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MULTILATERATION IN VOLUMETRIC VERIFICATION OF MACHINE TOOL

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Abstract: This paper presents the role played by multilateration when laser trackers (LTs) are used in volumetric verification of long range machine tools.

A comprehensive study on the influence of LTs positioning has been done. The parameters taken into account during the positioning of the LTs are: the viewing angle, the distance to the work space and the LT measurement errors.

Also, different self-calibration techniques for calculating the LTs positioning are explained in order to show their influence in the correction improvement using multilateration techniques.

Finally, it is shown how obtained results improve the effectiveness of this procedure in terms of the final error correction when LTs are used in long range machine tool volumetric verification.

Keywords: Machine tool volumetric verification, Laser tracker, Multilateration, Laser tracker self-calibration.

1. INTRODUCTION

Traditionally, machine tool geometric errors have been characterized using geometric verification. The poor adaptation of geometric verification techniques in long range machine tools makes of the verification procedure a slow and expensive process. A reduction of up two thirds of measurement time can be achieved using volumetric verification [1-3] for global error correction on the studied work space using large scale measurement systems like LTs.

A laser tracker (LT) is a portable measurement system which provides the spherical coordinates of a point measured by a laser beam. The points capture is done tracking the laser beam of a moving retro-reflector.

The greatest contribution to measurement uncertainty is due to the angular measurement noise. It depends on LT encoder's resolution. The contribution of angular measurement uncertainty to overall uncertainty is much bigger than the other component due to the radial measurement provided by the LT interferometer. Multilateration eliminates the angular error contribution. However, in practical applications, measurement errors are still present due to a bad LTs positioning or a bad LTs self-calibration process.

This paper presents the influence of the LTs self-calibration, and the importance of LTs location in multilateration error reduction.

2. THEORETICAL PRINCIPLES OF MULTILATERATION

The use and disposal of one, three or four LTs, is an important parameter when allocating the different LTs available [1].

When a work space point in machine tool coordinate system (CSMT) is measured with three LTs, that point shall be measured at three different LT coordinate systems (CSLTi). where i represents the number of LT. If the measured point is transformed from CSLTi $_{(i=1,2,3)}$ to CSMT three different points will be obtained due to the measurement noise.

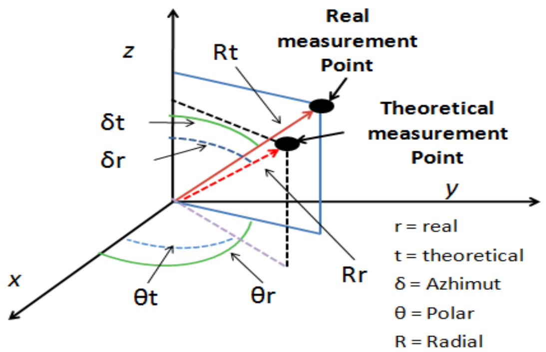

Figure1: Measurement noise

The measurement noise of each LT consists of two angular components azimuth and polar and a radial component (figure 1). Multilateration aims to achieve a lower measurement uncertainty. This is because only the radial component of each LT is used to determine the new multilateration point.

The new point is obtained by the intersection of the three spheres defined by the radial component of each LT. With (x_i y_i z_i) the centre of the spheres associated to LT$_i$ $_{(i=1,2,3)}$.

$$D_1^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 \quad (1)$$

$$D_2^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 \quad (2)$$

$$D_3^2 = (x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 \quad (3)$$

One of the processes most widely used to calculate the intersection of three spheres consists in transform the quadratic equations system in a lineal equations system. To do that, it is necessary to generate a new reference coordinate system (NCS), which has the following characteristic (figure 2-3):

- The origin of the new NCS coordinate system coincides with one of the CSLTi system (for example, CSLT1)
- The union of the origins from CSLT1-CSLT2 gives the direction of X axis
- The Y axis is defined so that the origin of CSLT3 is included in the XY plane.

