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A NEW TRACEABILITY CHAIN FOR CAPILLARY VISCOMETERS

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Abstract: A new viscosity traceability chain composed of three capillary viscometers at each chain level is proposed. The correlation matrix between the viscometers at each level can be determined, which enables one to lower the uncertainty relative to the traditional "single-threaded" traceability chain.

Keywords: traceability, viscometry, uncertainty.

1. INTRODUCTION

Capillary viscometers are extensively used to determine the viscosity of liquids. The established way of doing this is by first calibrating, for instance, one master Ubbelohde viscometer 0 (i.e. determining its constant C_0) using water as the reference liquid [1]. Once the constant is known, one is able to use the instrument to measure the viscosity ν of another liquid such as oil. This second liquid can in turn be used to calibrate the constant C_{0C} of a new (reference) viscometer 0C, thus propagating a traceability chain from the original master down to a n-th viscometer 0B, $\cdots$, 4B, 5. In this way a whole range of viscosity measurements can be made.

In this work we propose a new way of realizing the traceability chain that, besides being more reliable and robust, may improve on the uncertainties obtained from the "single-threaded" chain described above. The new "multi-threaded" chain is composed of n master capillary viscometers at its top, and again n intermediate ones at each subsequent lower level in the chain. An uncertainty reduction may be possible to achieve because one can determine the correlation matrix between the instruments at each level in the chain.

Note that each level can in principle contain an arbitrary number n of viscometers. However, for the sake of illustration, we will discuss here a chain with three instruments only.

2. THE MULTI-THREADED CHAIN

According to the norm ISO 3105, there are several models for glass capillary kinematic viscometers [2], each having different capillary diameters, and consequently different viscosity measurement ranges. At each level of the chain proposed here, three viscometers of the same model are put side to side in a thermostatic bath. The first two levels in the chain are shown in Fig.(1).

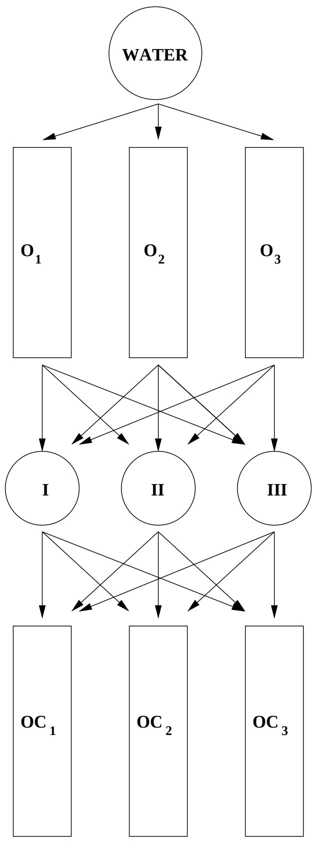

Figure 1: The first two levels in the chain.

In this figure, $0_1, 0_2$ and 0_3 are the master viscometers, while $0C_1, 0C_2$ and $0C_3$ are the reference ones, at the second chain level. The masters are calibrated with water, but the reference instruments are now cross-calibrated using different liquids I, II and III with viscosities ν_I, ν_{II} and ν_{III}, which are measured using all the masters, as indicated by the arrows in Fig. (1).

3. UNCERTAINTY REDUCTION

As an example of how the uncertainty in measurement might be reduced, let us consider the measurement of ν_I, the viscosity of liquid I. The discussion below can be easily applied to either ν_{II} or ν_{III}.

The combined uncertainty of a given master constant, $u(C_{0_k})$, $k = \{1, 2, 3\}$, can be written in terms of Type A and B uncertainties as follows [3],

$$u(C_{0_k}) = \sqrt{u_{cA}(C_{0_k})^2 + u_{cB}(C_{0_k})^2}\,, \quad (1)$$

where $u_{cA}(C_{0_k})$ is estimated using statistical methods (i.e. the standard deviation of the mean of C_{0_k}) and $u_{cB}(C_{0_k})$ is estimated in heuristic ways (i.e. non-statistical methods like numerical data from handbooks, calibration certificates and so on).

