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RECONFIGURATION OF A DYNAMICALLY DECOUPLED SYSTEM AFTER ACTUATOR FAULT

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Abstract: In the paper problems with reconfiguration of a dynamically decoupled system after actuator fault are considered. On the basis of a universal algorithm for synthesis of control system for proper, square, right and left–invertible multi-input multi-output LTI dynamic (MIMO) plants necessary calculations after fault occurs are discussed. It is presented a reconfiguration algorithm defining all calculations that need to be done to adapt decoupler to new working conditions ensuring stability and physical reliability of all parts of the system.

Keywords: dynamic decoupling, left-invertible, pole assignment, polynomial matrix equations.

1. INTRODUCTION

Diagnostics of industrial processes is one of the most important design issues. Faults may be connected with not proper working conditions of whole process, but very often the source of the faults could be in one of the process components. If they are dangerous the process should be emergency stopped. However, it could happen that failures of one or even more basic process elements let the process still last. For complex processes several methods may be used for proper diagnosing of the whole process [17; 20; 21; 25]. Basing on chosen methods diagnostic system fed information to overriding system about necessity of doing control system reconfiguration and allows continuing process after detecting fault, after losing control on some actuators [4; 11; 15].

Multi-input multi-output (MIMO) systems give process engineer opportunity to reconfigure the system in such way that influence of fault will be minimized. In the most optimistic case system after reconfiguration may still work in a fully automatic way.

In a design of the control systems decoupling methods are used which goal is to lead a system to the situation when a specific group of inputs affects a specific group of outputs and no element of this input group have influence on any other output component of the system. However, dynamic decoupling of MIMO systems is one of the most difficult problems especially for non-square plants which can have non minimum phase transmission zeros.

An idea of dynamic decoupling for multivariable (MIMO) systems has been considered by many authors since 60's beginning with [23]. Solution to Morgan problem was given in [5] and its stability conditions in [27]. General decoupling problem with stability for square and right-invertible plants has been intensively studied in the past (see e.g. [3; 6; 27; 29; 35]) and it still arouse considerable interest [10; 24; 28; 30; 33]. However, most of the proposed methods allow one to exist in the decoupled system some fixed poles which (if there are unstable) can result in its instability. Moreover there are as well often limited to the square or right invertible plants with minimum phase zeros only. Additionally the everyday practice shows that often there is a need to maintain and control processes with more outputs than inputs. It may happen e.g. during system failure when one lose some actuators and the precise control of all outputs is not possible. An universal decoupling algorithm which covers all these problems for square, right and left-invertible plants has been presented in [6].

In the paper problems with reconfiguration of a dynamically decoupled system after actuator fault are discussed. The main goal of the paper is not to present all details of algorithm described in [6] but rather to discuss under what conditions is it possible to omit some parts of calculations to make the reconfiguration procedure fully automatic and faster.

The paper is organized as follows. The problem statement and decoupling concept have been brought in section 2 and 3. The main results, reconfiguration issues after actuator fault are presented in section 4. Finally an example and concluding remarks are given.

2. PROBLEM STATEMENT

2.1. Decoupling

We consider a controllable and observable linear, LTI MIMO, model of the plant Σ defined by the state and output equations

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)\in R^n$, $\boldsymbol{u}(t)\in R^m$ and $\boldsymbol{y}(t)\in R^l$ are the state, input and output vectors respectively. In the polynomial matrix approach transfer matrices of all elements of the system are defined by pairs of polynomial matrices either relatively right prime (*r.r.p.*) for plants, or relatively left prime (*r.l.p.*) for other elements. Applying this approach, the plant model (1) can be transformed to the relatively prime matrix fraction description in the frequency s-domain as follows

$$\boldsymbol{y} = \boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s)\boldsymbol{u} \tag{2}$$

where

$$\boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s) = \boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{3}$$

