

UNCERTAINTY OF ESTIMATES OBTAINED WITH THE HISTOGRAM TEST OF ADCS

F. Corrêa Alegria¹, A. Cruz Serra²

Instituto de Telecomunicações, Av. Rovisco Pais 1, 1049-001 Lisbon, Portugal
¹ Telephone: +351-218418485, Fax: +351-218417672, e-mail: falegria@lx.it.pt
² Telephone: +351-218418490, Fax: +351-218417672, e-mail: acserra@ist.utl.pt

Abstract: We present here expressions to determine the uncertainty interval of the estimates of transition voltages and code bin widths of an analogue to digital converter obtained with the Standard Histogram Test. We consider the presence of additive noise as it influences the precision and causes a bias on the test results.

Keywords: Analogue to digital converter, Histogram Test, Uncertainty.

1 INTRODUCTION

The Standard Histogram Test of analogue to digital converters (ADCs) [1] allows the estimation of several of its parameters, namely the transition voltages and code bin widths, integral and differential non linearity, gain and offset errors.

This test consists of applying a sine wave to the ADC input, acquiring a large number of samples and comparing the frequency of the codes obtained with the one expected from an ideal ADC.

Due to several non ideal aspects of ADC testing, like the presence of additive noise, phase noise, jitter and error on the frequency of stimulus signal or sampling clock, the estimates have an increase uncertainty [5, 6]. To quantify that uncertainty, several studies were carried out which are used here to justify the expressions used to calculate that uncertainty.

2 HISTOGRAM TEST METHOD

In the Histogram Test of ADCs, a sinusoidal stimulus signal with amplitude A , offset C and phase at the origin φ , is applied to the input:

$$v(t) = C - A \cdot \cos(2\pi f \cdot t + \varphi). \quad (1)$$

This signal is sampled by the ADC at a rate f_s and M samples are acquired. The sample instants are given by

$$t_j = \frac{j}{f_s}, \quad j = 0, 1, \dots, M-1. \quad (2)$$

The digital output codes are used to compute the cumulative histogram, c_k , which is the number of samples that have an output equal to or less than k . This histogram

is then used to compute the estimated value of the transition voltages:

$$\hat{T}_k = C - A \cdot \cos\left(\frac{c_{k-1}}{M} \pi\right), \quad k = 1, 2, \dots, 2^{n_b} - 1, \quad (3)$$

in which n_b is the number of bits of the ADC. The transition voltages determine the ADC transfer function, that is, the relation between input voltage and output digital words as depicted in Fig. 1 [7].

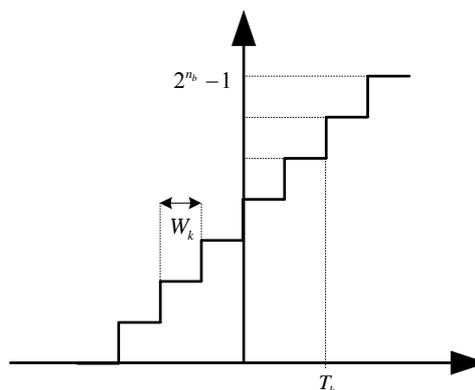


Fig. 1 – Illustration of the transfer function of an n_b -bit ADC.

The definition of transition voltage T_k , according to [1], is the value of DC voltage that when applied to the ADC input leads to half the samples having an output code of $k-1$ or less. From the estimation of the transition voltages it is possible to obtain other parameters of interest like the code bin widths defined as

$$\hat{W}_k = \hat{T}_{k+1} - \hat{T}_k, \quad k = 1, 2, \dots, 2^{n_b} - 2. \quad (4)$$

3 TRANSITION VOLTAGES

The presence of noise in the test setup or the ADC itself adds uncertainty in the estimation of the ADC parameters. We consider here additive noise, caused, for instance, by thermal fluctuations with a normal distribution with zero mean and standard deviation σ .

