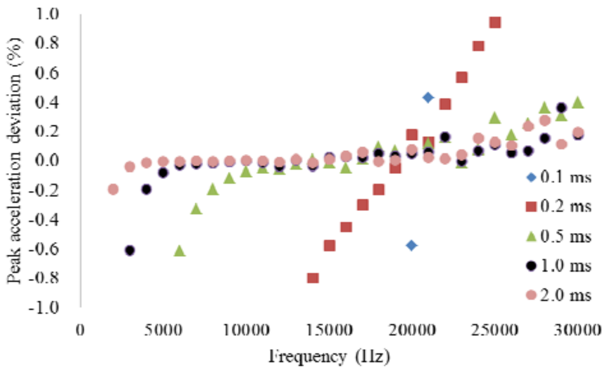

Figure 6 Dependence of peak acceleration deviation on the cut-off frequency in low shock for 5 different durations.

Figure 7 shows the computed differences between 4th BW (red line) and S-G (green line) filters in the cases of high shock with a peak acceleration of 100 km/s^2 and a duration of 160 μs. Also, this graph visually implies that a S-G filter with 7 orders and 440 side points achieves smaller deviations of the acceleration waveform than the optimal 4th BW filter.

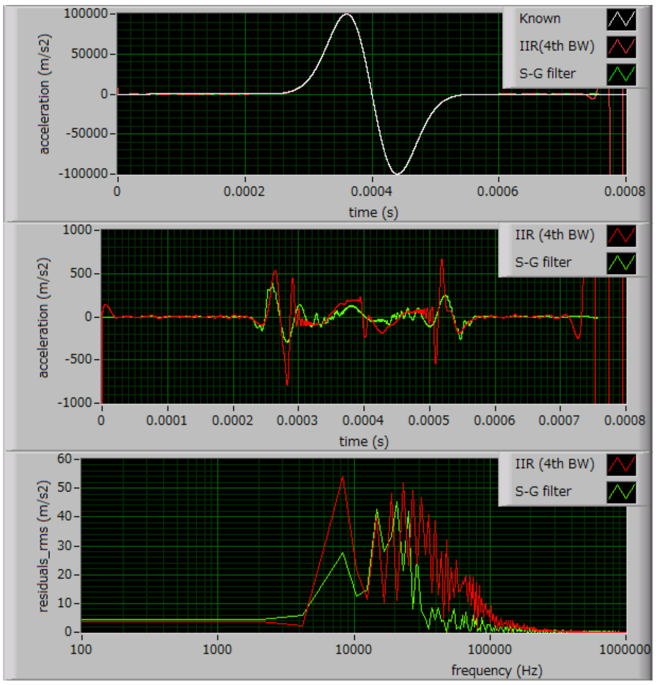

Figure 7 Comparison results of residuals between optimized 4th BW and S-G filters in high shock with a duration of 160 μs.

5. SAVITZKY-GOLAY FILTER

Figure 8 shows the dependence of the sum of residuals on the order and number of side point of the S-G filter in low shock with a peak acceleration of 5000 m/s^2 and a duration of 0.5 ms. The sum of residuals using the 4th BW filter is minimized by optimizing the cut-off frequency that is 10 kHz as shown in figure 5. The sum of residuals indicates that the S-G filter with 7 order and 1300 side points is optimal under the chosen conditions.

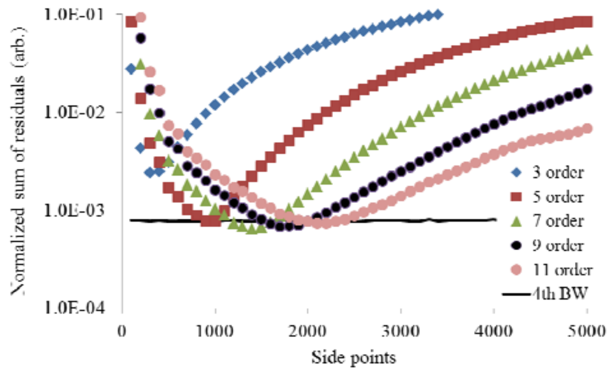

Figure 8 Sum of residuals between optimized 4th BW filter and S-G filters with 5 different orders in low shocks with a duration of 0.5 ms.