Assuming dynamic block decoupling of the designed control system we group output and a vector of exogenous signals into "*k*" blocks according to the partitions

$$\boldsymbol{y}(t) = \begin{bmatrix} \boldsymbol{y}_1(t) \\ \vdots \\ \boldsymbol{y}_i(t) \\ \vdots \\ \boldsymbol{y}_k(t) \end{bmatrix}, \ \boldsymbol{q}(t) = \begin{bmatrix} \boldsymbol{q}_1(t) \\ \vdots \\ \boldsymbol{q}_i(t) \\ \vdots \\ \boldsymbol{q}_k(t) \end{bmatrix}, \tag{4}$$

where

$$\boldsymbol{y}_i(t) \in R^{l_i}, \ \sum_{i=1}^{k} l_i = l, \ \boldsymbol{q}_i(t) \in R^{m_i}, \ \sum_{i=1}^{k} m_i = m. \tag{5}$$

We want to design a decoupled system in which each part (loop) $i = 1, 2, ..., k$ of a system defined by pairs of signals $\boldsymbol{q}_i(t)$, $\boldsymbol{y}_i(t)$ could be controlled independently of other parts $j \neq i$. Moreover, each part of the system should be designed with individually supposed dynamic properties according to the requirements.

2.2 Actuator faults

Definition 1 [26]

An actuator fault f is an event that changes the nominal input matrix $\boldsymbol{B} \in R^{n \times m}$ to the faulty input matrix $\boldsymbol{B}_f \in R^{n \times m}$ of the same dimensions. Faults appear abruptly and remain effective once they have occurred. The pair $(\boldsymbol{A}, \boldsymbol{B}_f)$ is assumed to be stabilizable.

Actuator fault change the nominal system (1) to the faulty system $\boldsymbol{\Sigma}_1$

$$\begin{aligned} \dot{\boldsymbol{x}}_f(t) &= \boldsymbol{A}\boldsymbol{x}_f(t) + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{b}_i d_f(t) \\ \boldsymbol{y}_f(t) &= \boldsymbol{C}\boldsymbol{x}_f(t) + \boldsymbol{D}\boldsymbol{u}_f(t) \end{aligned} \tag{6}$$

where after [26] $\boldsymbol{b}_i$ is the column of $\boldsymbol{B}$ that corresponds to the blocked actuator and $d_f(t)$ is a constant determining blocking position.

Very often a fault of actuator brings the system to a state in which we can use fewer inputs to operate the process. In this case rank of the input matrix of the plant model decreases and it is legitimate to propose another definition of the actuator fault.

Definition 2

An actuator fault f is an event that changes the nominal input matrix $\boldsymbol{B} \in R^{n \times m}$ to the faulty input matrix $\boldsymbol{B}_{fr} \in R^{n \times r}$ of less dimension than the original $\boldsymbol{B}$.

As in Definition 1 it is assumed that faults appear abruptly and remain effective once they have occurred and that the pair $(\boldsymbol{A}, \boldsymbol{B}_{fr})$ is stabilizable.

Actuator fault change in this case the nominal system (1) to the faulty system $\boldsymbol{\Sigma}_2$

$$\begin{aligned} \dot{\boldsymbol{x}}_f(t) &= \boldsymbol{A}\boldsymbol{x}_f(t) + \boldsymbol{B}_{fr}\boldsymbol{u}_f(t) + \boldsymbol{b}_i d_f(t) \\ \boldsymbol{y}_f(t) &= \boldsymbol{C}\boldsymbol{x}_f(t) + \boldsymbol{D}\boldsymbol{u}_f(t). \end{aligned} \tag{7}$$

The influence of the $\boldsymbol{b}_i d_f(t)$ should be minimized be using any disturbance decoupling method, e.g. [12-14; 18; 34] and will not be considered in this paper.

