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TEMPERATURE PROFILES PRODUCED BY A SPHERICAL HEAT SOURCE IMMERSED IN AN INFINITE LIQUID MEDIUM

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Abstract: Spherical heat source probes, which generate heat and monitor the temperature response with time, have been studied to measure thermal conductivity of different materials. However, for gases and liquids, the measurement of the thermal conductivity has to consider specific parameters. The aim of this paper is to model the physical phenomenon of a spherical device used to measure the thermal conductivity of liquids by means of the finite difference method, using commercial software, and then compare it to analytical equations. The influence of the thermal contact resistance in the process was also analyzed.

Keywords: Thermal conductivity, temperature, liquids, spherical probe.

1. INTRODUCTION

Each day, the rational use and saving of thermal energy become more and more relevant, especially at industrial sectors. Thus, the transfer of heat with the best efficiency is as important as minimizing the heat loss through the use of geometries and appropriate materials. Therefore, the thermal conductivity of materials is essential in many engineering applications where heat transfer plays a fundamental role. The knowledge of the thermal conductivity of the materials involved in a certain process enables to estimate the amount of energy, and consequently the selection and proper sizing of the equipments.

The thermal conductivity is a transport property that furnishes an indication of the rate at which energy is transferred by the conduction process (net transfer of energy by random molecular motion). The measurements methods of thermal conductivity are usually classified according to two categories: steady-state (the material properties, at each point of the medium, do not vary with time) and transient (the properties vary with time). The steady-state method uses the Fourier law, based on the measurement of heat flux and a temperature difference between the opposite surfaces of a sample. The transient method replaces the temperature difference between the surfaces of the sample by a temperature measurement as a function of time at only one position. The guarded hot plate, the concentric cylinders and the parallel plates techniques are examples of steady-state methods; while the hot wire, the hot strip and the laser flash techniques are examples of transient methods. The main differences between the methods are the accuracy and the time required for the measurement. Steady-state methods are generally more accurate, but normally require a long time. Transient methods require experimental apparatus less sophisticated and the measurements are faster, however the results are not so reliable. Transient methods can also be used to obtain specific heat, thermal diffusivity, and thermal conductivity within a single measurement.

Concerning the thermal conductivity of liquids, for a long while researchers have been working on its theoretical estimation, by linking it to other physical property, usually using temperature as a variable, or by deriving a model from the existing ones. Also, several experiments have been developed, using different techniques based on steady-state and transient methods, in order to measure it properly.

The line heat source probe (hot wire method) is usually recognized as the most accurate technique for measuring thermal conductivity of liquids. The concept and the initial experiments with heated wires started about 1780, and the first transient hot wire instrument was proposed by Stâlhane and Pyk in 1931 to measure the thermal conductivity of solids, powders and some liquids. Since 1780, the method has been studied by several researchers [1]. The standard test method for the determination of thermal conductivity of nonmetallic liquids of the American Society for Testing and Materials (ASTM) is based on this method [2]. An extensive uncertainty assessment elaborated for the hot wire method obtained the value of 5.8% for the thermal conductivity [3], although others literatures claim for uncertainty values better than 5%.

Spherical heat source probes have also been studied to measure thermal conductivity of different materials (solid, powder, paste etc.). In this technique, the spherical device simultaneously generates heat and monitors the temperature response with time. Heat sources of spherical symmetry are free of lateral thermal effects and they yield to the steady-state regime at long times, which is used to measure the thermal conductivity. The method was developed by Chato in 1968 for measuring thermal conductivity and thermal diffusivity of biological materials [4]. Balasubramaniam [5], Valvano [6] and Zhang *et al* [7] also used the method for measuring thermal properties of biomaterials. Woodbury used the method to measure thermal conductivity of building insulation with varying degree of wetness [8]. Fujii *et al* employed the technique to measure thermal conductivities of liquid mixtures of water-ethanol, water-methanol and R113-oil [9]. Dougherty used the method to perform thermal conductivity measurements in materials ranging from low viscosity fluids to insulation ones [10]. Kravets used the technique to measure thermal conductivity of milk and cream over the range of 25 °C to 125 °C [11]. Gelder employed the method to measure thermal properties of moist food materials at high temperatures [12]. Holeschovsky *et al* modified the technique to measure the thermal conductivity of liquids and silica gel [13]. Kubicar *et al* dealt with the theory and performance of the method, and used it to measure thermal conductivity of some materials [14].

