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BAYES FILTER FOR IMPROVING AND FUSING DYNAMIC COORDINATE MEASUREMENTS

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Abstract: This paper presents a novel methodology to improve the measurement accuracy of dynamic measurements. This is achieved by deducing an online Bayes optimal estimate of the true measurand given uncertain, noisy or incomplete measurements within the framework of sequential Monte Carlo methods. The estimation problem is formulated as a general Bayesian inference problem for nonlinear dynamic systems. The optimal estimate is represented by probability density functions, which enable an online, probabilistic data fusion as well as measurement uncertainty evaluation completely conform to the "Guide to the expression of uncertainty in measurement". The efficiency and performance of the proposed methodology is verified and shown by dynamic coordinate measurements.

Keywords: Dynamic coordinate measurements, Bayesian filtering, Particle filter, Sequential Monte Carlo methods, Online measurement uncertainty evaluation and data fusion

1. INTRODUCTION

Traditional coordinate measuring machines (CMM) were increasingly completed or superseded by mobile, dynamic coordinate measuring machines (DMM), such as laser tracker or photogrammetry systems (i.e. camera-based systems). They allow non-contact, in-situ and online coordinate measurements of complex moving objects over large measurement volumes along with excellent precision and high measurement rate directly in the shop-floor [1, 2].

Typical applications are dynamic motion measurements (traveled paths, positions, displacements, velocities, accelerations, jerks, orientations and vibrations), guidance and calibration of machines, robot trajectory measurements, deformation analysis as well as assembly and inspection of parts and structures.

Often, DMM are portable optical measurement systems, which perform online shop-floor measurements, mostly in unstable environments. However, it is in general not possible to model, quantify or compensate significant error sources, such as optical aberrations (e.g. reflection errors, changing refractive index), imaging errors or dynamic errors (e.g. motion artifacts), during dynamic measurements. As a result the obtained measurements exhibit a lower signal-to-noise ratio and reduced measurement accuracy. Furthermore it is not possible to evaluate the measurement uncertainty according to the "Guide to the expression of uncertainty in measurement" (GUM) due to the model imperfection and dynamic characteristic of the measurement process[1].

To overcome these shortcomings and to extend the application spectrum of DMM, this paper presents a methodology to increase the measurement accuracy by deducing the Bayes optimal estimate of the true measurand given uncertain, noisy or incomplete measurements in real-time. In every time step the measurand is represented by a probability density function (PDF). Thus, the Bayesian framework enables an online, probabilistic data fusion as well as measurement uncertainty evaluations completely conform to the GUM.

The rest of the paper is structured as follows. In section 2 the basic principles of measurement uncertainty evaluation as described in the GUM and in the "Supplement 1 to the GUM" (GS1) [3, 4] are briefly restated. Section 3 introduces the Bayesian inference and the probabilistic model of the measurement process using the state-space representation. The Bayesian estimation problem is formulated and a general solution to this problem within the framework of sequential Monte Carlo methods is devised. Section 4 presents experimental results of Bayes filter applied to dynamic coordinate measurements. In the end a summary and conclusion are given.

2. BASIC PRINCIPLES OF UNCERTAINTY EVALUATION

A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty [5, definition 2.9]. The de facto international standards to evaluate, calculate and express uncertainty in all kinds of measurements are the GUM and its extension the GS1 [3, 4]. These documents provide guidance on the uncertainty evaluation as a two-stage process: formulation and calculation. The formulation stage involves developing a measurement model relating output (measurand) Y to input

[1] Neither the GUM nor the GS1 deal with dynamic measurements.

quantities $X_1,\ldots, X_N$, incorporating corrections and other effects as necessary and finally assigning probability distributions to the input quantities on the basis of available knowledge. The calculation stage consists of propagating the probability distributions for the input quantities through the measurement model $Y=f(X_1,\ldots, X_N)$ to obtain the probability distribution for the output quantity. From this PDF the expected value and the standard deviation of the output quantity as well as the coverage interval are calculated. Several approaches are considered for the propagation of distributions:

a) the GUM uncertainty framework, constituting the application of the law of propagation of uncertainty,
b) analytic methods, in which mathematical analysis is used to derive an algebraic form for the probability distribution for the output quantity and
c) a Monte Carlo method (MCM), in which an approximation to the distribution function for the output quantity is established numerically by making random draws from the probability distributions for the input quantities, and evaluating the model at the resulting values.