3. DECOUPLING CONCEPT

The goal of decoupling of the LTI dynamic system can be achieved in a control system structure which contains the dynamic feedforward compensator with state feedback matrix $\boldsymbol{F}$. Feedback law, employed to decouple the system (the linear state variable feedback along with dynamic feedforward) is described by

$$\boldsymbol{u}(s) = \boldsymbol{G}^{-1}(s)\boldsymbol{L}_0(s)\boldsymbol{f}(s) + \boldsymbol{G}^{-1}(s)\boldsymbol{L}(s)\boldsymbol{q}(s), \tag{8}$$

where

$$\boldsymbol{f}(s) = \boldsymbol{F}(s)\boldsymbol{x}_p(s) \triangleq \boldsymbol{F}\boldsymbol{x}(t) \tag{9}$$

$\boldsymbol{x}_p(s)$ is a partial state vector of the plant, $\boldsymbol{G}(s) \in \mathbb{R}[s]^{m \times m}$, $\boldsymbol{L}(s) \in \mathbb{R}[s]^{m \times l}$, $\boldsymbol{L}_0(s) \in \mathbb{R}[s]^{m \times m}$, $\boldsymbol{F}(s) \in \mathbb{R}[s]^{m \times m}$ - polynomial matrices such that $\boldsymbol{G}^{-1}(s)\boldsymbol{L}_0(s)$ and $\boldsymbol{G}^{-1}(s)\boldsymbol{L}(s)$ are proper and $\boldsymbol{F}(s)\boldsymbol{A}_1^{-1}(s)$ is strictly proper. Without any loss of generality the matrix $\boldsymbol{L}_0(s)$ may be taken as $\boldsymbol{L}_0(s) = \boldsymbol{I}_m$.

The main problem is to find a method for block decoupling of the control system (between the signals $\boldsymbol{q}$ and $\boldsymbol{y}$) so as to obtain the transfer matrix $\boldsymbol{T}_{yq}(s)$ free of cancellation of unstable "hidden" modes. For the applied decoupling law this transfer matrix takes the form

$$\begin{aligned} \boldsymbol{T}_{yq}(s) &= \boldsymbol{B}_1(s)\left[\boldsymbol{G}(s)\boldsymbol{A}_1(s) - \boldsymbol{F}(s)\right]^{-1}\boldsymbol{L}(s) = \\ &= \boldsymbol{N}(s)\boldsymbol{D}^{-1}(s) \end{aligned} \tag{10}$$

with

$$\boldsymbol{N}(s) = \text{block diag}[\boldsymbol{N}_{ij}(s)] \in R[s]^{l \times m} \tag{11}$$

$$\boldsymbol{D}(s) = \text{block diag}[\boldsymbol{D}_{ii}(s)] \in R[s]^{m \times m} \tag{12}$$

where $i = 1, 2, ..., k$ and $j = 1, 2, ..., k$ according to the partition (4).

The algorithm starts with determination of the numerator matrix of the system. It is taken as a block diagonal matrix $N(s) = \text{block diag}[N_{ii}(s), i = 1, 2, ..., k]$, where particular blocks $N_{ii}(s)$ are the greatest common left divisors (*g.c.l.d.*) of columns of *i*-th row-block of $B_1(s)$ caused by the partition (4). Then $B_1(s)$ takes the form

$$B_1(s) = N(s)B(s) . \tag{13}$$

Then the algorithm allows one to calculate ranks of the remaining system elements which means number of poles of the system which values are assumed during calculations. The details of an universal algorithm for decoupling square, right and left-invertible plants has been presented in [8]. Utilizing theorems given in [16; 31], which describe stability of the decoupled system and property of all its elements the algorithm guarantees free location of all poles of the system and guarantees that all designed elements (parts of the system) are proper (or strictly proper), so they are able to be physically realizable.

4. RECONFIGURATION AFTER ACTUATOR FAULT

The algorithm outlined in the previous section allows one to fully automate process of the synthesis of decoupled system which may be used in an adaptive control for non-linear or reconfigurable systems. 13 steps of the algorithm allow one to check all necessary conditions and describe a calculation which has to be made to ensure stability and property. According to a type of fault full reconfiguration may require to carry out all calculations form the decoupling algorithm. However some of them serve only to determine e.g. orders of matrices of the transfer matrices of particular elements of the system. Some types of faults do not change structure of the systems itself and appropriate steps of the algorithm may be omitted. This will be discussed in this section from the point of view of the reconfiguration after actuator fault.

