XX IMEKO World Congress
Metrology for Green Growth
September 9–14, 2012, Busan, Republic of Korea

A METHOD FOR ESTIMATION OF FORM DEVIATIONS OF FREE-FORM SURFACES

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Abstract: In the majority of cases coordinate measurements of free-form surfaces are carried out along a regular $u \times v$ grid. The local geometric deviation, i.e. the distance of a particular measurement point from the CAD model of the nominal surface, is established for each point. The measurement aims at evaluating the form deviation and thus the greatest deviation of the actual surface from the CAD model. Obtaining data of a discrete character is inseparably connected with losing information on the surface properties. The present paper proposes an approach to estimating form deviation of free-form surface, consisting in using CAD model built on the observed geometric deviations and representing surface of form deviations.

Keywords: Free-form surface, Form deviation, CAD model.

1. INTRODUCTION

In the majority of cases coordinate measurements of free-form surfaces are carried out along a regular $u \times v$ grid with the use of *UV SCANNING* option, which is inbuilt in CMM software. The result of a measurement is a set of measurement points of a specified distribution on the measured surface. For each measurement point, the value of the local geometric deviation, i.e. the distance of this measurement point from the CAD model of the nominal surface in the normal direction is established. A form deviation is determined as a doubled highest absolute value out of the obtained set of local deviations [1]. Measurements can be made with reference to the datum features, and then the measurement results include also deviations of location and orientation. In order to reduce deviations in location and orientation, fitting the measurement data to the CAD model needs to be performed [2]; if it is the case, then local geometric deviations only represent surface irregularities and it is possible to assess a (simple) form deviation from them [1].

Obtaining data of a discrete character is inseparably connected with losing information on the surface properties. Measuring along a regular grid tends to omit critical points, e.g. points of greatest deviations. The aim of coordinate measurement is to determine a smooth surface of form deviations superimposed on the nominal surface. However, in the measurement process, the undesirable component connected with random phenomena and the deterministic component representing smooth surface overlap each other. In consequence, the spatial coordinates collected at each measurement point include two separate components. Therefore, in order to increase the accuracy of estimating form deviations, it is advisable to remove random effects from measurement data, and to establish a smooth surface approximating the information lost as a result of discretisation.

Because curvature is spatially variable at each point of a free-form surface, the distribution of machining forces and other phenomena occurring during machining are also spatially variable [3]. As an effect of this, the distribution of geometric deviations is of the same character. Deviations of a systematic (deterministic) character, as well as deviations of a random character, are observed on a surface. Deterministic deviations are spatially correlated however a lack of spatial correlation indicates spatial randomness of deviations [4,5]. Spatial statistics methods, including the most popular Moran's *I* statistics which measures spatial autocorrelation, can be applied to research on dependency of spatial data [6] (Section 2.2).

Identifying spatial autocorrelation in measurement data proves the existence of deterministic deviations. In such a case, advanced CAD software for surface modelling may be applied to fitting a surface regression model representing these deviations (Section 2.1). The first step in model diagnosing is to examine the model residuals for the probability distribution and the existence of spatial autocorrelation [6].

The present paper proposes an approach to estimating form deviation of free-form surface, consisting in using CAD model built on the observed geometric deviations and representing surface of form deviations. For this purpose, the random component must be removed from the measurement data. The spatial nature of deviations determined the type of approach to solving the issue. The data obtained from measurements made along a regular $u \times v$ grid of points were used as a basis to create the model. The regression analysis, an iterative procedure, spatial statistics methods, and NURBS modelling [7] were used for establishing the model.

2. ESTIMATING THE SURFACE OF FORM DEVIATIONS

In coordinate measurements of free-form surfaces, spatial data is obtained which provides information on the machining and on geometric deviations in the spatial aspect.

If a measurement is to take into consideration form deviation without reference to the datum features, the procedure of fitting the data to the CAD model must be performed [2]. Then, the determined local geometric deviations only represent surface irregularities which can be divided into three components of different lengths: form deviations, waviness, and roughness of the surface [3]. Spatial coordinates of each measurement point include all the three components at different proportions.