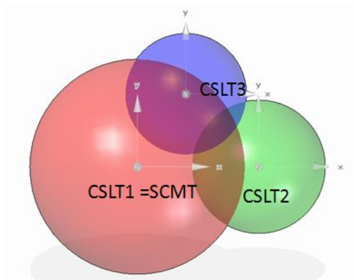

Figure 2: Multilateration technique

Solving the equations system the point coordinates can be obtained:

$$x = (D_1^2 - D_2^2 + x_2^2)/(2x_2) \quad (4)$$

$$y = (D_1^2 - D_3^2 + x_3^2 + y_3^2 - 2xx_3)/(2y_3) \quad (5)$$

$$z = (D_1^2 - x^2 - y^2)^{1/2} \quad (6)$$

The sign of the z component can be determined using a fourth LT, or an appropriate positioning of the three LTs and the study of the valid z component of each point.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} \frac{1}{x_1} & 0 & 0 \\ \frac{-x_2}{x_1 y_2} & \frac{1}{y_2} & 0 \\ \frac{-x_3}{x_1 z_3} + \frac{x_2 y_3}{x_1 y_2 z_3} & \frac{-y_3}{y_2 z_3} & \frac{1}{z_3} \end{bmatrix} \begin{bmatrix} r_1^2 - r_0^2 - x_1^2 \\ r_2^2 - r_0^2 - x_2^2 - y_2^2 \\ r_3^2 - r_0^2 - x_3^2 - y_3^2 - z_3^2 \end{bmatrix} \quad (7)$$

The uses of a fourth LT transforms the system of equations formed by eq 4, 5, 6 in a matrix system resolvable applying equation 7.

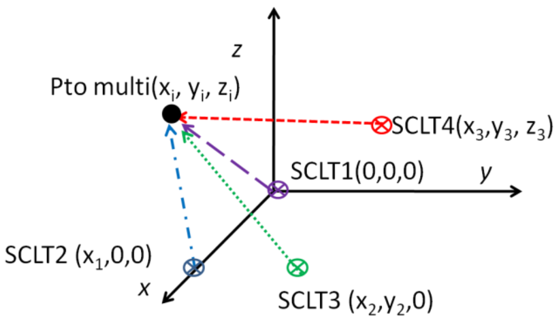

Figure 3: LT positioning in analytical multilateration and optimization self-calibration processes

3. TECHNIQUES FOR SELF-CALIBRATION OF THE LASER TRACKERS

The correct calculation of LTs positioning is very important when facing any measurement.

The volumetric verification technique needs to know the position of the machine tool points in machine tool coordinate system. However, points measured are known in LT coordinate system, and are affected by the measuring noise. When multilateration is applied to the measures obtained with three or four LTs, a new mesh/cloud of points is obtained. The new mesh/cloud is less affected by measurement noise.

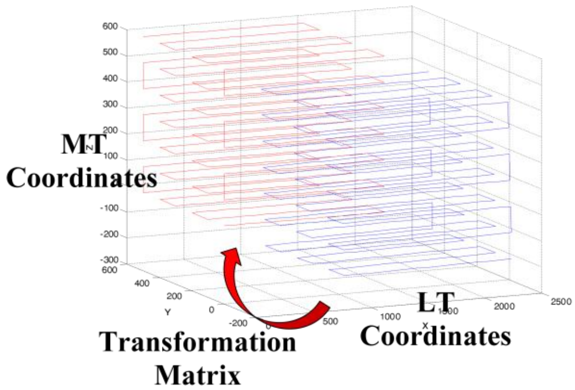

Figure 4: Obtaining transformation matrix

Adjustment of the different coordinate systems can be made using least-squares technique or optimization techniques such as quadrilaterion or trilateration (figure4).

Least-squares adjustment [4] obtains the rotation-translation matrix between CSLT system and CSMT reference system. It is the result of minimizing the difference between the measured mesh using both radial and angular components in $CSLTi_{(\ i=1,2,3)}$ and the reference mesh in the CSMT. Once the positions of LTs are obtained, they are used in conjunction with only the radial information of the LTs meshes to calculate the multilateralized mesh (figure 5).

