CALIBRATION TECHNIQUE OF ARTICULATED ARM COORDINATE MEASURING MACHINES BASED ON LARGE SCALE MULTILATERATION SYSTEM GAUGES

J. Santolaria, A. Brau , D. Samper, J. Mainz, J.J. Aguilar

Design and Manufacturing Engineering Dep., Universidad de Zaragoza, Zaragoza, SPAIN, jsmazo@unizar.es

Abstract: In this work we present a novel kinematic parameter identification procedure of an articulated arm coordinate measuring machine (AACMM). This procedure is based on a geometric multilateration system constructed from four Laser Trackers (LT's) that allow us to determine the spatial localization of a retroreflector target. Then, from the captured nominal data obtained by the LT's (used as a gauge instrument) and the data obtained by the AACMM, an objective function based on Euclidean distances and standard deviations is proposed. Hence, the self-calibration algorithm of the LT's positions and the coordinates calculated by means of multilateration, as well as the non-linear optimization algorithm proposed for the identification procedure of the AACMM are explained. Finally, we show how the obtained results show the effectiveness of this procedure in terms of using the measurements of the LT's as gauge instrument and in the final error committed by the measuring arm.

Keywords: Articulated arm coordinate measuring machine, Laser tracker, multilateration, parameter identification, kinematic calibration.

1. INTRODUCTION

Throughout the years, many researchers have proposed different approaches to address the problem of kinematic modeling and calibration of AACMM and robot arms. Some of these approaches include the ones presented by Denavit-Hartenberg [1], Hayati-Mirmirani [2], Sheth-Uicker [3], Stone [4] and Santolaria et. al. [5]. In general, the problem of kinematic modeling and calibration of AACMM and robot arms, aims to find the optimal kinematic model parameters from its (AACMM or robot arm) design values, where the error of the arm's calculated coordinates are as accurate as possible. A very common characteristic of most methods dealing with this is the mathematical approach towards the problem. Following the capturing procedure of a series of measured points and their respective nominal positions and orientations of the arm, the aforementioned methods obtain a set of parameters that minimize the positioning error in the captured positions. The set of parameters found, considerably increases the results obtained in both, the captured positions and the positions near the captured ones. Nevertheless a generalization of the evaluation of the positioning error at different arm positions

not included in the original identification procedure needs to be carried out. In this work, data has been captured by measuring simultaneously different points, distributed within the arm workspace, by means of the AACMM itself and four LT's. With the captured points by the LT's, a multilateration reference frame was constructed. Then, from the nominal coordinates expressed in the multilateration coordinate system and the captured positions of the AACCM at these points, a non-linear optimization procedure was done by means of least squares. The optimization objective function included the sum of distance errors between the captured points and the standard deviation of several captures positions in each considered point.

Moreover, we present the AACMM parameter identification based on nominal points obtained by means of multilateration of the captured coordinates with four large scale LT's measuring devices.

2. KINEMATIC MODEL

The kinematics of the measuring arm used in this work can be defined by the basic model developed by Denavit-Hartenberg (D-H). To this end, a kinematic model can be seen as a parametric model that accurately describes the arm kinematic behavior so that numerical relationships between the joint variables and the probe position for any arm posture can be established as shown in equation 1.

$$y = f(\theta_i, p) \tag{1}$$

The position and orientation of the end effector or probe (y) will be calculated according to the value of the joint variables (θ_i), $i = 1, \dots, n$ and to the equations of the model defined in f, which depend on the geometrical parameters vector p, making it possible to establish a relationship between the probes or end effector and the arm's global reference frame. This model is a mathematical representation of the direct kinematics which is characterized by four geometrical parameters in each of the arm joints (distances d_i, a_i and angles θ_i, α_i) defined in the basic D-H model. These parameters are used to calculate the transformation of coordinates between the successive coordinate frames associated with the arm joints. The homogeneous transformation matrix between frames i and i−1, which depends on the four

$$i^{-1}A_i = T_{z,d}\, T_{z,\theta}\, T_{x,a}\, T_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_i & -\operatorname{sen}\theta_i & 0 & 0 \\ \operatorname{sen}\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_i & -\operatorname{sen}\alpha_i & 0 \\ 0 & \operatorname{sen}\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos\theta_i & -\cos\alpha_i\operatorname{sen}\theta_i & \operatorname{sen}\alpha_i\operatorname{sen}\theta_i & a_i\cos\theta_i \\ \operatorname{sen}\theta_i & \cos\alpha_i\cos\theta_i & -\operatorname{sen}\alpha_i\cos\theta_i & a_i\operatorname{sen}\theta_i \\ 0 & \operatorname{sen}\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2)