The objective of this paper is to model the physical phenomenon of a spherical device used to measure thermal conductivity of liquids by means of the finite difference method, using commercial software, and then compare it to analytical equations. The influence of the thermal contact resistance in the process was also analyzed.

2. MODELING ASPECTS

For the modelling, a perfect sphere made of epoxy with radius of 1 mm was selected. The selected liquid medium was water at 20 °C. Epoxy resin was chosen because it can be used either for thermistor coating as for elements fixing (heater and temperature sensor, for instance). Although, its thermophysical properties range from a minimum value to a maximum one, an average value was used for each property.

The sphere may be internally heated by a thermistor, which can be used as a temperature sensor and a heating source, or by a heating device, which can be fixed together with a thermometer using epoxy resin. Nevertheless, it was assumed in this work that no heat loss through the connecting wires exists.

The heated sphere may induce density gradients in a fluid. The buoyancy force originated from these density gradients will be opposed by a combination of inertia and viscous forces from the liquid; depending on the interaction of these forces, natural convection may occur. So, fluids with low viscosity are more prone to natural convection then those with high viscosity. However, it was assumed in this work that the heat transfer is pure conduction.

When two materials are in physical contact, the temperature drop across their interface is due to what is known as “thermal contact resistance”. Its existence is due principally to surface roughness effects and its inverse is called “thermal contact conductance”. The thermal contact resistance between a solid material and a liquid medium is usually much smaller than that between two solid materials.

The model was based on some other assumptions, such as: the sphere has a perfect spherical shape; the sphere is made of isotropic and homogenous material; heat is uniformly generated within the sphere; and the medium is isotropic, homogeneous and infinite in extent.

The numerical simulation was realized to the transient and steady-state regimes in three-dimensional heat transfer by conduction, using commercial software. For interpolation, Lagrange polynomial of quadratic form was used. In the transient regime, the implicit finite difference method was applied and simulation was performed for a period time of 600 s, with an interval of 1 s.

The mesh consisted of 24171 tetrahedral elements in the medium, 2086 triangular elements in the boundary of the medium, 80 tetrahedral elements in the sphere and 80 triangular elements in the boundary of the sphere.

3. MATHEMATICAL MODEL

The governing heat conduction equations for the sphere with internal heat generation and the surrounding medium are respectively:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_s}{\partial r}\right)+\frac{\dot{q}}{k_s}=\frac{1}{\alpha_s}\frac{\partial T_s}{\partial t} \qquad 0\le r\le r_s \qquad t>0 \tag{1}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_m}{\partial r}\right)=\frac{1}{\alpha_m}\frac{\partial T_m}{\partial t} \qquad r\ge r_s \qquad t>0 \tag{2}$$

Where the subscripts *s* and *m* represent the sphere and the medium, respectively, r_s is the radius of the sphere, r is the radial distance, k is the thermal conductivity, α is the thermal diffusivity and $\dot{q}$ is the heating power per unit volume.

The governing differential equations are subject to the following initial and boundary conditions:

$$T_s = T_m = 20\ ^\circ\mathrm{C} \qquad t=0 \tag{3}$$

$$T_s \text{ is finite as } r\to 0 \qquad t>0 \tag{4}$$

$$T_m \text{ is finite as } r\to \infty \qquad t>0 \tag{5}$$

$$T_s = T_m \text{ at } r=r_s \qquad t>0 \tag{6}$$

$$k_s\frac{\partial T_s}{\partial r}=k_m\frac{\partial T_m}{\partial r} \text{ at } r=r_s \qquad t>0 \tag{7}$$

According to the solution of the heat integration given by Carslaw and Jaeger [15], the temperature distribution of a region bounded internally by a sphere, if the initial temperature is zero and there is constant heat flux in the interface, is characterized by the equation:

$$T_m=\frac{r_s^2 q''}{k_m r}\left\{erfc\left(\frac{r-r_s}{2\sqrt{\alpha_m t}}\right)-exp\left(\frac{r-r_s}{r_s}+\frac{\alpha_m t}{r_s^2}\right)\times\ldots\right.$$
$$\left.\ldots\times erfc\left(\frac{r-r_s}{2\sqrt{\alpha_m t}}+\frac{\sqrt{\alpha_m t}}{r_s}\right)\right\} \tag{8}$$

Where q'' is the heat transfer per unit area (heat flux) and $erfc(x)$ is the complementary error function.