4.1 Inputs outputs pairing

The first problem with decoupling using any decoupling method is pairing of the system inputs and outputs (4). This calculations are done in first step of the algorithm where the pairing is established by permutation matrix P. We permute rows of matrix $B_1(s)$ of the faulty system Σ_1 or Σ_2 and if it is necessary, group plant's outputs $y(s)$ (and $y_0(s)$). Substitute $B_1(s) := PB_1(s)$.

Proper pairing may be done by utilizing theory of RGA factors. Their analysis help to determine impact of the specific inputs on some outputs [19]. These simple calculations,

$$RGA(0) = G(0).* invG^T(0) , \tag{14}$$

which can be made automatically allows one to check how an actuator fault impacts the influence of its inputs. For the faults which cause loss of the actuator it is rather obvious that the change of input output pairing is necessary but even if we do not loss any inputs but their faults change its dynamics and gains the change of pairing may be a necessity.

Example 1

After change of the elements A(2,2) and A(3,1) of the system steady state gain matrix

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 3 & 7 & 4 \\ 3 & 2 & 4 \end{bmatrix} \quad RGA(A) = \begin{bmatrix} 1.8182 & 0 & -0.8182 \\ -0.5455 & 1.4 & 0.1455 \\ -0.2727 & -0.4 & 1.6727 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 3 & 1 & 4 \\ 1 & 2 & 4 \end{bmatrix} \quad RGA(A) = \begin{bmatrix} 0.5405 & 0.8649 & -0.4054 \\ 0.8108 & -0.4595 & 0.6486 \\ -0.3514 & 0.5946 & 0.7568 \end{bmatrix}$$

one should consider to change the pairing from $(u_1 - y_1; u_2 - y_2; u_3 - y_3)$ to $(u_1 - y_2; u_2 - y_1; u_3 - y_3)$.

RGA analysis may also be useful for analysis of the left invertible systems. If the original system Σ or the faulty one Σ_2 has more outputs than inputs $l > m$ then it has to be verified whether full dynamic decoupling is even possible. To check that a theorem given in [31] may be utilized.

Theorem 1 [31]

A left-invertible plant with the transfer matrix (10) of rank m can be block decoupled according to the partition (4) by use of linear state variable feedback and dynamic feedforward if and only if $rank B_{1i}(s) = m_i$, $i = 1, 2, ..., k$.

If theorem 1 is satisfied then a standard procedure may be utilized [8; 31] else only partial decoupling is possible and all decoupling procedure may be done for a squared down plant. Outputs which may be omitted in this calculations may be pointed out by a procedure given in [19]. To select a square subsystem we eliminate the output with the smallest row sum in the RGA matrix.

4.2 Interconnection transmission zeros

According to theories given in [16; 22; 31] if the polynomial matrix $\tilde{G}(s) \in R[s]^{l \times l}$, a *g.c.l.d.* of all columns $B(s)$ defined by the relation

$$B(s) = \tilde{G}(s)\tilde{B}(s) , \tag{15}$$

is not unimodular and if its zeros lie in the unstable region of the complex plane, the (unobservable) poles of decoupled system corresponding to these zeros are fixed and unstable. These so called 'interconnection' transmission zeros cannot be eliminated by a feedforward compensator of "zero order". To remove these unobservable poles we can use the compensation scheme together with an additional dynamic feedforward compensator obtained by augmenting the plant model with a serial dynamic element $R_a(s)P_a^{-1}(s)$.

After calculations of the element $R_a(s)P_a^{-1}(s)$ the "standard" procedure with an "augmented plant" can be used and a decoupled system $T_{yq}(s)$ without fixed poles is automatically obtained. Finally $\boldsymbol{R}_a(s)\boldsymbol{P}_a^{-1}(s)$ should be "shifted" into the structure of dynamic feedforward compensator [1; 16]. An algorithm which may be used to calculate this additional dynamics was given by [1; 16] and has been modified in [7] to make it more reliable and efficient.