mentioned parameters, is given by equation 2. This way, it is possible as stated before; to obtain the position and orientation of the end effector or probe of the arm for any given articulation values with respect to the instrument's global reference coordinate system.
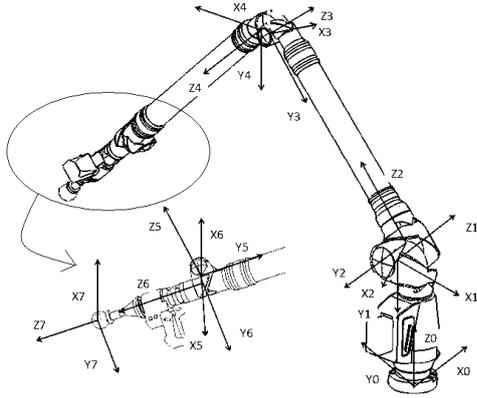

Fig. 1. Initial position of the measuring arm.

JOINT	θ_i	α_i	a_i	d_i
1	-90	-90	-40	300
2	135	-90	-40	0
3	180	-90	-34	590
4	-90	90	34	0
5	-90	-90	-34	588
6	180	-90	-34	0
7	0	0	0	318,2

Tabla 1. Initial parameters of measuring arm.

3. MULTILATERATION

In geometry, multilateration is the process of determining absolute or relative locations of points through the measurement of distances using the geometry of circles, spheres or triangles. To determine the *xyz* coordinates of a point in the multilateration reference system, three reference points are needed. These points can be located arbitrarily. For simplicity in the calculations, the three points have been located in coordinates $(0, 0, 0)$, $(x1, 0, 0)$ and $(x2, y2, 0)$ with respect to the global coordinate system. Therefore, the measured diagonal distances between the different systems and the lens can be expressed as follows:

$$r_0^2 = x^2 + y^2 + z^2 \qquad (3)$$
$$r_1^2 = (x - x_1)^2 + y^2 + z^2 \qquad (4)$$
$$r_2^2 = (x - x_2)^2 + (y - y_2)^2 + z^2 \qquad (5)$$

By grouping the solution equations, the coordinates of the lens in the global reference system are derived in equations $6 - 8$:

$$x = \frac{r_0^2 - r_1^2 + x_1^2}{2\,x_1} \qquad (6)$$

$$y = \frac{r_0^2 - r_2^2 + x_2^2 + y_2^2}{2\,y_2} - \frac{x_2}{y_2}\,x \qquad (7)$$

$$z = \pm\sqrt{r_0^2 - (x^2 + y^2)} \qquad (8)$$

To avoid the sign ambiguity in equation 8 in the Z coordinate, a fourth reference point located at (x3, y3, z3) must be used, not in the XY plane formed by the first three reference points where z3 = 0, as shown in figure 2. Then, through algebraic manipulation, the xyz coordinates of the target are derived in linear matrix form as expressed in (9). Since nominal dimensions are usually materialized by a physical gauge in these types of procedures, the obtained coordinates by means of multilateration have been used as a calibrated gauge, obtained from measurements made with the 4 LT's devices. A LT is a large scale measuring instrument with higher precision in the measuring of longitudinal distances, although they are around the same order of magnitude than that of AACMM. When using techniques based on multilateration with various measuring instruments measuring the same physical point, it is possible to capture points with an uncertainty much smaller than that obtained with an AACMM, thus allowing the consideration of these captured points to be used as gauge points in the calibration procedure of AACMMs.
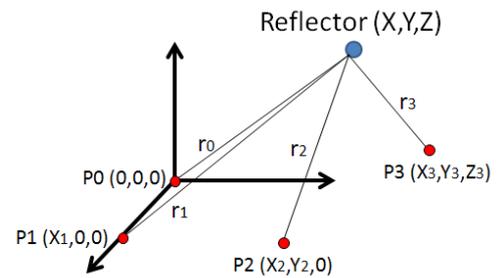

Fig.2. Multilateration geometry

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0,5 \cdot \begin{bmatrix} \frac{1}{x_1} & 0 & 0 \\ -\frac{x_2}{x_1 y_2} & \frac{1}{y_2} & 0 \\ -\left(\frac{x_3}{x_1 z_3}\right) + \left(\frac{x_2 y_3}{x_1 y_2 z_3}\right) & -\frac{y_3}{y_2 z_3} & \frac{1}{z_3} \end{bmatrix} \cdot (-1) \cdot \begin{bmatrix} r_1^2 - r_0^2 - x_1^2 \\ r_2^2 - r_0^2 - x_2^2 - y_2^2 \\ r_3^2 - r_0^2 - x_3^2 - y_3^2 - z_3^2 \end{bmatrix} \quad (9)$$