Table 1 summarizes each optimal value of cut-off frequency in 4th BW filters and side points in S-G filters. The "window" means window widths for polynomial approximation of a S-G filter and is the numerical value given by equation (3). From this result, peak accelerations of 1000 m/s^2 need lower optimal cut-off frequencies compared to 5000 m/s^2. That is why the effect of high frequency noise becomes larger in case of low peak acceleration.

$$window = (2 \times side\ points + 1) / decimation \ \times 1000 \qquad (3)$$

A relation between "cut-off" and "window" in low shock on the basis of table 1 is plotted in figure 9. The closed circles stand for a peak acceleration of 5000 m/s^2 among 4 different decimations from 0.5 MHz to 50 MHz. The open circles mean a peak acceleration of 1000 m/s^2. This result denotes that the relation between optimal cut-off frequencies and window widths is independence of three kinds of invariables: peak acceleration, duration and decimation.

Correspondingly, figure 10 presents a relation in low and high shocks. The open circles stand for high shock that has different waveform from low shock. The peak accelerations are respectively 20 km/s^2 and 100 km/s^2, and the duration ranges from 60 μs to 300μs. Thus, the relation concludes that a S-G filter purely works on low and high shocks as a low-pass filter (with differentiation).

Table 1 Optimal values of cut-off frequency in 4th BW filter and window width in S-G filter.

peak (ms^2)	*duration* (ms)	*decimation* (MHz)	*cut-off* (Hz)	*side points*	*window* (ms)
5000	0.1	50	21000	NA	
5000	0.2	50	16000	4500	0.18
5000	0.5	50	10000	NA	
5000	1.0	50	7000	NA	
5000	2.0	50	5000	NA	
5000	0.1	10	23000	600	0.12
5000	0.2	10	16000	900	0.18
5000	0.5	10	10000	1400	0.28
5000	1.0	10	7000	2000	0.40
5000	2.0	10	5000	2800	0.56
5000	0.1	1	22000	60	0.12
5000	0.2	1	17000	80	0.16
5000	0.5	1	10000	140	0.28
5000	1.0	1	6000	220	0.44
5000	2.0	1	4000	320	0.64
5000	0.1	0.5	24000	30	0.12
5000	0.2	0.5	15000	40	0.16
5000	0.5	0.5	9000	70	0.28
5000	1.0	0.5	6000	100	0.40
5000	2.0	0.5	4000	160	0.64
1000	0.1	10	14000	800	0.16
1000	0.2	10	10000	1300	0.26
1000	0.5	10	7000	2100	0.42
1000	1.0	10	5000	3000	0.60
1000	2.0	10	3000	4300	0.86

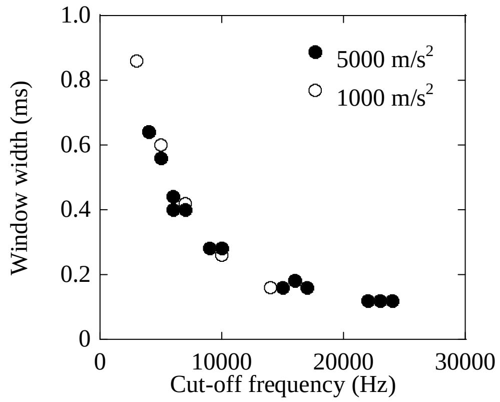

Figure 9 Relation between optimal cut-off frequencies and window widths in low shock.

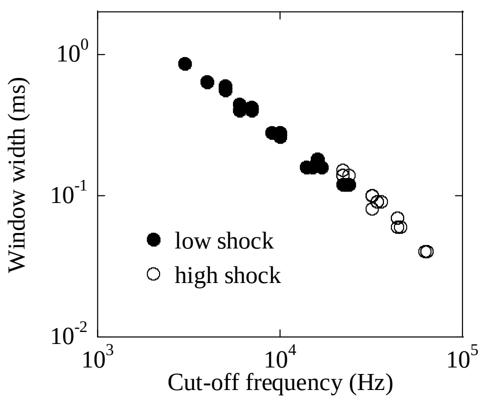

Figure 10 Relation between optimal cut-off frequencies and window widths in low and high shocks.

6. SUMMARY

An approach for the calculation of derivatives using S-G filter to obtain appropriate waveforms of acceleration in low and high shocks was proposed and discussed based on computer simulation. The obtained waveform was compared with waveforms that were twice differentiated and smoothed using 4th BW filter according to the procedure proposed in [1]. As the results, S-G filter implied smaller deviations than the 4th BW filter in both cases low and high shocks. These result leads to more accurate measurements of shock and complex sensitivity for primary calibration of accelerometers according to [1].

A relation was derived from each optimal residual between 4th BW and S-G filters with 7 orders, and did not depend on the decimation, peak acceleration, duration and waveform in cases of low and high shocks. This means that S-G filter differentiates the displacement of low and high shocks with low-pass filtering. As the future work, the authors would investigate to validate the waveform of acceleration using experimental data.

7. ACKNOWLEDGEMENTS

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