Any of these procedures can be applied in many situations and leads to valid statements of uncertainty. But they also exhibit their respective merits and drawbacks as well as application limitations. The GUM uncertainty framework is a general approximation that might not be satisfactory for nonlinear measurement functions, asymmetric or non-Gaussian probability distributions for the input or output quantities or significantly different sensitivity of the uncertainty of input quantities. The analytic methods theoretically yield to exact solutions, but in practice it is often not possible to analytically derive an algebraic form for the probability distribution of the output quantity. Finally, the Monte-Carlo methods are only exact within the limits of the numerical accuracy and come up with the burden of computational complexity. In order to not go beyond the scope of this paper, it is referred to GUM and GS1 [3, 4] for further details of the propagation of distribution.

In summary, the GUM and the GS1 specify a method to calculate the measurement uncertainty by using the propagation of probability distributions through a mathematical model of the measurement to derive the resulting probability distribution for the output quantity. The measurement uncertainty is evaluated based on this PDF and stated as uncertainty budget, which should include the measurement model, estimates, and measurement uncertainties associated with the quantities in the measurement model, covariances, type of applied probability density functions, degrees of freedom, type of evaluation of measurement uncertainty and the coverage interval. Hereby, the method of choice would be an analytical solution of the output density function. But as this is not possible in general, especially for nonlinear dynamic processes, sequential Monte Carlo methods are utilized to achieve a Bayes optimal estimation of the output density, as described in the next section.

3. BAYESIAN INFERENCE

A measurement process generates disperse (uncertain) observations of a true (but unknown) measurand. This is an erroneous transformation due to ubiquitous errors. Thus, a complete, exact or unique reconstruction of the true measurand is always impossible. But using Bayes' theorem it is possible to estimate the true measurand from observations in an optimal manner, since the functional relation between both is preserved.

This estimation problem can be solved as a sequential probabilistic inference problem for nonlinear dynamic systems. The unknown measurand (e.g. coordinates) is modelled as a state of a dynamic system (e.g. moving object) that evolves over time and is observed with a particular measurement process. This allows the estimation of measurable states (e.g. position) as well as derived or not directly measurable states (e.g. velocities, acceleration and orientations) of arbitrary dynamic systems in an optimal and statistically sound way given uncertain, noisy or incomplete observations.

The nonlinear dynamic system is described by the general discrete-time stochastic state space model

$$\begin{aligned} \mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}), \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k), \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the hidden system state vector with dimension n_x evolving over time $k \in \mathbb{N}$ (discrete time index) according to the possibly nonlinear state transition function[2] $\mathbf{f}(\cdot)$, the state $\mathbf{x}_{k-1}$ and the process noise $\mathbf{v}_{k-1}$. The process noise is used to describe uncertainties or mismodeling effects in the process model. The observations $\mathbf{y}_k \in \mathbb{R}^{n_y}$ with dimension n_y are related to the state vector via the nonlinear observation function $\mathbf{h}(\cdot)$ and the measurement or observation noise $\mathbf{w}_k$, which is corrupting the observation of the state. The dynamic equation $\mathbf{f}(\cdot)$ describes the process dynamic or objects motion and the measurement equation $\mathbf{h}(\cdot)$ defines the mapping between object states and measurements. This state space model corresponds to a first order hidden Markov model [6, 7].
The objective is to optimally estimate the hidden system states $\mathbf{x}_k$ in a recursive fashion (i.e. sequential update of previous estimate) as measurement $\mathbf{y}_k$ becomes available[3]. This is the central issue of the sequential probabilistic inference theory. From a Bayesian perspective, the complete solution to this problem is given by the conditional posterior density $p(\mathbf{x}_k \mid \mathbf{y}_{1:k})$ of the state $\mathbf{x}_k$, taking all observations $\mathbf{y}_{1:k} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k\}$ into account (see Fig. 1). The knowledge of the posterior allows one to calculate an optimal state estimate with respect to any criterion. For example the minimum mean-square error estimate is the

[2] The state transition function is also named system, process, dynamic, evolution or likelihood function.
[3] This is also referred to as online, sequential or iterative filtering.

conditional mean $\hat{\mathbf{x}}_k = E\left[\mathbf{x}_k \mid \mathbf{y}_{1:k}\right] = \int \mathbf{x}_k \, p(\mathbf{x}_k \mid \mathbf{y}_{1:k}) \, d\mathbf{x}_k$. Other estimates, such as median, modes, confidence intervals, etc., can also be computed from the posterior. That enables one to evaluate the measurement uncertainty according to the GUM and its supplement.