4. TEST SETUP

The data capture process consists of measuring a cloud of points located within the arm workspace simultaneously with the LT's, that conform the multilateralized system, and the AACMM, looking to both, cover positions distributed throughout the workspace of the arm and reached different arm angle values.
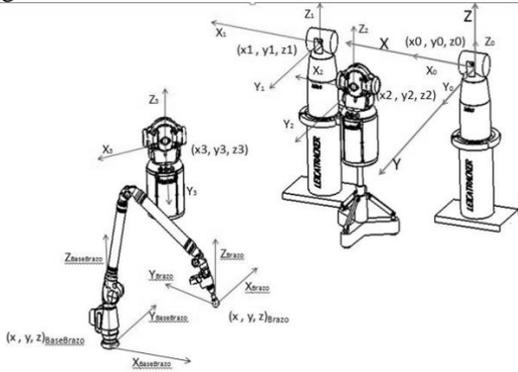
Fig.3. LT and AACMM used in tests

The LT's have been distributed in order to form the multilateration global coordinate system, forming 3 of them the global XY plane, and the fourth LT is set in a position not contained in the same plane. Finally the AACMM has been arranged in a position that maximizes the visibility of the LT's, as shown in Fig.3.

In the realization of the tests, 23 different positions of the retro-reflector were captured, arranged in such way that the first 21 have been used in the parameter identification procedure and the remaining 2 were used for evaluation. In the capturing data process, the 4 LT's simultaneously evaluate the distance from the captured point to the local coordinate system. With these distances, the reference systems of the 4 LT's are identified and expressed in the multilateration reference system, to then obtain the captured point coordinates in the same reference system. In regards to the AACMM, for each identifying position, 5 different configurations of angles were captured and 15 for evaluation of positions.
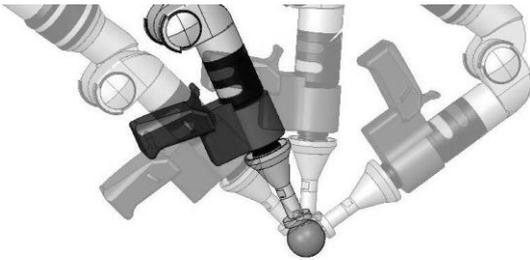

Fig. 4. Positions captured of the same physical point.

In order to perform the measurements continously with the AACMM, a self-centering probe has been used. This probe allows the direct probing of the retro-reflector center without the need of probing discrete points on the reflector surface.
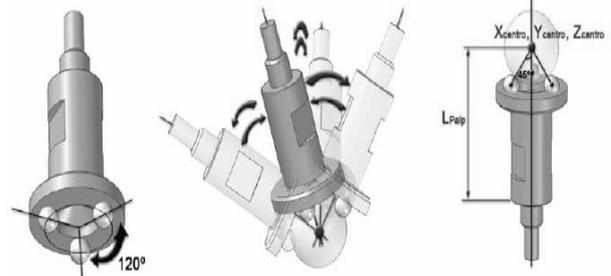

Fig.5. Self-centering probe

By means of the self-centering probe, different articulations angles configurations of the arm for the same retroflector position have been captured. All these points should theoretically correspond to the same retroreflector center invariably of the articulation angles, although, due to the initial set of considered arm parameters, the captured data in the AACMM global reference system materialized different points that depend on the articulation configurations while during the capturing procedure. The considered point on each reflector position captured in the AACMM reference system will be the average of all them in the corresponding position.

$$\overline{X_{IJ}} = \frac{\sum_{m=1}^{n_{ij}} X(m)_{ij}}{n_{ij}} \quad (10)$$

5. PARAMETER IDENTIFICATION

The parameter identification procedure of the AACMM starts off from the initial kinematic parameters defined in table 1. Afterwards, the optimum values that approximate the distances between captured points are calculated by means of least squares through the minimization of the error in the objective function shown in (11), using as a reference the points obtained by the multilateration procedure. This way, using the distances in the objective function avoids the coordinate transformation error, previous to a direct comparison in terms of the points coordinates.