The components connected with the form deviations and waviness, are surface irregularities superimposed on the nominal surface most often deterministic in character. The component connected with the surface roughness and measurement noise is random in character [3]. The component connected with the deterministic deviations is spatially correlated. The random component, on the other hand, is weakly correlated and is considered to be of a spatially random character [5]. The different spatial nature of effects may be the basis for removing the random component from measurement data. If a test for spatial autocorrelation shows spatial dependence of the data, a spatial model representing form/deterministic deviations can be prepared [5,6].

2.1. Modelling of the surface

In order to create the surface model representing deterministic deviations, the NURBS method was applied. The NURBS surface of the *p* degree in the *u* direction and the *q* degree in the *v* direction is a vector function of two variables [7].

The input data in surface interpolation is a set of points forming a spatial grid of points. In the case under concern, the data were obtained from coordinate measurements during which a two-direction grid of measurement points was obtained. In developing the geometric model, the method of global surface interpolation was used. The process is carried out in two stages [7]:

– at the first stage, a series of isoparametric curves located on the surface patch is created. These curves are interpolated on the subsequent rows of the pre-set points of one of the parameterization directions, *u* or *v*; the value of the other parameter describing the surface is then constant. A spatial grid of control points is obtained this way;
– at the second stage, coordinates of surface control points are determined. It is performed by interpolating curves through the control points of the curves which were interpolated earlier. The interpolation is made in the other parameterization direction. The surface is lofted on the series of curves which was determined earlier.

After the interpolation stage was completed, shape modification iteration of the created surface patch was applied in the subsequent stages (the approximation stages). These operations aimed at obtaining an adequate model of regression surface, which would represent deterministic deviations. In this case, popular procedures were applied of changing the NURBS surface shape [7]. Reducing the number of knots (at the subsequent iteration stages) results in reducing the number of control points of the surface. A less complex shape can be obtained this way.

2.2. Model's adequacy verification

The method of deterministic deviations surface adequate model designing consists in iterative modelling of the surface regression model and in testing the spatial randomness of the model residuals at the consecutive iteration stages (as in [4,5]). In the subsequently constructed models, the number of control points and the surface degrees in both directions are changed. The model residuals are examined at each step, and the arithmetic mean, probability distribution, and the *I* spatial autocorrelation coefficient (1) are determined. The spatial autocorrelation coefficient *I* has the following form [6]:

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} r_i r_j}{\sum_{i=1}^{n} r_i^2} \tag{1}$$

where:
n – number of measurement points, r_i, r_j – model residuals in i and j locations, c_{ij} – weighting coefficients, elements of weighting matrices reflecting spatial relations between residuals in i and j locations, $S_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \ (i \neq j)$.

It is assumed that the dependence between the data values at the i and j points decreases when the distance d_{ij} increases, this relation can be described in the following way [6]:

$$c_{ij} = d_{ij}^{\ -f} \tag{2}$$

where:
$c_{ij} = 0$ for $i = j$, f – constant ($f \geq 1$), in this work $f = 3$ is assumed, the correctness of the choice was investigated in previous works of the author [8].

After having determined the coefficient *I*, a test of significance for its value needs to be conducted. Positive and significant value of the *I* statistics implies the existence of positive spatial autocorrelation. Otherwise, lack of spatial correlation indicates spatial randomness of residuals [6].

The model with the smallest number of control points and the lowest surface degrees in the *u* and *v* directions (Section 2.2), for which the model residuals met the criteria of a normal probability distribution and of spatial randomness, is adopted as an adequate one.

3. EXPERIMENTAL RESEARCH

3.1. Measurements

The experiment was performed on a free-form surface of a workpiece made of aluminium alloy with the base measuring 50×50 mm (Fig. 1a), obtained in the milling process, the measurements were carried out on a Global Performance CMM (PC DMIS software).

In the first stage, the measurement along a regular grid with the *UV SCANNING* was made, in which 625 uniformly distributed measurement points were scanned (25 rows × 25 columns), and geometric deviations were computed. A map of the observed geometric deviations is shown in Fig. 1b. The measurement results are presented in Tab. 1, results at 10,000 measurement points are included as reference.

a)

b)