To simplify the derivations we chose to normalize the voltage values by dividing them by the stimulus signal

amplitude. This leads to the normalized ADC transition voltage, U , given by

$$U = \frac{T-C}{A}, \quad (5)$$

the normalized code bin width given by

$$L = \frac{W}{A}, \quad (6)$$

and the normalized additive noise standard deviation

$$\sigma_n = \frac{\sigma}{A}. \quad (7)$$

3.1 Error

The presence of noise causes a deterministic error in the transition voltages estimate [6] as can be seen in Fig. 2. It is not, however, an error that can be easily corrected since it depends on the noise standard deviation and the stimulus signal amplitude which would need to be known with a high degree of accuracy. On the other hand the value of the expression used to compute the error introduced by additive noise is not given in a closed form. Approximate expressions have been derived in [6] but are not appropriate to correct the error. They are useful however to calculate the amount of overdrive to use. Overdriving the ADC consists in using a sinusoidal stimulus signal with amplitude greater than what would be strictly required to stimulate all the ADC codes. With this technique the error caused by the presence of additive noise is limited to a chosen value. We will consider here the case where no overdrive is used. When overdrive is used a simple change of the expression presented here is possible. The expression derived in [6] for the maximum error caused by the presence of additive noise with a normalized standard deviation σ_n is

$$e_{\hat{U}} \leq \frac{\sigma_n}{5} \text{ for } \sigma_n \leq 0.1. \quad (8)$$

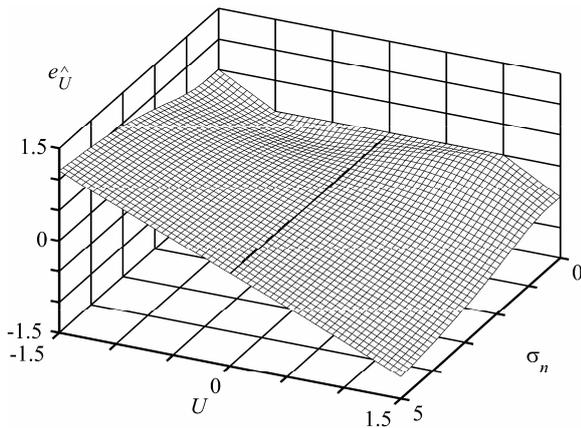


Fig. 2 – Representation of the error in the estimation of the ADC transition voltage (normalized) as a function of the normalized actual transition voltage and noise standard deviation.

This approximate expression is only valid when the additive noise standard deviation is smaller than 10% of the stimulus signal amplitude which is usually the case in practice.

3.2 Experimental Validation of the Error

To validate the approach presented in the previous section, several experimental ADC test were carried out and the mean of the estimated transition voltages was computed. The tests were carried out on a 12-bit National Instruments 6024E data acquisition board at 10 V full scale. Only the 8 most significant bits were used so that the ADC under test presented an ideal behaviour since the data acquisition board presented an INL lower than 0.5 LSB when tested with the full 12 bits. The function generator used was a Stanford DS360 ($A = 10$ V and $f = 10$ Hz), The noise generator used was Hewlett-Packard 33120A ($\sigma = 610$ mV). The test was repeated 15.000 times with 10.000 samples each.

Fig. 3 shows a good agreement between theoretical values and experimental results. The maximum error in this case was 117.5 mV which is in agreement with (8) since it is smaller than 122 mV (610 mV / 5). Note that the expression for the error of the transition voltages, $e_{\hat{T}}$, is the same as (8) if we substitute the normalized standard deviation of additive noise by its real value.

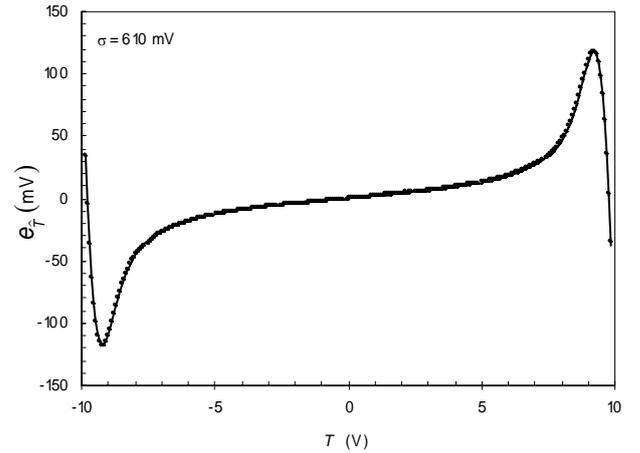


Fig. 3 – Representation of the error of the estimated transition voltages in the presence of additive noise as a function of the transition voltage. Theoretical values given by the solid line and experimental values are represented by dots.

The difference between the theoretical and experimental values, $\Delta e_{\hat{T}}$, is represented in Fig. 4. Apart from the uncertainty in the estimation of the estimation error due to the finite number of tests performed the difference is not exactly zero due to experimental difficulties namely in controlling the noise level and sinusoid amplitude and offset. The theoretical approach is however validated from the different tests carried out at different values of noise standard deviation, in particular in the case of $\sigma = 610$ mV represented here in Fig. 4.