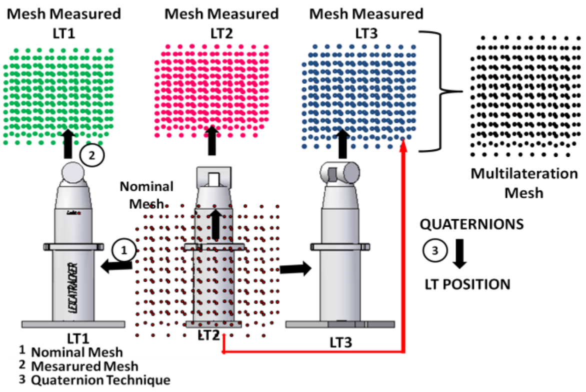

Figure 5: Quaternions self-calibration

Points captured by LT and used to LT self-calibration are affected by MT geometrical errors and LT measurement noise. The effect of geometrical error in the matrix obtained results in a centering of them self in the new coordinate system. However, the presence of measurement noise results in a modification of the matrix obtained.

Quadrilateration and trilateration do not use any angular information obtained by LTs. These methods provide the LTs position and multilateralized points using only the radial component of points measured by each LT as a result of nonlinear optimization (figure 6).

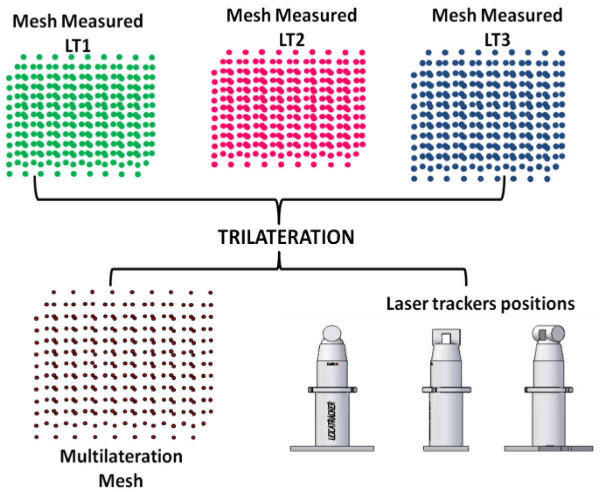

Figure 6: Trilateration self-calibration

In order to apply trilateration or quadrilateration LT self-calibration techniques, it is needed to create a new reference system as shows the third point of this paper.

If LTs positioning parameters ($x_1\, x_2\;\; y_2\, x_3\;\; y_3\;\; z_3$) are considered, the initial coordinates of the optimization points ($X_i\, Y_i\, Z_i$) and LT measured points coordinates provide the distance between LT coordinate system and measured point k_{ij} . With i number of LT and j the measured point,

$$k_{ij} = \left[\left(x_{ij} - X_i\right)^2 + (y_{ij} - Y_i)^2 + (z_{ij} - Z_i)^2\right]^{1/2} \qquad (8)$$

The calculated distance between CSLT$_i$ (k_{ij}) and CSRef (d_i), obtained from $X_i\, Y_i\, Z_i$, must be the same. If it does not, the assumptions for the positioning parameters ($x_1\, x_2\;\; y_2\, x_3\;\; y_3\;\; z_3$) are incorrect. The new parameters are obtained minimizing *Rest*, modifying LT positioning parameters an optimization points.

$$Rest^2 = \sum_{j=1}^{n} \sum_{i=1}^{4} \left\{\left[\left(x_{ij} - X_i\right)^2 + (y_{ij} - Y_i)^2 + (z_{ij} - Z_i)^2\right]^{1/2} - d_i\right\}^2 \qquad (9)$$

4. LASER TRACKER POSITIONING

Each point measured with each laser tracker is surrounded by a characteristic error ellipse. It indicates the area of uncertainty for each measured point.