$$\phi = \sum_{i=1,j=1}^{n} [(D_{ij\_AACMM} - D_{ij\_MULT})^2] + \sum_{i=1}^{n} [(\sigma_{xi})^2 + (\sigma_{yi})^2 + (\sigma_{zi})^2] \quad (11)$$

Once the parameter values are calculated, it is possible to observe the distribution of the points expressed in the AACMM reference system in all the 23 captured positions

against the points obtained with the initial parameters considered.
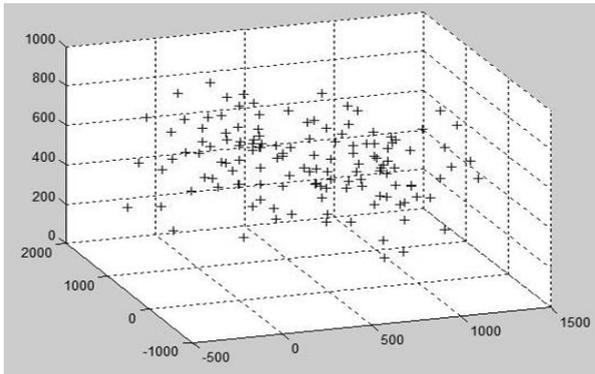

Fig.6. Points coordinates with initial parameters of AACMM.
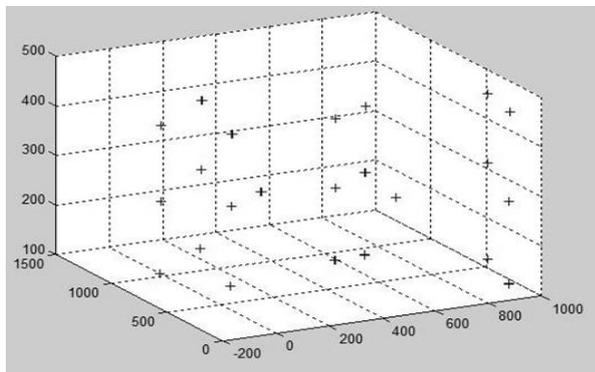

Fig.7. Points coordinates with optimized parameters of AACMM.

With the coordinates of the points optimized in the AACMM reference system and the distances corresponding to these values, the error in distances comparing the optimized and nominal values has been evaluated following the scheme presented in fig. 8. A mean error of 0.049 mm was obtained, with a maximum error in all the positions evaluated of 0.128 mm.
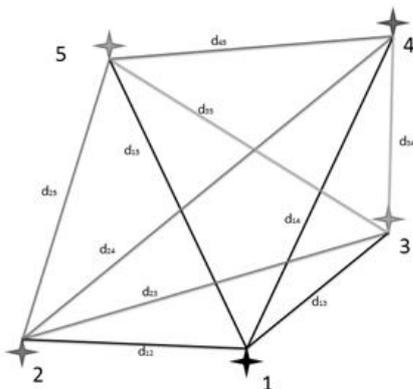

Fig.8. Distances comparison scheme.

6. CONCLUSIONS

In this work a novel calibration technique for parameter kinematic identification of a AACMM is presented. This procedure uses as reference values the materialized distances of the coordinates obtained through simultaneous measurements of four long range measuring systems, substituting the use of physical gauges.

Moreover, the use of multilateration system was validated for these types of procedures due to its high precision, sufficiently to be used as a gauge in identification procedure.

7. ACKNOWLEDGEMENTS

8. REFERENCES

[1] Denavit J., Hartengerg R.S., "*A kinematic notation for lower pair mechanisms based on matrices. s.l.*" Journal of Applied Mechanics, 1955, Vol. v.22, págs. 215-221.

[2] Ibarra R., Perreira N.D. Determination of linkage parameter and pair variable errors in open chain kinematic linkages using minimal set of pose measurement data. s.l. : Journal of Mechanisms, Transmissions, and Automation in Design, 1986, págs. 159-166.

[3] Hayati S.A., Mirmirani M. Improving the absolute positioning accuracy of robot manipulators. s.l. : Journal of Robotics Systems, 1985, Vol. 2, págs. 397-413.

[4] Seung-Woo K., Hyug-Gyo R., Ji-Young Ch. Volumetric phase-measuring interferometer for three-dimensional coordinate metrology. Korea Advanced Institute of Science and Technology, Science Town, 2002.

[5] J. Santolaria, A. Brau, J. Velázquez, J.J. Aguilar, A self-centering active probing technique for kinematic parameter identification and verification for articulated arms coordinate measuring machines, *Measurement Science & Technology*, 21, 055101 (2010) 11pp.