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MODELLING THE CALIBRATION OF THE IPRT ACCORDING TO THE ISO UNCERTAINTY GUIDE

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Abstract: According to the ISO guide to the expression of uncertainty in measurement, the calibration process and its uncertainty evaluation should be expressed in terms of mathematical function(s) of input quantities. However, in practice, expressing measurement or calibration in a way that is fully compliant to the ISO guide might be unrealistic and requires a clear definition of the calibration process itself. Depending on the point of view on the calibration process, different modelling equation with various complexities can be written. In this paper, four different approaches are given to model the calibration process of industrial platinum resistance thermometers.

Keywords: GUM, Uncertainty, IPRT, Modelling of measurements

1. INTRODUCTION

According to the ISO Guide to the expression of uncertainty in measurement (GUM) [1], when expressing uncertainties, output quantities of a measurement (or measurands) should be written as mathematical functions of input quantities of the measurement. And the uncertainty assessment should be performed based on this modelling equations. This process is intuitive and practical when the measurement is relatively simple. Consider a simple measurement of volume V of cylinder through dimensional measurements of a radius r and a height h of the cylinder. In this case, the input quantities are r and h, and the output quantity is V. The modelling equation is then,

$$V = V(r, h) = \pi r^2 h. \quad (1)$$

The standard uncertainty u_V of V is then determined by the uncertainty of the two input quantities through

$$u_V^2 = \left(\frac{\partial V}{\partial r}\right)^2 u_r^2 + \left(\frac{\partial V}{\partial h}\right)^2 u_h^2 \quad (2)$$

where

$$\frac{\partial V}{\partial r} = 2\pi r h,\ \frac{\partial V}{\partial h} = \pi r^2, \quad (3)$$

and u_r and u_h are the standard uncertainties of r and h. However, for many real-world situations, the strict implementation of GUM is sometimes neither intuitive nor practical. Especially when data reduction process such as the least squares fit is involved, the strict application of the GUM method might be impractical. It should be also noted that depending on the point of view of how the calibration process is conceived, the set of input and output quantities of the calibration might be different.

In this paper, four approaches of the modelling the calibration of the industrial platinum resistance thermometer (IPRT) were suggested. For each point of view, the input quantities and output quantities are defined, and the modelling equations are accordingly established. Then we will discuss on the degree of complexity and practicality of each approach.

2. INPUT AND OUTPUT QUANTTIES AND THE MODELLING EQUATION

In the usual calibration of an IPRT, the measurement of resistance of the IPRT is done at temperatures near defined calibration temperatures $t_{i,0}$ (i = 1, 2, …, N), and the actual temperature t_i at the i-th calibration temperature is measured by a calibrated standard platinum resistance thermometer (SPRT). The resistance $R_{t,i}$ of the IPRT at t_i is measured by measuring the resistance ratio $\beta_{t,i}$ to a standard resistor whose resistance value is known to be R_s. The temperature t_i is also measured through a measured resistance ratio $\beta_{r,i}$ to R_s, then the resistance value $\beta_{r,i}R_s$ is converted to the temperature t_i, applying to the calibration certificate of SPRT.

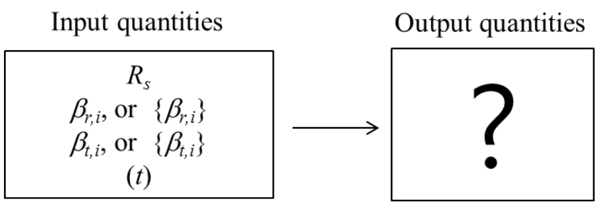

Figure 1. Input and output quantities of the calibration of IPRTs.

Figure 1 describes the problem discussed in this paper. In addition to R_s, depending on the approaches we take, one set of ($\beta_{r,i}$, $\beta_{t,i}$) or N sets of ($\beta_{r,i}$,$\beta_{t,i}$) can be included in the input quantities. For one approach, an arbitrary temperature t in the calibration range can be an input quantity. The output quantities can vary depending on the point of view on the calibration. We will discuss on four approaches in the following sections.

3. DEFFERENT APPROACHES

Approach 1: In this approach, the calibration is considered as a procedure to generate several pairs of (t_i, R_i) such that the resistance of the IPRT at temperature t_i is R_i. By calibrating at N temperatures, N pairs of (t_i, R_i) are generated. The modelling equations in this case are

$$t_i = t(R_s\beta_{r,i}) \tag{4a}$$
$$R_i = R_s\beta_{t,i}. \tag{4b}$$

The function $t(x)$ is the ITS-90 reference function plus the deviation function whose coefficients are stated on the calibration certificate. The explicit form of the $t(x)$ varies with the range of calibration of the SPRT.