The computation of the aforementioned Bayes posterior density requires the computation with all measurements $\mathbf{y}_{1:k}$ in one step (batch processing). The optimal method to recursively update the posterior density as new observations arrive is given by the recursive Bayesian estimation algorithm, which is faster and allows an online processing of data with lower storage costs and a quick adaption to changing data characteristics.

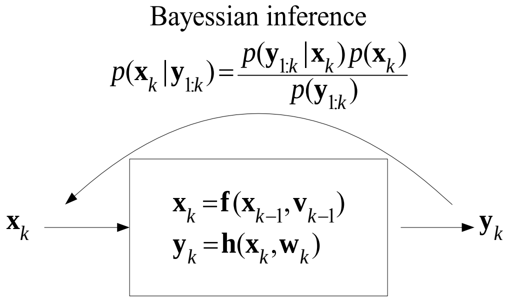

Fig. 1 Bayesian inference problem for dynamic systems

Using the Bayes theorem and the discrete-time stochastic state space model (1), the posterior density can be derived and factored into the following recursive update form

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k}) = \frac{p(\mathbf{y}_{1:k} \mid \mathbf{x}_k) \, p(\mathbf{x}_k)}{p(\mathbf{y}_{1:k})} = \frac{p(\mathbf{y}_k \mid \mathbf{x}_k) \, p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1})}{p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1})}. \quad (2)$$

The computation of the Bayesian recursion essentially consists of two steps: the prediction step $p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}) \rightarrow p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1})$ and the update step $p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}, \mathbf{y}_k) \rightarrow p(\mathbf{x}_k \mid \mathbf{y}_{1:k})$. The prediction step involves using the process function and the previous posterior density[4] $p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1})$ to calculate the prior density of the state $\mathbf{x}_k$ at time k via the Chapman-Kolmogorov equation

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) \, p(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}) d\mathbf{x}_{k-1}, \quad (3)$$

where the state transition PDF is given by[5]

$$p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) = \int \delta(\mathbf{x}_k - \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{v}_k)) \, p(\mathbf{v}_k) d\mathbf{v}_k. \quad (4)$$

The prior of the state $\mathbf{x}_k$ at time k does not incorporate the latest measurement $\mathbf{y}_k$, i.e. the probability for $\mathbf{x}_k$ is only based on previous observations. When the new uncertain measurement $\mathbf{y}_k$ becomes available the update step is carried out and the posterior PDF of the current state $\mathbf{x}_k$ is computed from the predicted prior (3) and the new measurement via the Bayes theorem (2), where the observation likelihood PDF and the normalizing constant in the denominator are given by

$$p(\mathbf{y}_k \mid \mathbf{x}_k) = \int \delta(\mathbf{y}_k - \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k)) \, p(\mathbf{w}_k) d\mathbf{w}_k \quad (5)$$

and $p(\mathbf{y}_k \mid \mathbf{y}_{1:k-1}) = \int p(\mathbf{y}_k \mid \mathbf{x}_k) \, p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}) d\mathbf{x}_k$. (6)

The equations (2-6) formulate the optimal Bayesian solution to recursively estimate the unknown system states (measurands) of a nonlinear dynamic system from uncertain, noisy or incomplete measurements. But this is only a conceptual solution in the sense that in general the integrals cannot be determined analytically. Furthermore, the algorithmic implementation of this solution requires the storage of the entire (non-Gaussian) posterior PDF as an infinite dimensional matrix, because generally it cannot be completely described by a sufficient statistic of finite dimension. Only in some restricted cases a closed-form recursive solution is possible. For example with restriction to linear, Gaussian systems a closed-form recursive solution is given by the famous Kalman filter [8]. In most situations, either both the dynamic and the measurement process or only one of them are nonlinear. Thus the multi-dimensional integrals are not tractable and approximative solutions such as sequential Monte Carlo methods must be used [9]. They make no explicit assumption about the form of the posterior density and approximate the Bayesian integrals with finite sums. These methods have the advantage of not being subject to the curse of dimensionality as well as not being constrained to linear or Gaussian models. The resulting algorithmic filter implementation is called particle filter [10].

