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SAMPLING STRATEGIES AND LONG TERM VARIATION MODELLING FOR A STATISTICAL FEED-FORWARD CONTROLLER

C.Hernández [1], *R.Tutsch* [1]

[1] Institut für Produktionsmesstechnik, Technische Universität Braunschweig,
Schleinitzstraße 20, D-38106 Braunschweig, Germany , carlos.hernandez@tu-bs.de

Abstract: The Statistical Feed-Forward Control Model (SFFCM) relies on a sequence of specification adjustments made on subsets of a population to counter the influence of the long time component of the variation. The difficulty strives in finding a proper estimate for the measure of the central tendency of each subset to minimize the number of the required adjustments. By means of simulating the assembly of two components having high dimensional variation, forty experiments were designed to compare the individual influence of different factors such as the number of adjustments, the sampling strategy and two measures of central tendency: the sample mean and the cumulative de-noised average. Simulation results showed that, regardless of the sampling strategy but keeping the inspection rate at 20%, the use of the cumulative de-noised average instead of the sample mean made possible to reduce the number of adjustments by 20%. Thus, while the shift mean of the resulting assembly was decreased by 90%; the standard deviation was reduced by 15%. Hence, the selection of a proper central tendency measure is crucial when modeling the long time variation. The cumulative de-noised average proved to be a valid alternative.

Keywords: Statistical Feed-Forward Control Model, Statistical Dynamic Specifications Method, Sampling Strategy, Subset Size, Central Tendency Measures.

1. INTRODUCTION

SFFCM has been conceived as an alternative to help controlling those processes in which the cumulative dimensional variation of the assembled components is in the order of magnitude of the nominal tolerance of the whole assembly. However, different from classical approaches, SFFCM focuses on managing the specifications and the tolerances of the inner components rather than on neutralizing the influence of the sources of variation.

To apply SFFCM the population under study has to be divided in subsets of units produced consecutively in a short time interval from which some will be drawn for inspection. Through this work the individual influence of the subset size, the sampling strategy and the central tendency measure on the detection and modeling of the long time variation was revealed with the help of forty specific experiments that were replicated seven hundred times each.

To carry out the experiments the manufacture of a lot of one thousand assemblies made of two types of components was simulated. The numbers corresponding to the normally distributed dimensions of the components were generated using Monte Carlo methods.

The influence of the factors was measured in terms of the process' ability to produce assemblies within the required assembly specifications. In practice, it was quantified by the reduction achieved in both the mean shift and the standard deviation of the dimensional values of the resulting assemblies and by the number of the required adjustments.

Simulation results showed that, independently of the sampling strategy but keeping the inspection rate at 20%, the mere use of the cumulative de-noised average as central tendency measure of the population's subsets made possible to reduce the number of adjustments by 20% and still keep the effectiveness of the controller.

2. STATISTICAL DYNAMIC SPECIFICATIONS METHOD (SDSM)

SDSM is a collection of steps to help managing dimensional specifications and tolerances of the inner components of an assembly [1], [5], [6].

Let L_{assy} and t_{assy} be the target and tolerance of a given assembly whose two components' specifications have been set to L_i and t_i [7 p.14]. Let the variation of L_1 be composed by a random and a long term component (Fig.1).

$$L_{assy} = L_1 + L_2 \tag{1}$$

$$t_{assy} = \sqrt{t_1^2 + t_2^2} \tag{2}$$

If a small subset were taken from the population of Component 1 (Fig. 1) [8 p.192], it would be discovered that 99.73% of the units lie in a band of width $6\sigma_{1,sub}$ which is centered at $\mu_{1,sub}$. Since the influence of the long time variation is only partial, the standard variation of the subset is expected to be smaller that the one of the whole lot.

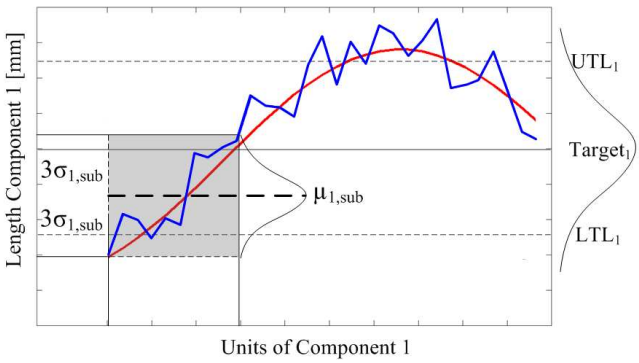


Fig. 1 Subset mean and standard deviation

It is reasonable to think that, at least for this subset, the nominal tolerance t_1 had not been fully used and that part of it could have been spared to complement the nominal tolerance t_2 of a matching subset of Component 2. In fact, it would have been possible to have another tolerance $t_{2,sub}$