Multilateration reduces the uncertainty area by spheres intersections obtaining a lower uncertainty area. The size of the zone is determined by the relative spatial position between each LT, specifically the spatial angle formed between them (figure 6). The position of the LTs determines the maximum possible noise reduction. However, when real measurements are made, the space to place LTs is limited. It is necessary to study the work space available to select the LTs distribution in which the error reduction is greater.

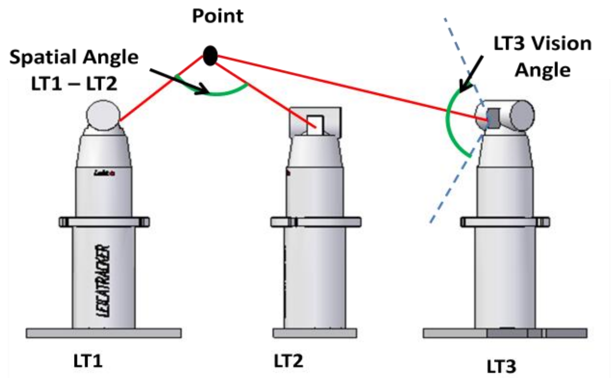

Figure 6: Angles of interest in a LT

Similarly, the positioning of the LTs also determines the amount of work space that is possible to be studied. This is determined by the tracking capability of the reflector for each LT. The viewing angle of the reflectors used is ± 30°. So that, the angle between any measured point and the LT position must be within this range. The ability to turn the head of each LT should be taken into account when positioning the LTs to avoid cutting the laser beam.

5. TESTS AND RESULTS

In order to check the influence of measurement noise in multilateration, the nominal LT measurement noise was modelled as a uniform distribution with values of table 1, for each one of the LT [5].

Table 1: Measurement noise

LTs Noise		
Azimuth	3,00E-05	µrad
Polar	3,00E-05	µrad
Radial	4	µm
	0,8	µm/m

The LT positioning study determines the influence of spatial angles in measurement noise reduction. In a work space with 0 ≤ X ≤ 1400, 0 ≤ Y ≤ 500, 0 ≤ Z ≤ 600, and maintaining all radial distance constant. A total of 20.000 measurements of the gravity centre point of the work space P = [0, 250, 300] were simulated. At first, the three LTs are at the same point, when the spatial angle between LTs is modified maintaining the same spatial angle between each LT $\alpha_{12} = \alpha_{13} = \alpha_{23}$. LTs look like a tripod.

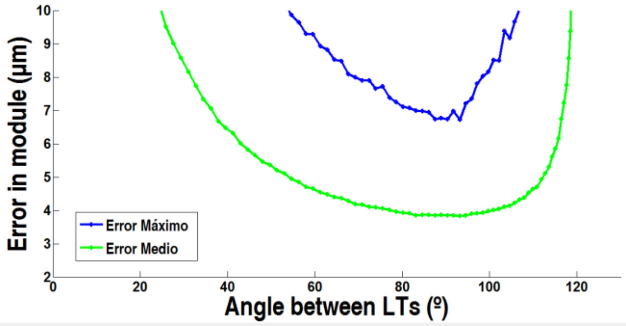

Figure 7: Influence of spatial angle in error measurement error reduction.

It is an ideal situation in which there are not restriction is LT positioning. The aim of this test is to study the scope of multilateration technique as well as its tendency. (figure 7) The figure 7 shows how the best result is produced whit a spatial angle of 90° between LTs. At this point, a complete elimination of the angular noise is obtained. The multilateration achieves a noise reduction of 94.2%. Moreover, a rapid increase of the error can be observer when spatial angle is bigger than 100º (figure 9).

The actual spatial arrangement of LTs is limited by the usable space around the machine tool, as well as the characteristics of the LTs. Therefore, a new test was made modifying the spatial angle between the LTs. Two LTs were positioned at the same height LT2-LT3 while the LT1 was placed at several heights in a range up to two meters higher than LT2-LT3. Firstly, all of them are in the same position. After then, LT2 and LT3 modify its spatial angle between 10º and 340º. Subsequently, the height of LT1 is modified changing the angle $\alpha_{12} = \alpha_{13}$.