From the reconfiguration point of view all these calculations are necessary only if the plant after a fault has any 'interconnection' transmission zeros, which real parts lie in the left-half plane of the s-domain E.g. for the system with $\boldsymbol{B}_1(s)$ of zero order the additional dynamics is not necessary at all.

4.3. Stability of the system

Stability of the decoupled system describes the following lemma and theorem.

Lemma 1 [16]

The block diagonal matrix $\boldsymbol{D}(s) \in R[s]^{l\times l}$ that satisfies the relation (10) exists if there exist polynomial matrices $\bar{\boldsymbol{L}}(s) \in R[s]^{m\times(m-l)}$ and $\bar{\boldsymbol{B}}(s) \in R[s]^{m\times(m-l)}$ of full rank such that

$$\boldsymbol{G}(s)\boldsymbol{A}_1(s) - \boldsymbol{F}(s) - \boldsymbol{L}(s)\boldsymbol{D}(s)\boldsymbol{B}(s) = \bar{\boldsymbol{L}}(s)\bar{\boldsymbol{B}}(s).$$

Theorem 2 [16]

The closed-loop poles of the decoupled system $\boldsymbol{T}_{yq}(s)$ realized by linear state variable feedback (l.s.v.f.) with dynamic feedforward consist of the zeros of $\left|[\boldsymbol{L}(s), \bar{\boldsymbol{L}}(s)]\right|$, which are uncontrollable, the zeros of $\left|[\boldsymbol{B}^{\mathrm{T}}(s), \bar{\boldsymbol{B}}^{\mathrm{T}}(s)]^{\mathrm{T}}\right|$, which are unobservable and the zeros of $\left|\boldsymbol{D}(s)\right|$, which are controllable and observable.

All this matrices and their determinants zeros are arbitrary set in appropriate steps of the decoupling algorithm. Recalculation of the necessary number and values of all poles of the decoupled system depends naturally on a type of actuator fault.

According to *Theorem 2* to eliminate any unobservable poles of the decoupled system for the plants with more inputs than outputs $m > l$ one has to determine matrix $\bar{\boldsymbol{B}}(s)$ so as to get $[\boldsymbol{B}^{\mathrm{T}}(s), \bar{\boldsymbol{B}}^{\mathrm{T}}(s)]^{\mathrm{T}}$ unimodular. So for system Σ_1 or Σ_2 which after fault still has $m > l$ to ensure that the reconfigured system will not have any unobservable poles these calculations must be done any time unless the fault does not change $\boldsymbol{B}(s)$ so much that zeros $\left|[\boldsymbol{B}^{\mathrm{T}}(s), \bar{\boldsymbol{B}}^{\mathrm{T}}(s)]^{\mathrm{T}}\right|$ with the old $\bar{\boldsymbol{B}}(s)$ will still be stable. For actuator faults according to the *Definition 2* these calculations are obligatory.

The rest of calculations depends on the system structure and to be more precise on the structure of matrices (2). The fault which does not change their structures does not need to recalculate rank of the matrices $\boldsymbol{D}(s)$ and $[\boldsymbol{L}(s), \bar{\boldsymbol{L}}(s)]$ which do not change number of uncontrollable and controllable and observable poles of the decoupled system. The assumed matrices $\boldsymbol{D}(s)$ and $[\boldsymbol{L}(s), \bar{\boldsymbol{L}}(s)]$ leave the same and the only calculation necessary to be carried out regardless of the type of damage is right matrix division

$$[\boldsymbol{L}(s)\boldsymbol{D}(s)\boldsymbol{B}(s) + \bar{\boldsymbol{L}}(s)\bar{\boldsymbol{B}}(s)]\boldsymbol{A}_1^{-1}(s) = \boldsymbol{G}(s) - \boldsymbol{F}(s)\boldsymbol{A}_1^{-1}(s) \quad (16)$$

where $\boldsymbol{G}(s) \in R[s]^{m\times m}$ is the quotient and $-\boldsymbol{F}(s) \in R[s]^{m\times m}$ is the remainder. That and the time domain realization of the designed control system which details may be found in [2; 9; 32] ends the algorithm.