$$t_{1,sub} = 3\sigma_{1,sub} \tag{3}$$

$$t_{2,sub} = \sqrt{t_{assy}^2 - t_{1,sub}^2} \tag{4}$$

On the other hand, if the subset mean $\mu_{1,sub}$, had been known then it would have been possible to define an adjusted target $L_{2,sub}$ for a matching subset of Component 2 to help meeting the desired L_{assy}.

$$L_{2,sub} = L_{assy} - \mu_{1,sub} \tag{5}$$

3. STATISTICAL FEED-FORWARD CONTROL MODEL (SFFCM)

SFFCM divides the system in two parts, a feeding and a controlled sub-system, to place an intermediate observation point between them (Fig. 2). Thus, by means of applying iteratively SDSM any possible deviation detected in the output of the feeding Sub-system A could be countered by altering the input to the controlled Sub-system B to prevent the occurrence of defective units from within the system. The implementation of SFFCM only makes sense if a long time scale component in the variation can be detected in the output of the feeding Sub-system A [5], [6].

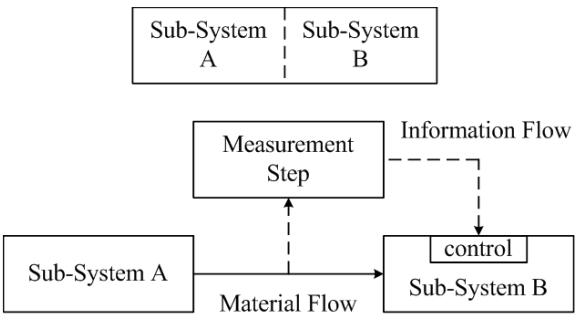


Fig. 2 Statistical feed-forward control model

4.1 Subset Size

The subset size is defined by the number of consecutive units coming out of the feeding Sub-system A from which a sample will be drawn to determine the adjustments to the input of the controlled Sub-system B. The subset size determines the number of adjustments.

4.2 Sampling Strategy

The sampling strategy comprises two aspects: the number of observations per subset and the selection method in which the sample will be drawn: either simple or systematic random sampling [10 p.52] with individual or common selection pattern for all subsets [3], [4].

4.3 Long Term Variation Modelling

Since SFFCM exerts the control by means of determining the values of $L_{2,sub}$ that counter the drift experienced by $\mu_{1,sub}$ through time, finding a proper central tendency measure for each subset is crucial to succeed. In this work, two approaches were considered: the sample mean and the cumulative de-noised average.

The cumulative de-noised average, $\bar{x}_{1,sub,cdna}$, is based on the accumulation of knowledge about the population. Basically, the observations of the current and the previous subsets are processed by a noise reduction algorithm that delivers a new set of points which is used then to construct a smoother curve by interpolating consecutive points. The points of the new curve corresponding to the current subset are finally averaged to obtain $\bar{x}_{1,sub,cdna}$ as the central tendency measure of the current subset (Fig 3).

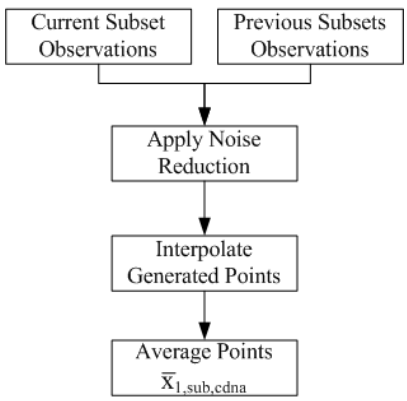


Fig. 3 Cumulative de-noised average.

4. EXPERIMENTS

Forty experiments were designed considering three factors: number of adjustments (subset size), sampling strategy and central tendency measure.

