

# COMPARISON EXPERIMENT FOR PULSE REPETITION INTERVAL BASED LENGTH MEASUREMENT LINKED TO A FEMTOSECOND OPTICAL FREQUENCY COMB

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**Abstract:** As an alternative to the conventional method, which length measurement is performed as a function of the wavelength of a monochromatic laser source, we proposed the method of measuring arbitrary and absolute length using the pulse repetition interval of a femtosecond optical frequency comb (FOFC - the national standard tool for measuring length in Japan). A helium-neon laser evaluation system was developed for distance determination comparison experiment and its uncertainty estimation was performed in order to validate the proposed method.

**Keywords:** Length measurement, Optical frequency comb, Pulse train, Interference, Repetition interval.

## 1. INTRODUCTION

Development of novel length measurement methods directly linked to a femtosecond optical frequency comb (FOFC) or FOFCs is currently in progress. For example, see Ref. [1-6]. The basic ideas of these length measurement methods are simple. One is from the traditional heterodyne method [7, 8], which is to treat a distance as a function of wavelength. Another is to measure a length based on the adjacent pulse repetition interval length (APRIL). Our results [1-6] suggested that in air there is no difference between the usage of wavelength or APRIL as a scale in air. The present investigation was a review and an exploration of the APRIL-based length measurement idea.

This paper is organized as follows. First, a short of literature review is given in Section 2. Next, an FOFC and its coherence character are described in Section 3. Next, Comparison between the stability of a wavelength and the stability of an APRIL is described in Section 4. Then, the basic scheme of the experiment and the result of the preliminary experiments are shown in Section 5. Finally, the main conclusions is summarized in Section 6.

## 2. A SHORT LITERATURE REVIEW

The literature review of length measurement using pulse lasers were performed in Ref. [9]. This review was specializing in the FOFC-based measurement methods.

For distance measurement, displacement measurement [10, 11] [12] and absolute length measurement [13] [14] [15] are different. Next, short distance ( $\leq 1 \mu\text{m}$ ) measurement [16] and long ( $> 1 \text{mm}$ ) distance measurement [17] are different. In the case of a short distance measurement, the influence on the length measurement by the refractive index of air ( $10^{-6}$  order) can be disregarded. When the measurement object is long, we can suppress the atmospheric turbulence, for example, performing the experiment in an underground tunnel [18] or using a vacuum pump [19] to shut out the air. We can do a few things [20] [21] to the experimental environment (distribution of temperature and atmospheric phase fluctuation [22] etc.) at the measurement of a long length distance in an open air.

## 3. FOFC AND ITS TEMPORAL COHERENCE CHARACTER

For convenience of explanation, let us briefly review the essence of an FOFC and its temporal coherence character [6].

The features of an FOFC can be summarized as follows [23, 24]: In the frequency domain, a mode-locked laser generates equidistant frequency comb lines with the pulse repetition frequency  $f_{\text{rep}}$ , and the whole equidistant frequency comb is shifted by offset frequency  $f_{\text{CEO}}$  from zero frequency. In the time domain, when the electric field packet repeats at the pulse repetition period  $T_R = 1/f_{\text{rep}}$ , due to offset frequency  $f_{\text{CEO}}$ , the carrier phase slips by  $\Delta\varphi_{\text{ce}} = 2\pi f_{\text{CEO}} / f_{\text{rep}}$  to the carrier-envelope phase.

The features of temporal coherence character of an FOFC can be summarized as follows. The power spectrum of an FOFC light source can be expressed as the multiplication of a comb function and a Gaussian function. Since, the Fourier transform of a comb function is a comb function, and the Fourier transform of a Gaussian function is also a Gaussian function. Based on the Wiener-Khinchine

theorem, the interferometric signal of the autocorrelation function is the convolution integral of a comb function and a Gaussian function. (Fig. 1.)

As a result, it is understood that the temporal coherence function periodically displays a high temporal coherence peak with the pulse repetition period where the pulse trains display a high-intensity peak.