Like it was shown on the basis of the deliberation even huge part of a calculation of the described decoupling algorithm may be omitted due to the fact that for some failures the system structure is constant.

4.4 Outline of the reconfiguration algorithm

An universal algorithm for decoupling square, right and left-invertible plants is presented in [8]. The detailed description of all steps of the standard decoupling procedure for square and right invertible plants $m \geq l$ may be as well found in [2, 9]. The algorithm guarantees free location of all poles of the system and guarantees that all designed elements (parts of the system) are proper (or strictly proper). Moreover it allows one to fully automate process of the synthesis of decoupled system which may be used in an adaptive control for non-linear or reconfigurable systems. Below an outline of the discussed procedure is presented. The algorithm has been divided into 4 general stages of calculations. An offline analysis of the possible faults allows one to prepare matrices $\boldsymbol{D}(s)$ and $[\boldsymbol{L}(s), \bar{\boldsymbol{L}}(s)]$ which are used in Stage 4 finalizing calculations.

The algorithm

Stage 1.

Given the plant description (1) derive its transfer matrix $\boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s)$. Group plant's outputs $\boldsymbol{y}(s)$ (and $\boldsymbol{y}_0(s)$).

If the system has more outputs than inputs $l > m$ check whether full dynamic decoupling is possible and calculate a new numerator matrix $\boldsymbol{B}_m(s)$. Substitute $\boldsymbol{T}_m(s) = \boldsymbol{B}_m(s)\boldsymbol{A}_1^{-1}(s)$.

Stage 2.

Calculate numerator matrix (11) and determine $\tilde{\boldsymbol{G}}(s) \in R[s]^{l\times l}$, a *g.c.l.d.* of all columns of the matrix $\boldsymbol{B}(s) = \tilde{\boldsymbol{G}}(s)\tilde{\boldsymbol{B}}(s)$. If $\tilde{\boldsymbol{G}}(s)$ is not unimodular (or stable) calculate the additional dynamic element.

Connect (in series) additional dynamic element $\boldsymbol{R}_a(s)\boldsymbol{P}_a^{-1}(s)$ with the plant (1). Run the "standard" procedure again with an "augmented plant".

Stage 3.

According to *Theorem 2* eliminate any unobservable poles of the decoupled system by finding a $\bar{\boldsymbol{B}}(s)$ matrix which makes $[\boldsymbol{B}^{\mathrm{T}}(s), \bar{\boldsymbol{B}}^{\mathrm{T}}(s)]^{\mathrm{T}}$ unimodular.

Stage 4.

Determine the structure and chose matrices $\boldsymbol{D}(s)$ and $[\boldsymbol{L}(s),\bar{\boldsymbol{L}}(s)]$. Execute right matrix division (16) and determine the time domain realization of the designed control system.

5 EXAMPLE

In order to illustrate the theoretical considerations an example of reconfiguration of a decoupling control system is presented. Let assume a plant (of $n=5$ order with $m=3$ inputs and $l=3$ outputs) defined by the following matrices of the state and output equations (1)

$$\boldsymbol{A}=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\-1&-2&-1&0&0\\0&0&0&-1&0\\0&0&0&-1&2\end{bmatrix},\ \boldsymbol{B}=\begin{bmatrix}1&0&0\\0&0&0\\0&1&1\\0&0&1\\0&1&0\end{bmatrix},$$

$$\boldsymbol{C}=\begin{bmatrix}1&0&1&0&0\\0&1&0&0&1\\0&1&0&0&0\end{bmatrix},\ \boldsymbol{D}=\boldsymbol{0}.$$

This plant can be described in the *r.r.p.* matrix fraction as follows

$$\boldsymbol{B}_1(s)=\begin{bmatrix}s-2&s-8&4\\1&s+4&-1\\0&1&0\end{bmatrix},$$

$$\boldsymbol{A}_1(s)=\begin{bmatrix}s^2-2s&-8s-1&4s\\s-2&s^2+s-6&-s+3\\0&0&s+1\end{bmatrix}.$$