Table 1 Experiments Design

	Random Sampling		Subset Pattern		Tendency Measure	
#	Simple	Systematic	Common.	Indiv.	$\bar{x}_{1,sub}$	$\bar{x}_{1,sub,cdna}$
1	√		√		√	
2	√		√			√
3	√			√	√	
4	√			√		√
5		√	√		√	
6		√	√			√
7		√		√	√	
8		√		√		√

5. SIMULATION

The production of 1,000 assemblies made of two components was simulated and replicated 700 times. Every replication was made employing a new population generated randomly using Monte Carlo Methods. Whereas the inspection rate was set to 20%; the number of adjustments was shifted between 2 and 10. The numbers in Table 4 and Table 5 average the result of 700,000 tries.

Following the definition of SFFCM, Component 1 represented the feeding Sub-system A and Component 2 the controlled Sub-system B. Thus, the variable to control was the length of the latter.

The initial simulation conditions, nominal specifications and manufacture process characteristics, are summarized in Table 2.

Table 2 Specifications and Process Characteristics [mm]

	Target	Tol.	Mean	St. Dev.	C_p
Assembly	30.00	1.00	29.55	0.29	1.14
Comp. 1	20.00	0.82	19.60	0.25	1.09
Comp. 2	10.00	0.58	9.95	0.15	1.29

The values above can be derived from the following formulae [8 p.339], [9 p.91].

$$\mu_{assy} = \mu_1 + \mu_2 \tag{6}$$

$$\sigma_{assy} = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{7}$$

$$C_{p,i} = t_i / 3\sigma_i \tag{8}$$

The following assumptions were taken as valid:

- **Normally distributed** and **homoscedastic** noise.
- **No correlation** between components' populations.
- **Process variation** separable into random noise and potentially controllable long time variation.
- **Process stability** to respond predictably to the adjustments.

5.1 Simulation Results

Table 3 Assembly average mean $\bar{\mu}_{assy,adj}$

Exp	2-Adj	4-Adj	5-Adj	8-Adj	10-Adj
1	29.9489	29.9494	29.9480	29.9485	29.9491
2	**29.9470**	**29.9511**	**29.9511**	**29.9555**	**29.9522**
3	29.9481	29.9489	29.9473	29.9484	29.9490
4	**29.9468**	**29.9518**	**29.9506**	**29.9546**	**29.9509**
5	29.9490	29.9487	29.9498	29.9493	29.9493
6	**29.9458**	**29.9511**	**29.9499**	**29.9543**	**29.9529**
7	29.9484	29.9501	29.9481	29.9499	29.9487
8	**29.9464**	**29.9513**	**29.9499**	**29.9564**	**29.9516**

Table 4 Assembly average standard deviation $\bar{\sigma}_{assy,adj}$

Exp	2-Adj	4-Adj	5-Adj	8-Adj	10-Adj
1	0.2494	0.2486	0.2474	0.2449	0.2456
2	**0.2495**	**0.2480**	**0.2473**	**0.2457**	**0.2457**
3	0.2496	0.2478	0.2473	0.2453	0.2454
4	**0.2495**	**0.2478**	**0.2479**	**0.2454**	**0.2462**
5	0.2494	0.2478	0.2472	0.2455	0.2455
6	**0.2492**	**0.2481**	**0.2474**	**0.2463**	**0.2462**
7	0.2491	0.2478	0.2468	0.2452	0.2454
8	**0.2494**	**0.2477**	**0.2475**	**0.2459**	**0.2458**

Table 5 Standard deviation of the mean values $\mu_{assy,adj}$

Exp	2-Adj	4-Adj	5-Adj	8-Adj	10-Adj
1	0.0134	0.0135	0.0137	0.0132	0.0132
2	**0.0136**	**0.0138**	**0.0151**	**0.0157**	**0.0151**
3	0.0143	0.0135	0.0134	0.0130	0.0130
4	**0.0136**	**0.0147**	**0.0151**	**0.0143**	**0.0152**
5	0.0131	0.0130	0.0126	0.0134	0.0131
6	**0.0137**	**0.0146**	**0.0154**	**0.0149**	**0.0146**
7	0.0128	0.0128	0.0130	0.0128	0.0130
8	**0.0137**	**0.0142**	**0.0145**	**0.0153**	**0.0145**

Table 6 Standard deviation of the Std. Dev. values $\sigma_{assy,adj}$

Exp	2-Adj	4-Adj	5-Adj	8-Adj	10-Adj
1	0.0050	0.0048	0.0050	0.0049	0.0051
2	**0.0048**	**0.0051**	**0.0054**	**0.0050**	**0.0052**
3	0.0048	0.0050	0.0052	0.0052	0.0050
4	**0.0047**	**0.0049**	**0.0048**	**0.0052**	**0.0053**
5	0.0048	0.0051	0.0049	0.0051	0.0049
6	**0.0049**	**0.0051**	**0.0052**	**0.0052**	**0.0053**
7	0.0047	0.0050	0.0050	0.0051	0.0047
8	**0.0048**	**0.0051**	**0.0051**	**0.0052**	**0.0051**