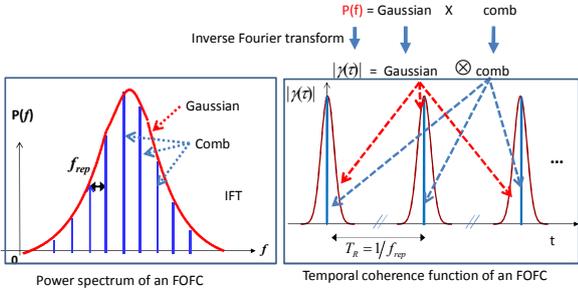


Fig. 1. Temporal coherence character of an FOFC

#### 4. WAVELENGTH VS APRIL

An APRIL is calculated by  $c / (f_{rep} \times n)$ , where  $c$  is a light velocity in the vacuum,  $n$  is the refractive index of light propagated medium. The stability of an APRIL depends on the stability of  $c$ ,  $f_{rep}$ , and  $n$  respectively. The light velocity in the vacuum is a constant as 299 792 458 m/s. The stability of  $f_{rep}$  is better than  $10^{-13}$  levels [25, 26], and is limited by the stability of the radio frequency standard [27] which the FOFC source was locked. The stability of the refractive index of light propagated medium is limited by the technology of measurement, generally is about  $10^{-8}$  order for air [28, 29] [30].

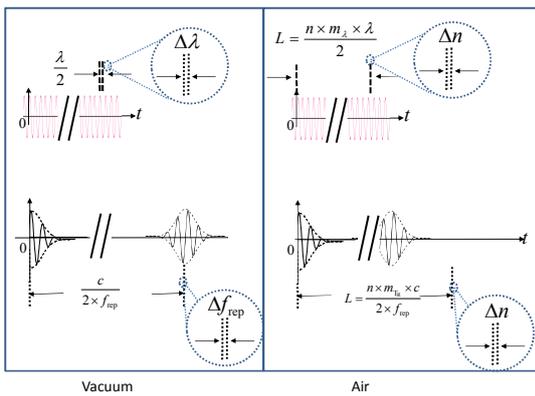


Fig. 2. Wavelength and APRIL used as a scale.

From the above-mentioned analysis, we can conclude that the APRIL of an FOFC and the wavelength  $f$  of a monochromatic laser source have stability at the same level as a fundamental length scale, because the stability of  $f_{rep}$  is smaller than the stability of  $n$ . In other words, the stability

of  $n$  decides the stability of  $c / (f \times n)$  and  $c / (f_{rep} \times n)$ . (Fig. 2.)

#### 5. EXPERIMENT SETUP

The basic scheme of the experiment was shown in Fig. 3. The measurement will be done in the next two steps.

1. In the first step, the translation stage will be set at the end of the translation stage near the beam splitter; we can record an interference fringe by the same pulse train.

2. In the second step, the translation stage will be moved to the far end of the translation stage, we can observe an interference fringe by the different pulse train.

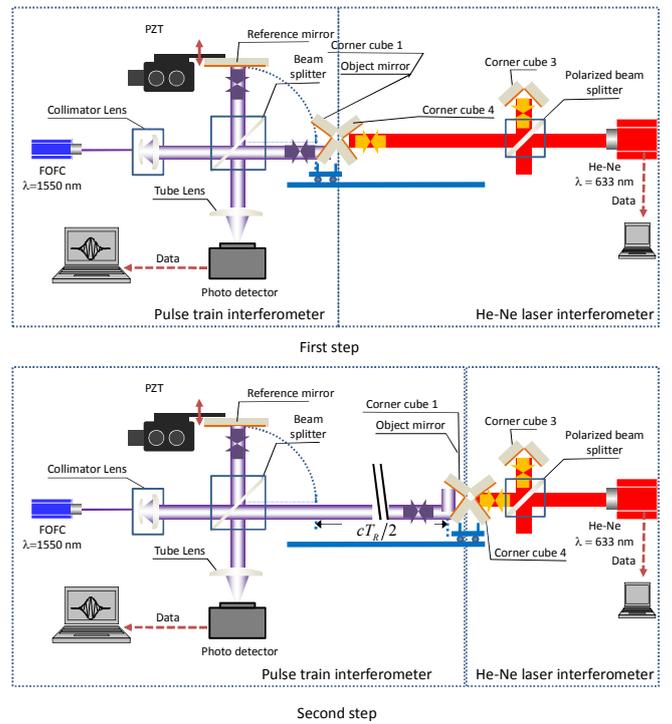


Fig. 3. Optical system and procedure for experiment.