The correlation coefficient r between two given constants $C_{0_i}, C_{0_j}, i \neq j$ is estimated by

$$r(C_{0_i}, C_{0_j}) = \frac{\mathrm{cov}(C_{0_i}\, C_{0_j})}{u(C_{0_i})\, u(C_{0_j})} \,. \qquad (2)$$

However, the covariance $\mathrm{cov}(C_{0_i}\, C_{0_j})$ relative to Type A contributions to the combined uncertainty $u(C_{0_k})$ in Eq.(1) is zero, because of their random character. This can be seen using a linear regression in a $C_{0_i} \times C_{0_j}$ plot,

$$y_i = (a\, x_i + b) + \epsilon_i \,.$$

Here y_i and x_i stand for a given constant C_{0_i} and the residual ϵ_i is a random variable with mean zero. The terms in parentheses represent systematic uncertainties of Type B (i.e. the best fit line), while ϵ_i represents Type A contributions, which are randomly and simmetrically distributed around the best fit line, so its covariance should statistically vanish as the number of measurements increases.

Therefore the covariance $\mathrm{cov}(C_{0_i}\, C_{0_j})$ in Eq.(2) has only Type B contributions. According to the Schwarz inequality, we can write it as

$$\left|\mathrm{cov}(C_{0_i}\, C_{0_j})\right| \leq u_{cB}(C_{0_i})\, u_{cB}(C_{0_j}) \,.$$

We can now be conservative and assume the worst case scenario: $\left|\mathrm{cov}(C_{0_i}\, C_{0_j})\right| = u_{cB}(C_{0_i})\, u_{cB}(C_{0_j})$. In this way, Eq.(2) can now be written as

$$r(C_{0_i}, C_{0_j}) = \frac{u_{cB}(C_{0_i})\, u_{cB}(C_{0_j})}{u(C_{0_i})\, u(C_{0_j})} \,. \qquad (3)$$

The correlation among the C_{0_k} implies in turn a non-zero correlation among the viscosities $\nu_{\mathrm{I}}, \nu_{\mathrm{II}}$ and ν_{III}, since each one is measured by all three viscometers. For instance, for liquid I we can construct Table (1), where $\nu_{I,0_k,n}$ means the n-th measurement of ν_I by the master 0_k.

measurement	0_1	0_2	0_3
# 1	$\nu_{I,0_1,1}$	$\nu_{I,0_2,1}$	$\nu_{I,0_3,1}$
# 2	$\nu_{I,0_1,2}$	$\nu_{I,0_2,2}$	$\nu_{I,0_3,2}$
...	...	...	...
# n	$\nu_{I,0_1,n}$	$\nu_{I,0_2,n}$	$\nu_{I,0_3,n}$
average	$\bar{\nu}_{I,0_1}$	$\bar{\nu}_{I,0_2}$	$\bar{\nu}_{I,0_3}$
std. dev.	$s(\nu_{I,0_1})$	$s(\nu_{I,0_2})$	$s(\nu_{I,0_3})$

Table 1: n measurements of ν_I using the masters $0_1, 0_2$ and 0_3.

From this table, we can find ν_I by the following functional relationship involving $\nu_{I,0_1}$, $\nu_{I,0_2}$ and $\nu_{I,0_3}$,

$$\nu_I = \frac{\nu_{I,0_1} + \nu_{I,0_2} + \nu_{I,0_3}}{3} \,. \qquad (4)$$

The uncertainty in ν_I is

$$u(\nu_I)^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{\partial \nu_I}{\partial \nu_{I,0_i}} \frac{\partial \nu_I}{\partial \nu_{I,0_j}}\, u(\nu_{I,0_i}, \nu_{I,0_j}) + s(\bar{\nu}_I)^2 \,,$$

where $u(\nu_{I,0_i}, \nu_{I,0_j})$ is the covariance between $\nu_{I,0_i}$ and $\nu_{I,0_j}$, and the partial derivatives are to be taken relative to Eq.(4). Here $s(\bar{\nu}_I)$ is the experimental variance of the average $\bar{\nu}_I$, making it the random contribution to the uncertainty $u(\nu_I)$ [4]. Expanding the expression above in terms of the correlation coefficient $r(\nu_{I,0_k}, \nu_{I,0_l})$, we have