This approach is very close to the actual calibration procedure, and indeed identical to that of a digital thermometer calibration where no suitable model for dR/dt is available, thus no interpolation is possible. However, in IPRTs, the modelling of temperature-resistance characteristics is possible (whether one chooses to use a CVD equation, higher order polynomials or ITS-90 type equations [2]), and one can indeed go beyond the pairs of (t_i, R_i). Furthermore, this leaves two output quantities, and therefore covariance matrix instead of scalar uncertainty per one calibration point. Customers of this calibration must find their own way to interpret the covariant matrix and to convert it to a single uncertainty in terms of temperature or resistance. The fact that t_i is not an exactly defined number, unlike $t_{i,0}$, may cause additional inconvenience to customers.

Approach 2: In this approach, the calibration is considered as measurement of $R_{i,0}$ at defined calibration temperatures $t_{i,0}$ (such as 100.000 ℃). This is done by measuring the resistance of the IPRT at $t_i = t(R_s\beta_{r,i})$ near $t_{i,0}$. (it is not usually practical to set the isothermal environment so that $t_i = t_{i,0}$ in a practical time scale.) The modelling equation is

$$R_{i,0} = R_s\beta_{t,i} + (t_{i,0} - t(R_s\beta_{r,i})) \times \left.\frac{dR_t}{dt}\right|_{t=t_{i,0}}. \tag{5}$$

This modelling equation is of moderate complexity and the information provided the equation is very close to what the customers want, unless they want an interpolation formula.

However, strictly speaking, $\left.\frac{dR_t}{dt}\right|_{t=t_{i,0}}$ should be determined for each individual IPRT by the calibration and with a specified interpolation scheme. Therefore, one has to include all of the calibration points (not just the ones at i-th calibration points), and the modelling equation may become much complicated. However, if the IPRT can be classified as one of the written standard (such as the ones that can be specified as α = 0.00385 or 0.00392 [3]), and if the difference $|t_{i,0} - t(R_s\beta_{r,i})|$ is small enough (such as < 10 mK as can be easily achieved), then using a predetermined table value for $\left.\frac{dR_t}{dt}\right|_{t=t_{i,0}}$ will lead only negligible amount of error and uncertainty. Just as the modelling equation in the approach 1, this modelling equation does not provide an interpolation scheme of the IPRT. Therefore, if customers want to know the resistance at arbitrary temperatures in the calibration range, they have to devise their own interpolation scheme.

Approach 3: In the next approach, calibration is considered as a procedure to find coefficients of an interpolation equation that represent temperature-resistance characteristics of the IPRT using the N pairs of (t_i, R_i). The coefficients are usually decided by a method of least squares. Assuming that the interpolation equation of choice is Callendar van Dusen (CVD) equation, the modelling equation for this approach is

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \sum_i t^2(R_s\beta_{r,i}) & \sum_i t^3(R_s\beta_{r,i}) \\ \sum_i t^3(R_s\beta_{r,i}) & \sum_i t^4(R_s\beta_{r,i}) \end{pmatrix}^{-1} \begin{pmatrix} \sum_i \left(\frac{\beta_{t,i}}{\beta_{t,ip}} - 1\right) t(R_s\beta_{r,i}) \\ \sum_i \left(\frac{\beta_{t,i}}{\beta_{t,ip}} - 1\right) t^2(R_s\beta_{r,i}) \end{pmatrix}. \tag{6}$$

It is assumed here that $t_i \geq 0$ for all i for the sake of simplicity of the argument, and therefore $C = 0$ in the CVD equation [4]. It is also assumed that the ice-point reading $\beta_{t,ip}$ is somehow found for the IPRT by a measurement and an interpolation similar to (5). An alternative way is to treat the ice-point resistance of IPRT as an independent fitting parameter to be determined by the method of least squares.

This is indeed very close to the description of actual practice for many calibration laboratories where the measurement of resistance at several calibration points is provided on the calibration certificate with a specified interpolation equation and coefficients of the equation. However, the modelling equation is impractically complex for uncertainty assessment that includes covariance matrix. Furthermore, because the output quantities of this modelling equation are the coefficients A and B, the uncertainty of the measurement will be calculated in terms of the uncertainties of A, B and their covariance. This information is not useful for most customers. This modelling also assumes that one interpolation scheme is more suitable than another, and provides a specific scheme to customers, but recent study shows that there are limitations for these interpolating equations [2]. The most serious and complex problem is that, due to the strong correlations among input quantities $\beta_{r,i}$'s and $\beta_{t,i}$'s, it is nearly impossible to correctly calculate the covariance terms of uncertainty. Even if it can be somehow done, it is questionable whether it is worthwhile to do so for industrial grade thermometers, whereas much smaller uncertainty is accessible with the use of SPRTs.