4. EXPERIMENTAL RESULTS FOR KNOWN DYNAMICS

Bayes filter is a general term for all algorithmic implementations of the Bayesian recursion equations (2-6). In this section the extended Kalman filter (EKF) and the particle filter (PF) - as the two mainly used Bayes filters - were analyzed and verified with respect to efficiency and performance applied to dynamic coordinate measurements of a marker-based close range photogrammetric (i.e. stereo vision based) CMM.

Fig. 2 Experimental setup consisting of close range photogrammetric CMM and high accuracy parallel kinematic (hexapod)

The experiments were conducted in a high precision measuring room (accuracy class 1 according to VDI/VDE 2627 part 1 [11]) by constant temperature $\vartheta = 20 \pm 0.1°\mathrm{C}$. The dynamic CMM consists of two Gigabit Ethernet interline progressive scan charge-coupled device (CCD)

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[4] At time $k = 1$ the previous posterior is the initial density of the state vector, i.e. $p(\mathbf{x}_0 \mid \varnothing) = p(\mathbf{x}_0)$.

[5] $\delta(\cdot)$ is the Dirac-delta function.

cameras using infrared illumination. To block out visible light, optical infrared band-pass filters, cutting off at 810 nm and a spectral bandwidth of 35 nm, were used. Both cameras are mounted on a mechanical frame fulfilling the normal stereo case. The system calibration, i.e. estimating the inner and outer orientation of each camera, was performed by bundle adjustment using a 3D field of calibrated reference points. Both cameras were synchronized by hardware trigger and acquire images with 776 × 582 pixels at maximal 64 frames per seconds. All tracking targets were calibrated with a traditional CMM that is specified with a bidirectional Maximum Permissible Error E3 = (0.75 + L/500) µm, where the measuring length L is given in mm comparable to ISO 10360 [12]. The targets were moved by a high accuracy parallel kinematic (hexapod) that is specified with a smallest linear and angular increment of 1 nm and 1 µrad as well as a linear and angular repeatability of 200 nm and 10 µrad. The target was moved along a circular trajectory r=5 mm in an inclined plane of 45° according to ISO 9283 [13]. The experimental setup is depicted in figure 2.

The measurement rate of the camera system was set to f = 30Hz. Three independent measurement runs were performed with v=1, 3, 5 mm/s. The PF and EKF were used for recursive forward-backward smoothing with a sliding window of 500 ms length of time. The measurement process was modelled as nonlinear estimation problem of 3-dimensional Cartesian coordinates. The measured quantity $\mathbf{y}_k$ (i.e. coordinates of the circular trajectory) is generated by

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r\cos\varphi_k \sin\theta_k \\ r\sin\varphi_k \sin\theta_k \\ r\cos\theta_k \end{bmatrix} + \mathbf{v}_k,$$

$$\mathbf{y}_k = \mathbf{x}_k + \mathbf{w}_k.$$

The PF was used with 200 particles, Gaussian process noise with standard deviation 5µm, and uniform measurement noise with standard deviation 50µm. The EKF always uses Gaussian noise with the same noise parameters as the PF. The applied process noise based on an uncertainty budget that incorporates the positioning error of the parallel kinematic, the calibration uncertainty and environmental influences. The measurement noise was determined by calibration measurements of the dynamic CMM according to ISO 10360 and VDI 2634 [11, 12]. The realized measurement error is given in Tab. 1. It was computed as Euclidean distance between the least square circles fitted in the measured coordinates as well as the unfiltered and filtered coordinates of the moved tracking target. The measured coordinates, the fitted circular trajectory and the histogram of deviations for a single measurement run are shown in figure 3. An example of an estimated posterior Bayes posterior PDF is given in figure 4.

The results show that the PF outperforms the EKF, especially in the case of high velocities. This is due to the increasing nonlinear dependence of consecutive measured coordinates, which yields to poor prediction accuracy of the Taylor approximation of the process dynamic of the EKF.