For a temperature measured at the surface of the sphere ($r = r_s$) and for long times ($t\to\infty$), Eq. (8) gives:

$$T_m=\frac{r_s q''}{k_m}=\frac{\dot{Q}}{4\pi r_s k_m} \tag{9}$$

Where $\dot{Q}$ is the heating power and T_m is the stabilized value of the temperature response.

Carslaw and Jaeger [15] also presented an equation for the temperature distribution within the sphere, if the initial temperature is zero, considering its heat capacity and the thermal contact resistance between the heated sphere and the surrounding medium:

$$T_s = \frac{\dot{Q}}{4\pi k_m r_s}\left[\frac{1+r_s h}{r_s h} - \frac{2r_s^2 f^2 h^2}{\pi} \times \ldots \right.$$

$$\left. \ldots \times \int_0^\infty \frac{exp\left(-\alpha_m u^2 t / r_s^2\right)}{\left[u^2(1+r_s h) - fr_s h\right]^2 + \left(u^3 - fr_s hu\right)^2} du \right] \qquad (10)$$

Where $f = 4\pi r_s^3 \rho_m \left(c_m / m_s c_s\right)$, ρ is the density, c is the specific heat, m is the mass and $h = H/k_m$. H is the thermal contact conductance, which is the inverse of the thermal contact resistance (R).

The thermal contact resistance between a solid material and a liquid medium is small ($R \to 0$). Thus, applying the limit, Eq. 10 gives:

$$T_s = \frac{\dot{Q}}{4\pi k_m r_s}\left[1 - \frac{2r_s^2 f^2}{\pi}\int_0^\infty \frac{exp\left(-\alpha_m u^2 t / r_s^2\right)}{\left(u^2 r_s - fr_s\right)^2 + \left(fr_s u\right)^2} du\right] \qquad (11)$$

For the steady-state regime, Carslaw and Jaeger [15] presented equations, if the initial temperature is zero, for the temperature distributions within the sphere and within the surrounding medium:

$$T_s = \dot{q}\frac{\left[r_s^2 - r^2 + 2Rr_s k_s + 2r_s^2\left(k_s / k_m\right)\right]}{6k_s} \qquad 0 \le r < r_s \qquad (12)$$

$$T_m = \frac{\dot{q} r_s^3}{3k_m r} \qquad r > r_s \qquad (13)$$

Applying the limit again for $R \to 0$, Eq. 12 gives:

$$T_s = \dot{q}\frac{\left[r_s^2 - r^2 + 2r_s^2\left(k_s / k_m\right)\right]}{6k_s} \qquad (14)$$

4. RESULTS

The transient temperature distribution of the medium can be evaluated by means of Eq. 8. However, this equation is only valid if the heat flux in the interface is constant. This condition was checked by means of the software for the upper pole of the sphere immersed in water, at the initial temperature of 20 °C, and with heating powers of 5, 10, 15 and 20 mW.

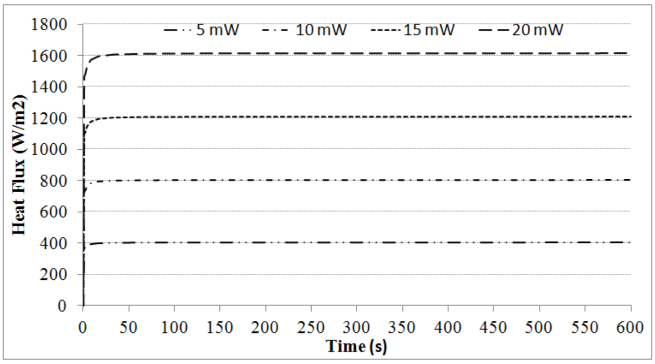

Figure 1 – Heat Flux in the Interface

The transient temperature at the upper pole of the surface of the sphere was analysed for water at the initial temperature of 20 °C, using the software (dashed line) and Eq. 11 (solid line), for four values of heating power. The highest differences between the software and Eq. 11 were very small (less than 0.1 °C), which occurred at the very beginning of the transient regime (at the 6th second). After approximately two minutes, the differences were smaller than 0.007 °C for 5 mW and 0.03 °C for 20 mW.