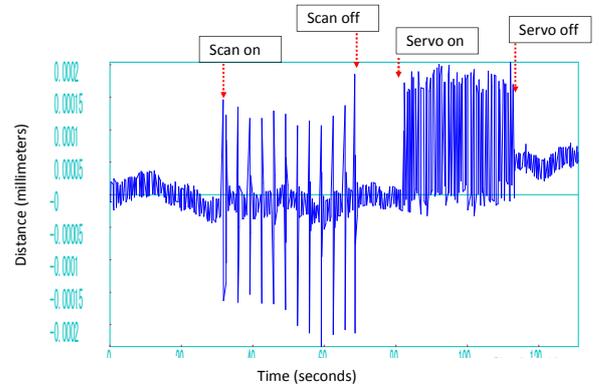


Fig. 4. The stability of the stage and vibration introduced by the servomechanism of the stage and scanning of the reference mirror.

When we move the translation stage, we record the displacement of the translation stage by the He-Ne laser interferometer.

Before obtaining the data, we examined the influence from the vibration by the servomechanism of the stage and scanning of the reference mirror using the He-Ne laser interferometer. A typical data set is shown in Fig. 4. The capture rate of data was 1000 Hz, and the acquisition time was 130 seconds. The following three facts can be understood from this data. First, the stage always vibrated by about  $\pm 30$  nanometers due to the experimental condition. Second, the vibration introduced by the servomechanism of the stage and scanning of the reference mirror was about  $\pm 200$  nanometers and about  $\pm 400$  nanometers, respectively. Third, no large shift of the stage due to these two kinds of vibration has been observed. In other words, after receiving the vibration, the stage returns to its former location.

A typical data set of the scanning of the stage is shown in Fig. 5. We confirmed that the scanning of the stage was linear and steady.

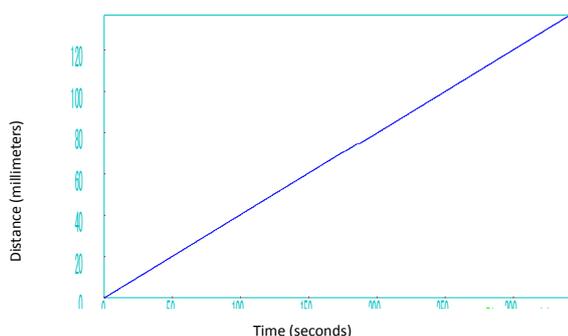


Fig. 5. The stability of the scan stage.

A typical data set of the change of measurement value by temperature change is shown in Fig. 6.

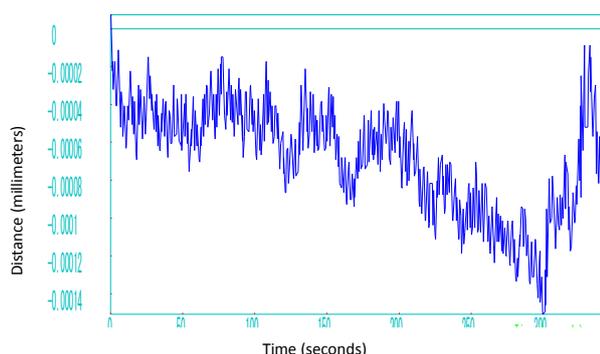


Fig. 6. The change of measurement value by temperature change.

The conclusion drawn from the results of the above data is that the proposal method can be verified by the meters order length measurement comparison experiment with the accuracy of a few hundred nanometers.

## 6. SUMMARY

As an alternative to the conventional method, which length measurement is performed as a function of the wavelength of a monochromatic laser source, we proposed the method of measuring arbitrary and absolute length using the adjacent pulse repetition interval length (APRIL) of an FOFC - the national standard tool for measuring length in Japan. We theoretically studied the stability of APRIL. A helium-neon laser evaluation system was developed for distance determination comparison experiment and its uncertainty estimation was performed in order to validate the proposed method. From the result of the preliminary experiments, we can conclude that the proposal method can be verified by the meters order length measurement comparison experiment with the accuracy of a few hundred nanometers.

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