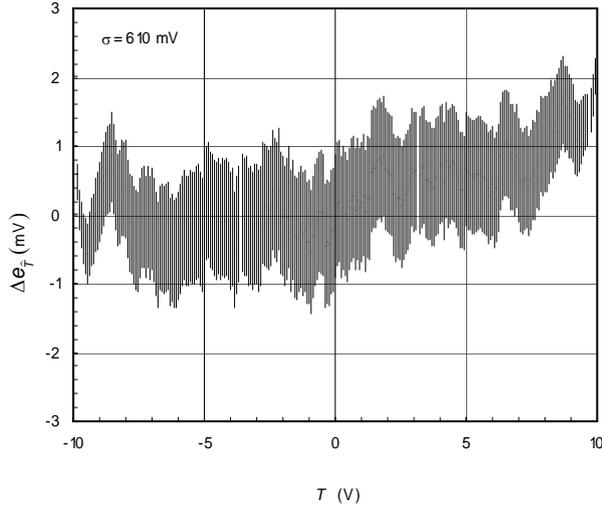


Fig. 4 – Representation of the difference between the theoretical and experimental values of transition voltage estimation error. The vertical lines represent the uncertainty in the determinations of the error due to the finite number of tests performed (99.9% confidence level).

3.3 Standard Deviation

On the other hand, the presence of additive noise causes also a random error on the transition voltages estimation. This kind of error is approximately given by its standard deviation [5]:

$$\sigma_{\hat{U}} \approx \frac{\pi}{M} \sqrt{\max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{\pi\sqrt{\pi}}\right)\right)}. \quad (9)$$

In Fig. 5 we can observe the dependence of this standard deviation with the actual transition voltage and noise standard deviation, both normalized.

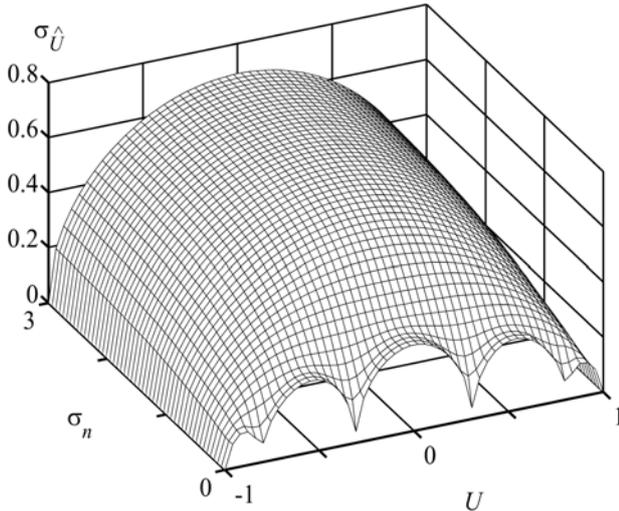


Fig. 5 – Representation of the normalized standard deviation of the estimated transition voltages as a function of the actual transition voltage and additive noise standard deviation. The number of samples acquired in this example is 5.

3.4 Experimental Validation of Standard Deviation

The same test carried out in 3.2 was also used to compute the standard deviation of the estimates of

transition voltage which can be seen in Fig. 6 together with its theoretical value (solid line).

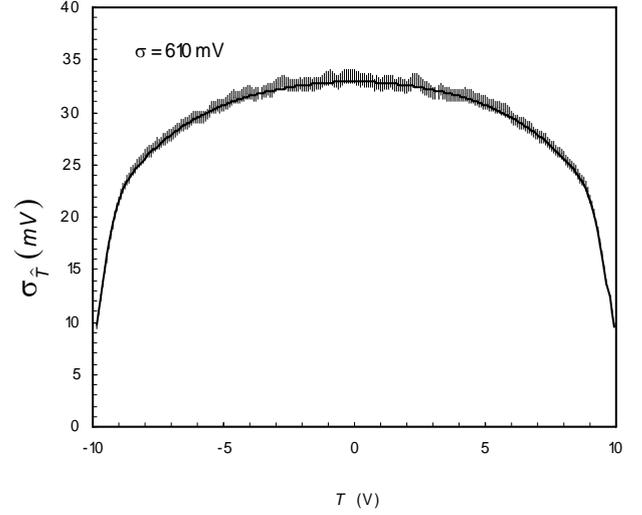


Fig. 6 – Representation of the standard deviation of the estimated transition voltages in the presence of additive noise as a function of the transition voltage. Theoretical values given by the solid line and experimental values are represented by vertical lines which represent the uncertainty in the determinations of the error due to the finite number of tests performed (99.9% confidence level).