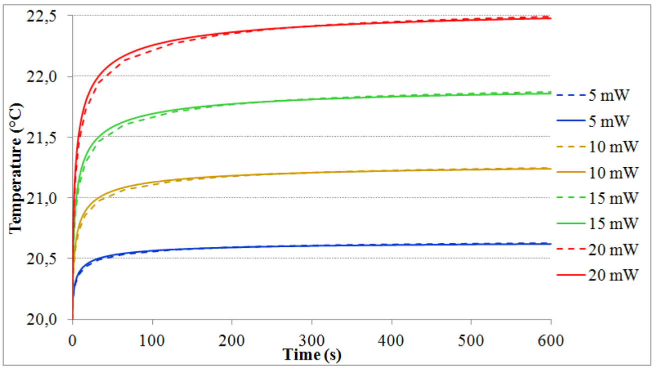

Figure 2 – Transient Temperature at the Surface of the Sphere

The steady-state temperature distribution within the sphere and the medium was analysed for water at the initial temperature of 20 ºC, using the software (dashed line) and Eq. 12 and Eq. 13 (solid line). The highest differences between the software and Eq. 12 were 0.014 °C for 5 mW and 0.058 for 20 mW, which occurred approximately in the middle of the radius of the sphere.

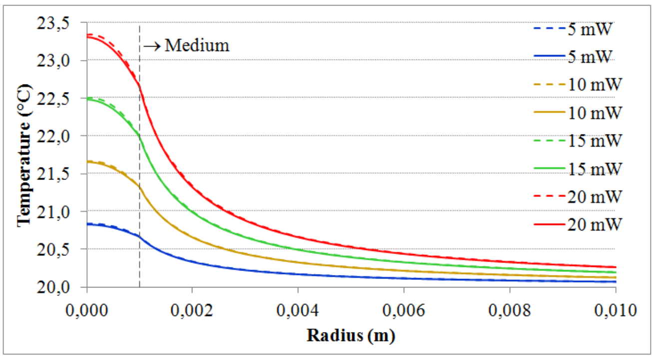

Figure 3 – Steady-state Temperature (Sphere and Medium)

The steady-state temperature distribution within the sphere was also analysed, at the initial temperature of 20 ºC, for different values of thermal conductivity (0.7, 1.2 and 1.7 W/m^2K), using the software (dashed line) and Eq. 12 (solid line), for the heating power of 5 mW. The highest differences between the software and Eq. 12 were 0.018, 0.015 and 0.013 °C for k = 0.7, 1.2 and 1.7 W/m^2K, respectively, which occurred approximately in the middle of the radius of the sphere.

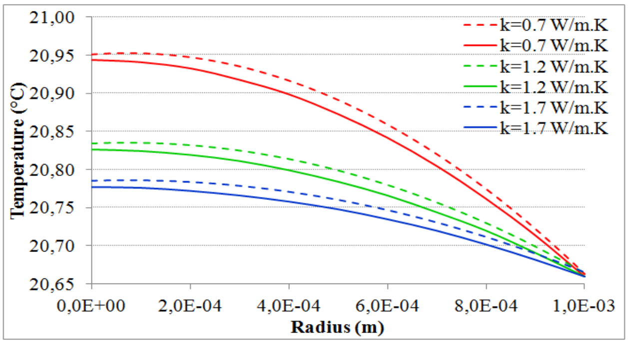

Figure 4 – Steady-state Temperature within the Sphere

The influence of R was analysed for the transient regime at the interface between the sphere and the medium, at the initial temperature of 20 °C, for the values of 10^{-3}, 0.18×10^{-3} and 0.10×10^{-3} m^2K/W, using Eq. 10, for 5 mW. The temperature at the interface is shifted upward as higher as R, which reduces the estimated value of thermal conductivity of the medium. Using the interface temperature for R = 0.0001 m^2K/W and Eq. 9 to determine the apparent k, a reduction of approximately 6% of the real value of k was found.

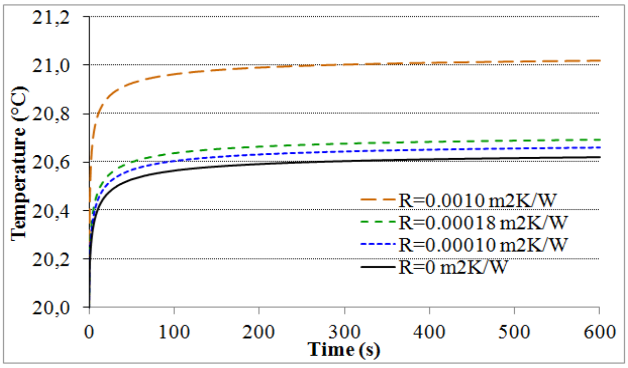

Figure 5 – Influence of R in the Transient Regime

4. CONCLUSIONS

The comparison of the results showed that the values obtained with the analytical equations and the software (using the selected configurations) agrees well, especially for the lower values of heating power.