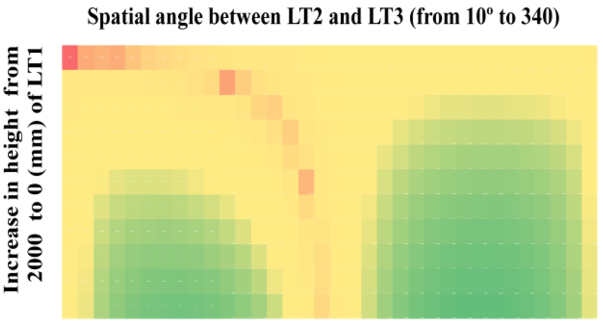

Figure 8: Measurement error reduction colour map (real LT positioning)

The colour map of figure 8 shows the relative reduction of error as a function of spatial angle between the LTs. The colour scale represents at green areas the grater error reduction capacity and at red the areas the lower error reduction (table 2). In the upper left corner, the height of the three LTs is the same and the spatial angle between LT2 and LT3 is 10º. Meanwhile, in the lower right corner, the spatial angle between LT2 and LT3 is 340º and LT1 height is 2 meters greater than LT2.

Table 2: Colour scale of figure 8

Residual Error	Min (µm)	Max (µm)
Green	5.65	12.4
Yellow	12.4	1504
Red	Higher	

The closer spatial angles are to the ideal position, the bigger noise reduction is obtained. Positioning the LTs in the ideal position is almost impossible as a result of the restrictions imposed.

Maintaining fixed the LTs spatial angles, the restriction on the angle of vision of the LTs and restriction on tracking angle have to be checked (figure 9).

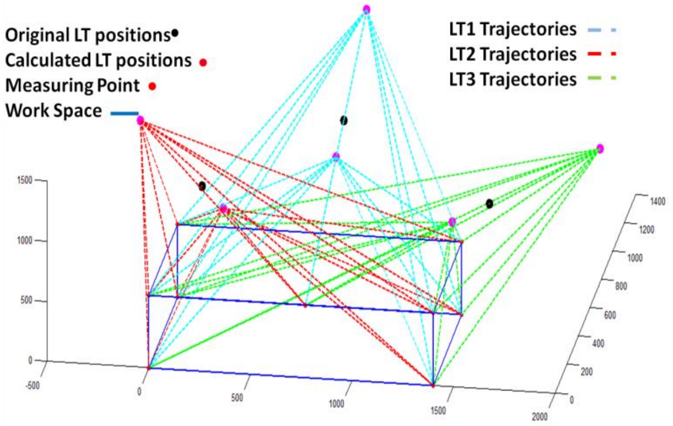

Figure 9: Influence of LT angles

Of all the possible positions of the LTs, the best option is the one which meets the visibility of the angles and the laser is located as close as possible to the workload. The improvement is a result of the reduction in the variable component of the radial noise, reducing the average error around 2-4% depending on the distance to the measured point.

Using a parametric synthetic data generator [2-3] points captures from each of three laser tracker were generated. Measured points were affected by geometric error of the machine depending on the position of the target reflector point within its workspace and measurement noise according to LTs positioning (table 3). The position of the LTs, which limits the scope of multilateration, was extracted of color map in figure 8. LTs positioning provides an error of 5.27µm in measuring the midpoint of the workspace.

Table 3: Laser tracker positioning

Nº LT	Coordenate X(mm)	Coordenate Y(mm)	Coordenate Z(mm)	Turn About X(º)	Turn About Y(º)	Turn About Z(º)
LT 1	700	2769	1950	15	35	20
LT 2	-1228	2548	550	60	5	35
LT 3	2628	2548	550	5	60	40
LT 4	700	250	1300	40	35	5

Table 4: Volumetric verification results

Nº LT	A. Initial E.M (µm)	Max. Initial Error(µm)	Noise Error %	A. Opt. E.M (µm)	Max. Opt.Error (µm)	Residual Error %
LT 1	466,12	1.37	9.0	58.09	390.07	12.5
LT multi	463.20	1.35		12.27	43.17	2.6

The volumetric error reduction and compensation of geometric errors was performed using the method of parameter identification [2] with the addition of an improvements package in the same [3]. The results show, how multilateration improve volumetric verification results improving machine tool accuracy (table 4).