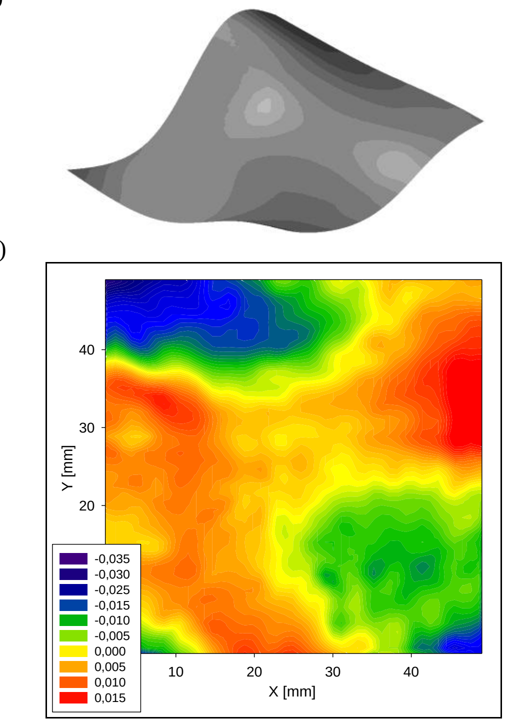

Fig. 1. Characteristics of the measured surface:
a) CAD model, b) the map of deviations

Table 1. Statistical parameters of geometric deviations

Statistical parameters of geometric deviations [mm]	10000 pts	625 pts
Mean	−0.0004	−0.0010
Standard deviation	0.0106	0.0094
Minimum Maximum	−0.0330 +0.0233	−0.0318 +0.0188

Statistical parameters of the obtained set of deviations were determined, and tests for randomness were performed. In all statistical tests a confidence level P=0.99 was adopted. The tests (Section 2.2) showed spatial autocorrelation of the measurement data, which indicated the presence of form/deterministic deviations on the surface under research.

3.2. Constructing the model

In the second stage, the regression surface estimating form deviations was modelled on the obtained measurement data using an iterative procedure, NURBS geometric modelling, and spatial statistics methods (Section 2.1, 2.2). The model with the smallest number of control points and the lowest surface degrees in the u and v directions, for which the model residuals met the criteria of a normal distribution and of spatial randomness, was adopted as an adequate one. The criterion was met for the number of control points amounting to 15×15, and the number of surface degrees being 3×3. The determined model represents deterministic/form deviations, whereas the residuals of the model constitute the random effects. Maps of deterministic and random deviations are shown in Fig. 2. The computation and modelling results are compiled in Tab. 2.

Table 2. Modelling and computation results

[mm]	Deterministic component	Random component
Mean	−0.0009	0.0000
Minimum	−0.0311	−0.0077
Maximum	+0.0190	+0.0067
Standard deviation	0.0096	0.0014
Form deviation	0.0622	

a)

b)

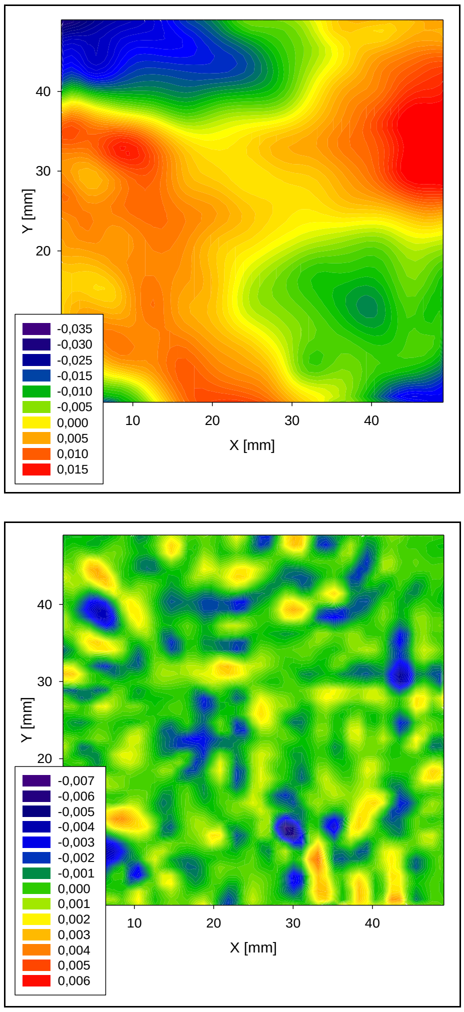

Fig. 2. Maps of distribution of deviations:
a) deterministic component, b) random component