Table 3 and Table 4 show that, regardless of the sampling strategy, the system output is sensitive to the application of the cumulative de-noised average $\bar{x}_{1,sub,cdna}$ as central tendency measure. Particularly, Experiments 2, 4, 6 and 8 systematically delivered better results when the number of adjustments was either 8 or 10 (Fig. 4).

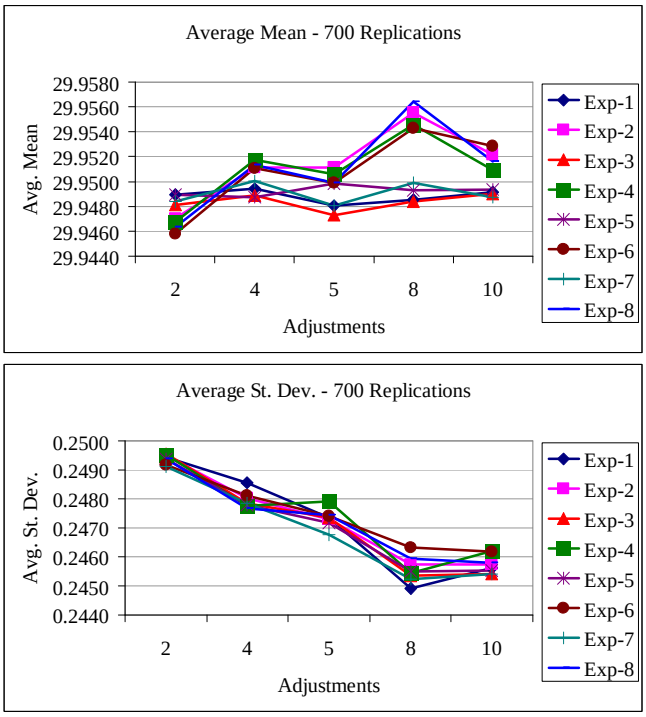
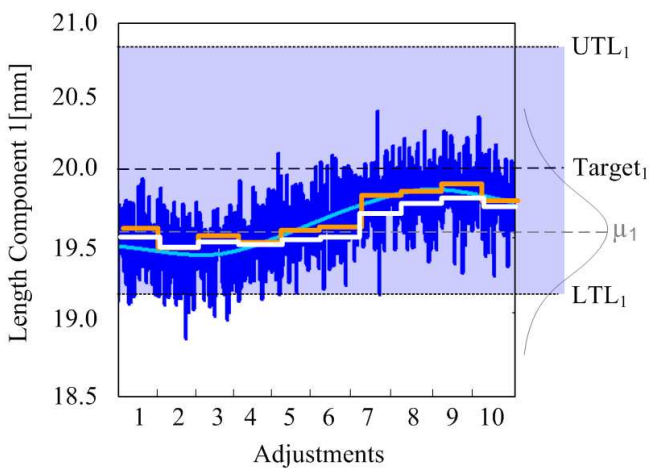


Fig. 4 Average mean and average standard deviation.

Differently from previous works [1], [2], [3] and [6] in which 10 adjustments along with the use of the sample mean as central tendency measure were customary, this simulation showed that only 8 adjustments combined with the cumulative de-noised average can deliver better results without sacrificing the output stability. In this case the shift of the mean of the resulting assembly was reduced by 90%, from 0.45 to 0.05. The resulting standard deviation was reduced by 15%, from 0.29 to 0.26. Thus the potential capability index C_p varied from 1.14 to 1.36.

The difference between using the sample mean (orange) and the cumulative de-noised average (white) to model the long time component of the variation (light blue) is shown in Fig. 6.



Fig, 6 Sample mean v/s Cumulative de-noised average (white)

6. CONCLUSIONS

While both the number of adjustments and the central tendency measure showed to impact significantly the simulation results; the sampling strategy did not show a definitive influence.

Acquiring and using knowledge about the variation through the time proved to be worthy. It made possible to reduce the number of adjustments, and therefore the cost and effort, without sacrificing the neither the effectiveness of the controller nor the stability of the process' output.

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