As demonstrated by the test presented above, multilateration reduces the measurement uncertainty of the measured points through the adequate positioning of the LT employees by providing a greater number of accuracy points. Therefore, laser tracker positioning should be as accurate as possible. An error in the LT positioning will influence the coordinates of the measured points. This error will affect the multilateration results as well as the approximation functions obtained in the verification.

The calibration tests using least-squares, trilateration and quadrilateration was performed measuring a synthetic mesh with four LTs in different positions with the same error noise modelled. To compare the different techniques two parameters can be checked:

- ➢ The coordinates difference between nominal point without measurement noise and multilateralized point in CSMT.
- ➢ Difference between the average module distance of each mesh point to all other points without LT measurement noise contribution and average module distance of each multilateralized mesh point to all other points.

The self-calibration method that provides the smallest value of the first parameter of control results in the final best relation between the theoretical and multilateration points but not necessarily the best self-calibration. The smallest difference between the average module distance of all of the points and the average of the same points without noise provides a better overall positioning of the measured points.

To eliminate random sources of error, 60 points uniformly distributed on the works space defined by table 5 were measured. The same test was repeated 10 times, performing the average of all results in table.

Table 5: Work Space

	X (mm)	Y (mm)	Z (mm)
Initial	0	0	0
Final	1400	500	600
Interval	100	100	100

Table 6: Parameters of control in self-calibration

	First Parameter	Second Parameter
LT nominal position (µm)	10.2	6.7
Least-squares (µm)	15.4	9.2
Trilateration (µm)	11.2	8.8
Quadrilateration (µm)	8.7	6.8

In the first row of table 6, the best values of each parameter of control are observed. It is due to the control parameters have been calculated using the nominal LT's positions. The second parameter of control presents the quadrilateration as the best technique to obtain the positions of the LTs. Self-calibration using least squares provides a variation of 37.3% respect the nominal position meanwhile the different between trilateration, quadrilateration and LT nominal position is of 31.3% and 1.5% respectively. The first parameter of control is calculated applying a least squares adjustment regardless of the LT self-calibration technique used. This is the reason because of the first parameter of control in quadrilateration is smaller than nominal one. Creating a mathematical instead of a physical correction.

6. CONCLUSION

The measurement uncertainty of measurement systems affects the volumetric verification correction calculated for the machine tool. The most influential factor is the angular noise which came from the angular encoders.

Applying multilateration techniques, the uncertainty may be reduced substantially. The scope of measurement noise reduction is the result of spatial angle between laser trackers, LT self-calibration technique used and LT specifications.

The results of our synthetic test show that a reduction in the measurement errors is directly related to the spatial angle formed by the measurement laser beams with the measuring point. If there are no restrictions on the LTs' positioning, they should be placed forming a spatial angle of $\alpha = 90^{o}$ between them. In this case, the residual error of the measurement noise will be similar to the radial error of the LTs. Otherwise, maintaining the same value for different spatial angles is recommended so that the spatial angle takes a value between 25º and 90º and not between 90º and 120º.

The self-calibration method was used to obtain the relative positioning of the laser tracker that directly influenced multilateration by modifying the precision of the measurement. Regarding the test results, the best method is quadrilateration with a difference of 1.5% respect nominal LTs positions results. It is followed by trilateration and least squares adjustment 31.3% and 35.5% respectively.

When the measurement noise is reduced, a volumetric error with more influence of geometrical errors is obtained. So, the approximation functions of geometrical errors obtained look more like the real ones. This is a new way of verify long range machine tools using existing LTs which are been used by now.

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