Tab. 1 Error in position measurements averaged over three independent runs for v=1, 3, 5 mm/s. The position error is defined as Euclidean distance in reference to the least square circle fitted in the measured coordinates of the CMM.

	Mean(error) in µm	Max(error) in µm	Std(error) in µm
CMM	10.1, 16.8, 24.1	19.8, 25.8, 40.1	0.9, 2.1, 3.1
EKF	8.96, 15.4, 20.7	15.9, 19.8, 30.3	0.7, 1.8, 2.1
PF	8.90, 13.8, 17.1	15.6, 19.8, 23.7	0.7, 1.5, 1.9

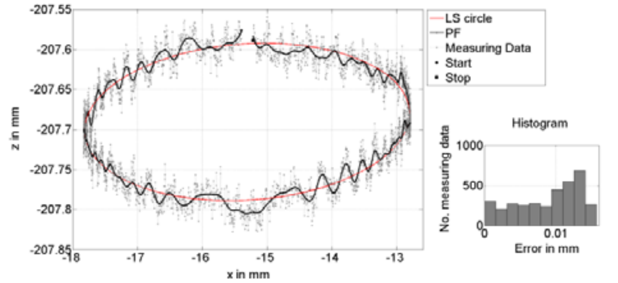

Fig. 3 Analysis of a measurement run. A least square circle was fitted in the measured coordinates of the CMM.

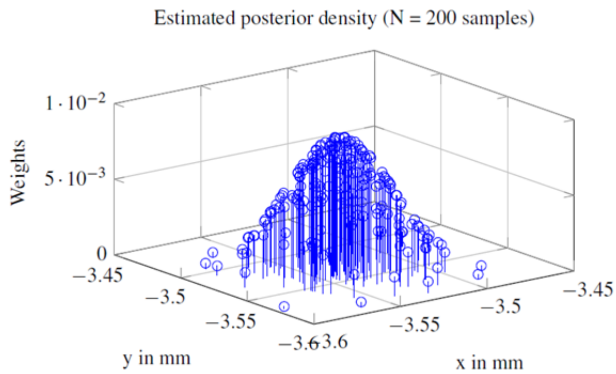

Fig. 4 Example of an estimated Bayes posterior PDF in a time step. Due to the sake of visualization the z-coordinate is neglected and only the 2D PDF approximation for the quantity $\mathbf{y}_k=[x_k,y_k]$ is shown.

5. EXPERIMENTAL RESULTS FOR UNKNOWN DYNAMICS

The Bayesian estimation is an extremely powerful and easy to implement method for inferencing in dynamic systems, i.e. data processing with the purpose of prediction, filtering or smoothing. However, it is very difficult to appropriately determine the process equation and even more to specify the noise sources, systematically in a general setting. Especially the dynamic function and the statistical characterization of the process noise were key design parameters with crucial impact on the achievable accuracy.

In case of coordinate measurements of standard geometrical features, as points, lines and circles[6], it is easily possible to derive an appropriate state space models, like autoregressive (AR) models [14,15] $\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{v}_k$, linear

[6] That means continuously measuring a point in space or measuring a linear or circular trajectories.

or circular models (e.g. as the one given in the last section). It is also quite feasible to develop process models in cases where the motion path is completely know a priori (e.g. drill trajectory) and will be realized by a mechatronic positioning system (e.g. robot). In such cases the dynamic equation $\mathbf{x}_k = \mathbf{f}(\cdot)$ can either be implemented as look-up table containing kinematic quantities for every time step or as analytical expression, e.g. by an *n*th-degree polynomial in Cartesian coordinates.

But for the general case of arbitrary coordinates along completely unknown trajectories it is almost impossible to determine a precise process model. Here it is only feasible to apply universally kinematic white noise models [16].

These models assume that a derivative of the position *s(t)* is a white noise process *w(t)*. The simplest model is the white-noise acceleration model, where the acceleration $\ddot{s}(t)$ is specified as white noise. The second simplest model is the Wiener-process acceleration model (WPAM). This model is commonly used as so called white-noise jerk model that assumes that the acceleration derivative, i.e. jerk *j(t)*, is a white noise process: $j(t) = \dot{a}(t) = \dddot{s}(t) = w(t)$. The corresponding state space representation for coordinate measurements is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{v}(t),$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \mathbf{x}(t) + \mathbf{w}(t),$$

with $\mathbf{x} = [s, v, a]^T$ along a generic Cartesian axis.