$$u(\nu_I)^2 = \sum_{k=1}^{3}\left(\frac{u(\nu_{I,0_k})}{3}\right)^2 + s(\bar{\nu}_I)^2 + 2\sum_{k=1}^{2}\sum_{l=k+1}^{3} \frac{u(\nu_{I,0_k})}{3}\frac{u(\nu_{I,0_l})}{3}\, r(\nu_{I,0_k}, \nu_{I,0_l}) \,. \qquad (5)$$

Now if we accept the worst case scenario and take the correlation between the viscosities as 1 (which maximizes the uncertainty), then we can simplify the above expression to

$$u(\nu_I)^2 = \left(\sum_{k=1}^{3}\frac{u(\nu_{I,0_k})}{3}\right)^2 + s(\bar{\nu}_I)^2 \,. \qquad (6)$$

If the uncertainties $u(\nu_{I,0_k})$ are equal to each other, then Eq.(6) is equivalent to the uncertainty of measurement of just *one* viscometer plus $s(\bar{\nu}_I)$. In other words, the viscosity uncertainty in a single-threaded chain is then equivalent to the $r = 1$ multi-threaded one, with the additional assumptions that all $u(\nu_{I,0_k})$ are equal and $s(\bar{\nu}_I) = 0$.

Therefore, by taking these assumptions we can compare both the single-threaded ($r = 1$) and the multi-threaded ($|r| \leq 1$) uncertainties. Since the latter case always implies a lower uncertainty than the former (by Eq.(5)), if one can determine exactly the correlation between the viscosities (instead of taking the usual conservative step of $r = 1$), it may be possible to find a "multi-threaded" uncertainty which is lower than that of the usual single-threaded chain. However, how can we determine this correlation?

4. DETERMINING THE CORRELATION

Again as in Eq.(1), the uncertainty in $\nu_{I,0_k}$ can be written as

$$u(\nu_{I,0_k}) = \sqrt{u_{cA}(\nu_{I,0_k})^2 + u_{cB}(\nu_{I,0_k})^2} \,, \qquad (7)$$

where $u_{cA}(\nu_{I,0_k})$ is the random uncertainty determined using the standard deviation of $\nu_{I,0_k}$. The Type B uncertainty again is estimated in a non-statistical fashion.

Therefore, following the steps leading from Eq.(1) to Eq.(3), the correlation $r(\nu_{I,0_k}, \nu_{I,0_l})$ is estimated as

$$r(\nu_{I,0_k}, \nu_{I,0_l}) = \frac{u_{cB}(\nu_{I,0_k})\, u_{cB}(\nu_{I,0_l})}{u(\nu_{I,0_k})\, u(\nu_{I,0_l})} \,. \qquad (8)$$

This correlation can be used in turn to calibrate the reference viscometers at the second level, and so on. The argument above can be extended to all other levels in the chain, thus allowing an overall uncertainty reduction in the whole multi-threaded chain.

5. CONCLUSION

In estimating uncertainty in measurement it is often difficult to quantify correctly the correlation between the influence quantities in a experiment. However, some (if not all) the input quantities may indeed be correlated, which can result in an increase or decrease of the combined uncertainty [5].

Here we have presented the basic idea of a new traceability chain which, by allowing an accurate estimative of the correlation, other than the usual conservative choice of r, may reduce the combined uncertainty in the traceability chain.

Furthermore, because the new chain can have an arbitrary number of viscometers at each level, it is more reliable and robust than the single-threaded one, since defects and measurements errors in a viscometer are more easily smoothed out between the correlated instruments.

This multi-threaded chain is currently being implemented at the Brazilian National Metrology Institute (Inmetro), using a chain of three viscometers at each level, just as described above. The measurement results are not yet ready, however, so the comparison of the uncertainties obtained with the new and old chain are not available yet.

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