The difference between the experimental values of standard deviation and the theoretical value, $\Delta\sigma_{\hat{T}}$, are represented in Fig. 7. Again we can see they are very close to zero.

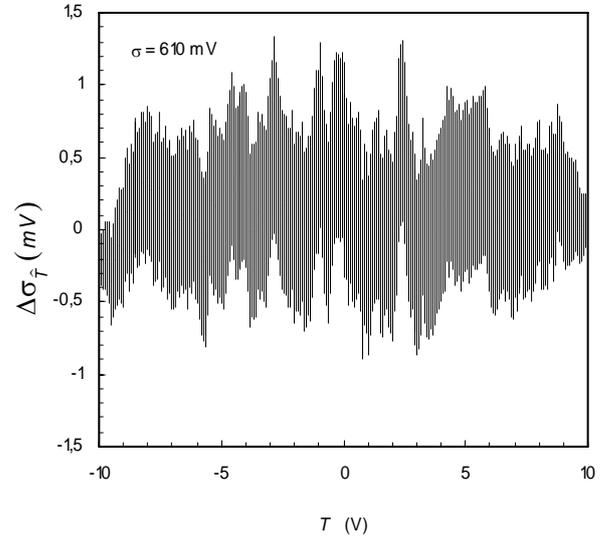


Fig. 7 – Representation of the difference between the theoretical and experimental values of transition voltage estimates standard deviation. The vertical lines represent the uncertainty in the determinations of the error due to the finite number of tests performed (99.9% confidence level).

3.5 Uncertainty of the Transition Voltages

Combining these two sources of uncertainty, using the expression recommended in [8] for this effect, leads to the following expression for the combined uncertainty

$$u_{\hat{U}} = \sqrt{e_{\hat{U}}^2 + \sigma_{\hat{U}}^2}. \quad (10)$$

Including (8) and (9) in (10) leads to

$$u_{\hat{U}} = \sqrt{\frac{\sigma_n^2}{5^2} + \frac{\pi^2}{M^2} \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{\pi\sqrt{\pi}}\right)\right)} \quad (11)$$

The estimation of the transition voltages can be obtained from their normalized values using (5). Considering that the sinusoidal stimulus signal is produced by a function generator, it will inevitably have an uncertainty in its amplitude and offset. These two sources of uncertainty should also be included in the uncertainty budget. The combined uncertainty of the ADC transition voltages is given by [7]:

$$u_{\hat{T}} = \sqrt{A^2 u_{\hat{U}}^2 + U^2 u_A^2 + u_C^2}. \quad (12)$$

Using (11) and noting that the maximum value of U^2 is 1, one gets the following expression for the uncertainty of the estimated transition voltages:

$$u_{\hat{T}} = \sqrt{\frac{\sigma^2}{25} + \frac{\pi^2}{M^2} \max\left(\frac{A^2}{4}, M \cdot \min\left(\frac{A^2}{4}, \frac{A \cdot \sigma}{\pi\sqrt{\pi}}\right)\right)} + u_A^2 + u_C^2 \quad (13)$$

4 CODE BIN WIDTHS

We now perform the same analysis but for the code bin widths using the fact that they are computed from the transition voltages using (4).

4.1 Error of the Estimates

The estimation of the ADC code bin widths is also affected by the presence of additive noise. In [9] we studied its relative error and concluded that its maximum, in absolute value, is 65% and occurs for extreme values of normalized transition voltage (± 1), regardless of the value of additive noise standard deviation:

$$\varepsilon_{\hat{L}} = 0.65 \Rightarrow e_{\hat{L}} = 0.65 \hat{L}. \quad (14)$$

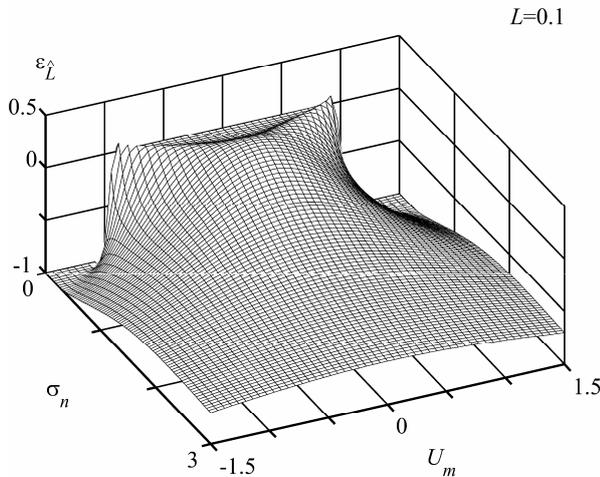


Fig. 8 – Representation of the relative error of the estimative of the code bin widths as a function of the normalized mean value of the two transition voltages that delimit the bin and of the normalized additive noise standard deviation.