The above models are simple but crude. Actual dynamics seldom have constant accelerations that are uncoupled across coordinate directions. Furthermore it is hardly to specify adequate noise sources. Usually, the process and measurement noise should be chosen such that they represent the dynamic uncertainty and the uncertainty of measurement system. But this information is mostly not available for all kinematic quantities and can also only be obtained to some extent for the quantity position by calibration measurements.

Therefore an adaptive particle filter (APF) for arbitrary trajectories and kinematics was developed. It based on the assumption that the measurement target (i.e. evolution of coordinates in time) behaves locally like a polynomial of some degree *d* in time. For an arbitrary time window $T = (t_1, \ldots, t_n)$ and a fixed degree *d* of a polynomial with $n \geq d + 1$, a least squares interpolation polynomial can be fitted in the measured coordinates, continuously. Then, the extrapolation of the polynomial and the Markov-chain information in the particles is used to generate a process function. This approach solved several problems at once: a) the complete state vector can be reconstructed (i.e. analytically calculate higher derivatives of the measured position), b) the standard deviations for the likelihood function can be computed sequentially, c) the process noise can be adapted via the extrapolation error, continuously and d) there is no need to develop or laboriously parameterize a state space model.

In order to analyze the proposed methodology a highly nonlinear helix trajectory was generated by cylinder coordinate transformation with sampling period *T=1/30s*, 3rd order polynomial, N_P*=1000* particles and initial true state $\mathbf{x}_0 = [\varphi, r, z, \dot{\varphi}, \dot{r}, \dot{z}, \ldots]^T = [0,1,0,\pi/(100T),0,1/(10T),0,1,0]^T$. Gaussian process and measurement noise with $\sigma = \begin{bmatrix} 10 & 10 & 50 \end{bmatrix} \mu m$ for *x* ,*y* and *z* were used, which were determined by calibration measurements of the photogrammetric CMM (see last section). In the conducted experiments PF with WPAM collapsed after a few seconds (see fig. 5), whereas the APF shows superior estimation accuracy (see fig. 6) for all dynamics and kinematics. In order to study the WPAM the experiments were repeated with a less nonlinear trajectory generated with the initial true state $\mathbf{x}_0 = [0,1,0,2\pi/(100T),0,1/(100T),0,0,0.4]^T$ and the same noise sources. The resulting root-mean-square error (RMSE) with increasing number of particles N_P*=300, …, 10000* is depicted in figure 7. The APF realized always significant better estimation accuracy compared to SIR-PF with WPAM. Especially for low number of particles the WPAM model failed to compute an estimate. That results in NaN estimations. These results proof the robustness, outstanding estimation accuracy and easiness of the proposed methodology for arbitrary nonlinear dynamics and kinematics.

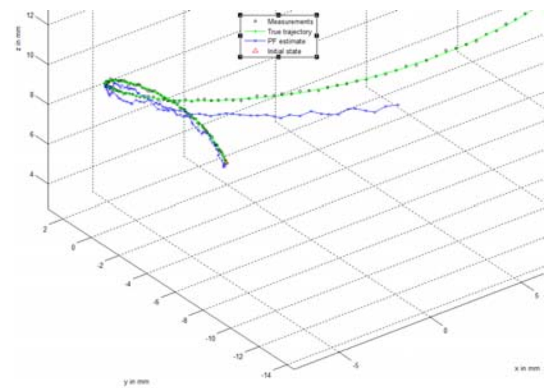

Fig. 5 Collapse of PF with WPAM after few time steps in the estimation of a highly nonlinear helix trajectory