3.3. Determining form deviation

The established model approximates geometric deviations between measurement points, making it possible to estimate the global form deviation. According to [1], in order to determine the global form deviation without reference to the datum features (not including location and direction deviations), it is necessary to fit the established surface (in this case, the surface model) to the nominal surface (in this case, to the nominal CAD model, Fig. 3a). While fitting the surfaces, the location and direction deviations were separated, and the values of these deviations can be derived from the transformation matrix presented in

Fig. 3b. The process of fitting the models was performed on a representation of 50,000 points in the Geomagic Qualify 12 programme. The accuracy of such an operation carried out on such a big number of points after the random variable has been removed, is definitely greater than the accuracy of fitting raw measurement data (625 points) to the nominal CAD model in CMM software. Comparison of results of establishing the form deviation from the raw measurement data and from the model of deviations is presented in Tab. 3.

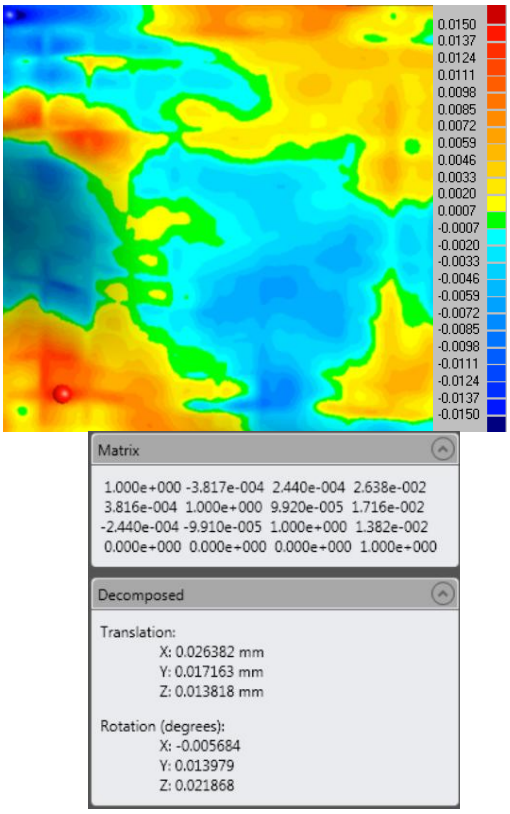

Fig. 3. a) the map of distribution of deterministic/form deviations witout reference to datums, b) transformation matrix (Geomagic Qualify 12)

Table 3. Comparison of results of establishing the form deviation from measurement results and from the model of deviations

[mm]	Measurement results	Model of form deviations		
		Deviation in measurement point	Global deviation	
		With reference to datums		Without reference to datums
Minimum	−0.0318	−0.0311	−0.0324	−0.0146
Maximum	+0.0188	+0.0190	+0.0193	+0.0127
Min.-Max.	0.0506	0.0501	0.0517	0.0273
Form deviation	0.0636	0.0622	0.0648	0.0292
Location (x,y)	0.500,47.736	0.500,47.736	0.500,48.408	0.506,48.414

When analysing the results listed in Tab. 3, it can be noticed that the differences between the values of the deviations established from the raw measurement data and these obtained from the deviation model, amount to tenths of a micrometre, which means that the model reproduces the surface well, and that the contribution of the random variable in the proximity of the greatest discrete deviations (the minimum and maximum ones) was small. The values of the global deviations obtained from the model are closer to the values of measurements at 10,000 measurement points (Tab. 1); the latter values can be adopted as reference values. According to the assumption, when approximating a surface between measurement points, a model produces an image that represents the form surface more closely than the one obtained from the raw measurement data on which it was built.

4. CONCLUSIONS

The determined form deviation is always an estimate which can be obtained from raw measurement data (a discrete representation of the determined surface) or from a surface model estimating form deviations. The advantage of applying the proposed model of deviations lies in the possibility of estimating the global deviation (and its location), which reduces the uncertainty of determining a form deviation. Moreover, separating the random component from measurement data and using much greater number of points determined on a model make it possible to reduce the fitting uncertainty in measurements without reference to datums. Diminishing the fitting uncertainty results in a lower uncertainty of determining deviations of form, location, and orientation. Therefore, the benefits of this method are a lower fitting uncertainty and a lower estimating error. The operation described above cannot be performed in the device's software; the measurement data need to be transferred to some special software. The data is then processed off-line, which does not block the CMM.

5. REFERENCES

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