Approach 4: In the last approach, the calibration is considered as a process to provide enough information, so that the resistance of IPRT can be calculated for a given (arbitrary) t in the temperature range of the calibration. The information comes from the selected interpolation equation and the coefficients calculated by the formula similar to (6). In this case, the modelling equation is

$$R = R_s \beta_{t,ip}(1 + At + Bt^2), \qquad (7)$$

where A and B (and C if $t_i < 0$ °C for some i) are from (6) or any other least squares fit. This is indeed the closest description to that most customers want to see in their calibration result. They want to know R of their IPRT and its uncertainty at a given t in a certain range. However, since this modelling equation uses the result of calculation described in approach 3, it inherits all of the problems described in approach 3, with even more complex formulation.

4. SUMMARY AND DISCUSSION

Table 1 summarizes all of four approaches with their input and output quantities and their characteristics. Following the approach 3 or 4 would be the most accurate description of the calibration process. However, they have many practical difficulties. It is not practical to assess the uncertainties using arithmetic process with such complexity as required by approach 3 or 4, noting that the level of uncertainty one can achieve with the best IPRT is about tens of millikelvins. In addition, due to limitation of the interpolating equations, it may not be good to limit the calibration result to a single interpolation scheme.

Perhaps, the most difficult problem in approach 3 or 4 is the correlation among $\beta_{r,i}$'s and $\beta_{t,i}$'s. Some correlation must exist among $\beta_{r,i}$'s due to the use of the same reference SPRT for the measurement of temperature. The correlation among $\beta_{t,i}$'s also exist due to the use of the same resistance ratio bridge throughout the calibration. For the type A evaluation of the covariance, these $\beta_{r,i}$'s and $\beta_{t,i}$'s must be measured simultaneously as voltage and current are measured simultaneously for the determination of their covariance terms as described in Annex H.2 of GUM. However, due to the very nature of the thermal measurement, it is impossible to do the simultaneous measurement where one has to wait until the thermometers to reach thermal equilibrium with the environment at one temperature to a certain degree before the measurement at the next temperature is performed. Therefore, the determination of the covariance matrix must somehow rely on our prior knowledge of the measurement, and must be calculated by the type B method. For IPRTs where the expected uncertainties are at least a few tens of millikelvins, this is not a practical approach.

Therefore, we believe that the approach 2 is the most suitable as a modelling equation of IPRTs. However, one has to modify the point of view to look at the calibration procedure as follows: The calibration of IPRTs is the measurement of resistance of the IPRTs at several defined temperatures. There is modelling equation and the uncertainty can be assessed at each calibration temperature. The calibration procedure as GUM describes ends there. However, for the convenience of customers, sometimes calibration laboratories may select a certain interpolation scheme and use the results of calibration to find the interpolation equation and give a temperature-resistance table. However, this should be considered outside of the "measurement" activity to which GUM can be applicable for all practical purposes. This should be regarded as a purely arithmetic activity and one should not expect the uncertainty assessment in conformity with GUM to be practical, if it is ever possible. The customers should find the uncertainty due to the interpolation from the uncertainty of each calibration point and the characteristics and results of the interpolation. However, in this case as well, some sort of upper limit of uncertainty may be found, and rigorous calculation may not be of practical interest.

Table 1. Summary of four approaches discussed in this paper with their input quantities X, output quantities Y, and their description.

	X	Y	Description
1	R_s, $\beta_{r,i}$, $\beta_{t,i}$	t_i, R	· Simple modelling equation · Two measurands, covariance matrix instead of scalar uncertainty
2	R_s, $\beta_{r,i}$, $\beta_{t,i}$	$R_{i,0}$	· Modelling equation of moderate complexity · Users must interpolate and assess the uncertainty due to the interpolation.
3	R_s $\beta_{r,1}$, $\beta_{r,2}$, …, $\beta_{r,N}$ $\beta_{t,1}$, $\beta_{t,2}$, …, $\beta_{t,N}$	A B (C)	· Close to the actual calibration process · Very complicate modelling equation · Describes the whole calibration process · The uncertainties of coefficients have minimal meaning to the users · Complexity due to the correlations between input quantities
4	R_s $\beta_{r,1}$, $\beta_{r,2}$, …, $\beta_{r,N}$ $\beta_{t,1}$, $\beta_{t,2}$, …, $\beta_{t,N}$ t	R	· Close to the actual calibration process and customer usage · Even more complex modelling equation · Describes the whole calibration process and the user process · Complexity due to the correlations between input quantities

5. REFERENCES

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