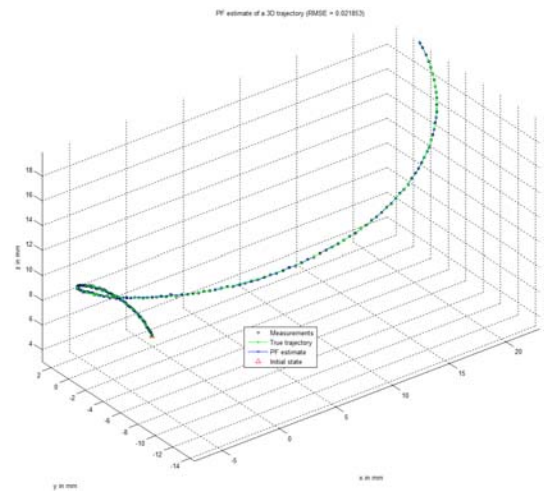

Fig. 6 Superior accuracy of proposed APF for the estimation of a highly nonlinear helix trajectory

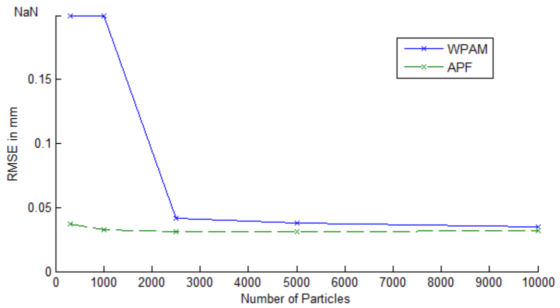

Fig. 7 Root-mean-square error (RMSE) with increasing number of particles for PF with WPAM and APF

6. FUSING DYNAMIC COORDINATE MEASUREMENTS

The usage of the Bayesian estimation for dynamical coordinate measurements requires the software integration in existing coordinate measurement systems as well as straightforward and easy to use models for various measurement tasks without laborious determination of model parameters. Since Bayes filter need as inputs only sequential coordinate measurements, their addition and integration in the data processing chain of existing coordinate measurement systems should be possible without problems.

The above given APF allows the measurement of arbitrary trajectories and kinematics of multiple objects and/or sensors by simply extending the state vector. This result automatically in Bayesian estimation for the combined state vector, i.e. a Bayesian data fusion is achieved. Hereby, the number of the state dimensions is only limited by the available computing power. The resulting workflow and data processing chain is given in figure 5.

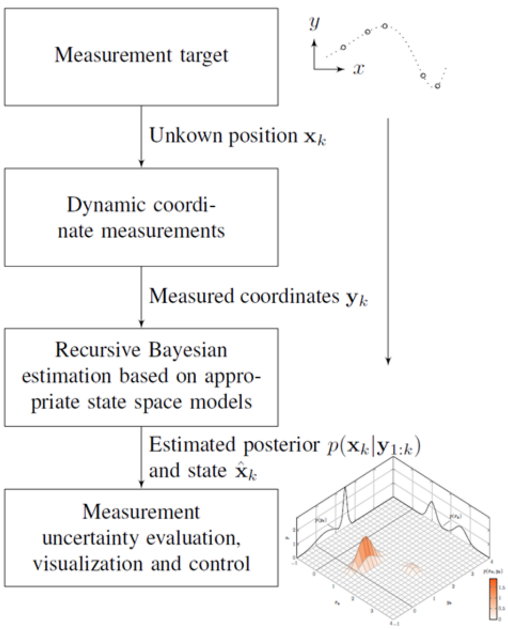

Fig. 8 Workflow of Bayesian filtering and data fusion for dynamic coordinate measurements. For the sake of visualization, the case of measuring a two-dimensional motion of two objects with a photogrammetric CMM is illustrated, i.e. the state vector $\mathbf{x}_k = [s_x, s_y]^T$.

7. SUMMARY AND CONCLUSIONS

In this paper the problem of improving and fusing coordinate measurements of an arbitrary dynamic process was considered. In order to deal with uncertainty and to optimally estimate the true measurand, it was solved as a Bayesian sequential probabilistic inference problem for nonlinear dynamic systems. Benefit of the developed methodology is a statistically sound, real-time and data-driven estimation of dynamic system variables of interest (measurand, parameters, etc.) in form of the Bayes posterior PDF. Based on the posterior density the measurement accuracy can be improved by filtering of ubiquitous noise and deducing an optimal quantity value with respect to any optimality criteria. Furthermore it is possible to realize a completely sensor-independent measurement uncertainty evaluation and probabilistic data fusion in every time step.

8. ACKNOWLEDGEMENTS

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