It can be observed by the modelling that the temperature at the surface of the sphere is always increasing, it means that the steady-state regime is never reached. Therefore, a quasi-steady-state regime is in fact used to calculate the thermal conductivity of the material. It has to be highlighted that, as time passes, this temperature increasing becomes smaller and smaller. So, depending on the requirements of the user, this increasing can be considered negligible from a certain point.

It was shown that the use of Eq. 8 for this kind of application is valid, once the heat flux is constant in the interface of the bodies.

The influence of the thermal contact resistance in the process is significant, which means that it is important to know this value so that the temperature at the surface of the sphere can be corrected. Applying this correction, the thermal conductivity of the medium can be estimated with a good accuracy.

The experimental measurements are in progress and will be published in the next future.

5. REFERENCES

[1] Assael, M. J.; Antoniadis, K. D.; Wakeham, W. A. “Historical Evolution of the Transient Hot-Wire Technique”. Int. J. of Thermophysics, Vol. 31, N^o 6, 2010.

[2] American Society for Testing and Materials (ASTM). D-2717-15: Standard test for thermal conductivity of liquids. 2009.

[3] Hammerschmidt, U.; Sabuga, W. “Transient Hot Wire (THW) Method: Uncertainty Assessment”. Int. Journal of Thermophysics, Vol. 21, N° 6, 2000.

[4] Chato, J.C. “A Method for the Measurement of the Thermal Properties of Biological Materials”. Thermal Problems in Biotechnology, ASME, pp. 16-25, 1968.

[5] Balasubramaniam, T.A. “Thermal Conductivity and Thermal Diffusivity of Biomaterials: A Simultaneous Single Measurement Technique”. Ph.D. Thesis, Northeastern University, 1975.

[6] Valvano, J. W. “The Use of Thermal Diffusivity to Quantify Tissue Perfusion”. Ph.D. Thesis, Harvard University, 1981.

[7] Zhang, H.; Cheng, S.; He, L, Zhang, A.; Zheng, Y.; Gao, D. “Determination of Thermal Conductivity of Biomaterials in the Temperature Range 233-313K Using a Tiny Detector Made of a Self-Heated Thermistor”. Cell Preservation Technology, Vol. 1, N^o 2, 2002.

[8] Woodbury, K. A. “An Experimental and Analytical Investigation of Liquid Moisture Distribution in Roof Insulating Systems”. Ph.D. Thesis, Virginia Polytechnic Institute and State University, 1984.

[9] Fujii, M.; Takamatsu, H.; Fujii, T. “Measurement of Thermal Conductivities of Liquid Mixtures by Using Small Bead Thermistor”. Proceedings of the 1st Asian Thermophysical Properties Conference, pp. 462-467, 1986.

[10] Dougherty, B. P. “An Automated Probe for Thermal Conductivity Measurements”. M.Sc. Thesis, Virginia Polytechnic Institute and State University, 1987.

[11] Kravets, R.R. “Determination of Thermal Conductivity of Food Materials Using a Bead Thermistor”. Ph.D. Thesis, Virginia Polytechnic Institute and State University, 1988.

[12] van Gelder, M. F. “A Thermistor Based Method for Measurement of Thermal Conductivity and Thermal Diffusivity of Moist Food Materials at High Temperatures”. Ph.D. Thesis, Virginia Polytechnic Institute and State University, 1998.

[13] Holeschovsky, U. B.; Martin, G. T.; Tester, J. W. “A Transient Spherical Source Method to Determine Thermal Conductivity of Liquids and Gels”. Int. Journal of Heat Mass Transfer, Vol. 39, N^o 6, pp. 1135-1140, 1996.

[14] Kubicar, L.; Vretenar, V.; Stofanik, V.; Bohac, V. “Hot-Ball Method for Measuring Thermal Conductivity”. Int. Journal of Thermophys, 2008.

[15] Carslaw, H. S.; Jaeger, J. C. “Conduction of Heat in Solids”. Oxford University Press, 2nd Ed, 1959.