It has the poles $s_1=2$, $s_{2,3}=-0.2150\pm i1.3071$, $s_4=-1$, $s_5=-0.5698$ and one transmission zero $s_1^o=-2$. So, it is unstable and minimum phase. A fully decoupled system contains four (1-1-2 in a loop) controllable and observable poles and the dynamic precompensator is of rank zero. So for any system Σ_1 with minimum phase zeros one has only do calculations of Stage 1 and 3, assume matrix $\boldsymbol{D}(s)$, $[\boldsymbol{L}(s),\bar{\boldsymbol{L}}(s)]=\boldsymbol{I}_3$ and carry out calculations of Stage 6.

The transfer matrix of the decoupled system may be described by

$$\boldsymbol{N}(s)=\boldsymbol{I}_3,\ D(s)=\begin{bmatrix}s+0.5&0&0\\0&s+0.4&0\\0&0&s^2+s+0.24\end{bmatrix}.$$

Assuming different possible actuator faults one can consider few possible plants to be decoupled and control systems to be reconfigured and recalculated. Let us present three options:

a) the value of transmission zero changes with change of actuator parameters. If the element $\boldsymbol{B}_{32}<-1$ the value of zero is greater than 0 and the plant changes to be a non-minimum phase one. Then one obtain decoupled system which consist of six controllable and observable poles and dynamic precompensator of rank two. In such a case calculations of all stages has to be carried out. Being prepared to such a fault matrices $\boldsymbol{D}(s)$ and $[\boldsymbol{L}(s),\bar{\boldsymbol{L}}(s)]$ may be assumed offline and used in Stage 4.

b) an fault of the first actuator brings the system to an left invertible plant and after grouping first and third output the system to be decoupled is described in the *r.r.p.* matrix fraction by matrices

$$\boldsymbol{B}_m(s)=\begin{bmatrix}s^2+4s+3.75&-2-2\\1&0\end{bmatrix},$$

$$\boldsymbol{A}_1(s)=\begin{bmatrix}s^3+s^2-2.25s-3.25&-s^2+s+2\\4.25s+4.25&s^2-s-2\end{bmatrix}.$$

In this case there is as well necessary to assume four poles but this time grouped in two sets (1-3 in a loop). The transfer matrix of the decoupled “square” system may be described by

$$\boldsymbol{N}(s)=\boldsymbol{I}_3,\ D(s)=\begin{bmatrix}s+0.5&0\\0&s^3+1.4s^2+0.64s+0.096\end{bmatrix}.$$

The system after decoupling has two independent outputs y_1 and y_2 with third output y_3 depending on them.

c) an fault of the third actuator brings the system to an left invertible with $\boldsymbol{B}_1(s)$ not satisfying theorem 1 so then a standard procedure may not be utilized. Then it is possible to square down the plant but depending on which output will be omitted different structures (minimum or non-minimum phase) have to be dealt with.

d) an fault of the third actuator makes the system uncontrollable and the decoupling is not possible.

As the example shows an offline analysis allows one to check the system structure after a fault and then prepare appropriate (sets of poles) matrices. Creating a type of graph of possible faults and structures of the control systems considerably simplifies and speeds up the calculations.

The system after reconfiguration is still partly decoupled and may allow to continue regulation or at least to safe disabling.

6. CONCLUSIONS AND FINAL REMARKS

In the paper problems with reconfiguration of a dynamically decoupled system after actuator fault were considered. The analysed algorithm allows one to fully automate the process of redesign the control system which may be used especially in adaptive decoupled, reconfigurable, fault tolerant MIMO systems.

Conditions were shown which indicates the possibility of carrying out simplified path of calculations in the case the system change does not change its structure. Such a way of reconfigurability considerably simplifies and accelerates process of designing of the control system. Full reconfiguration of the system in some cases may be accelerated approximately over 50% which may play a significant role for safety of operation of dangerous processes which after failures should be stopped as fast as possible in some specific way.

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