In Fig. 8 we represent the exact relative error as derived in [9] as a function of normalized additive noise standard deviation and normalized mean transition voltage which is the arithmetic mean between the two transition voltages that delimit a given code bin.

4.2 Experimental Validation of the Estimation Error

The same tests used before for the transition voltages estimation, enabled also the computation of the code bin widths estimation error due to the presence of additive noise. In Fig. 9 we can see the results represented together with the theoretical values. A good agreement is observed. Note that the maximum relative error occurs at the extreme values of transition voltage, that is, for the higher and lower ADC output codes. In the experimental test the value obtained was -49.61% which can not be seen in Fig. 9 since we choose a scale for the vertical axis starting at -15% in order to show clearly the data in the middle of the graph.

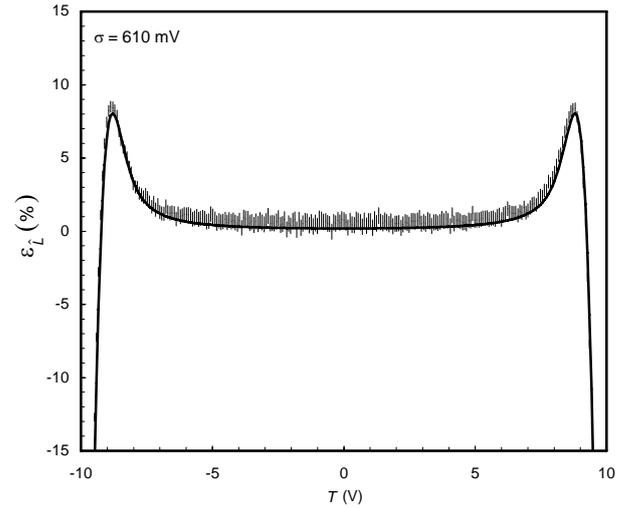


Fig. 9 – Representation of the relative error of the estimated normalized code bin widths in the presence of additive noise as a function of the normalized transition voltage. Theoretical values are represented by the solid line and experimental values are represented by vertical bars representing the 99.9% confidence interval.

To make more evident the agreement between theoretical and experimental values, we depict in Fig. 10 the difference between them ($\Delta\varepsilon_{\hat{w}}$).

Once again the disagreement observed at the graph edges is due to experimental difficulties in controlling the amplitude and offset of the stimulus signal.

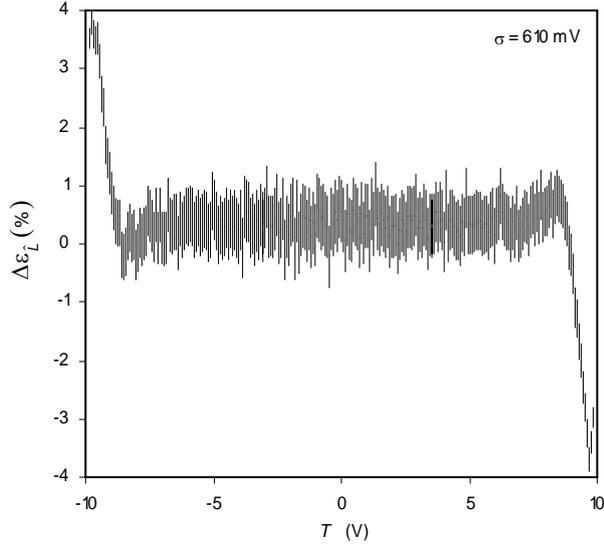


Fig. 10 – Representation of the difference between the theoretical and experimental values of the code bin width estimates relative error. The vertical lines represent the uncertainty in the determinations of the error due to the finite number of tests performed (99.9% confidence level).

4.3 Standard Deviation of the Estimates

For the random error caused by the presence of additive noise, the standard deviation of the estimates of the code bin widths can be computed from the standard deviation of the transition voltages that delimit the bin and their correlation coefficient. In [10] we proved experimentally that its value is positive and small as can be seen in Fig. 11.

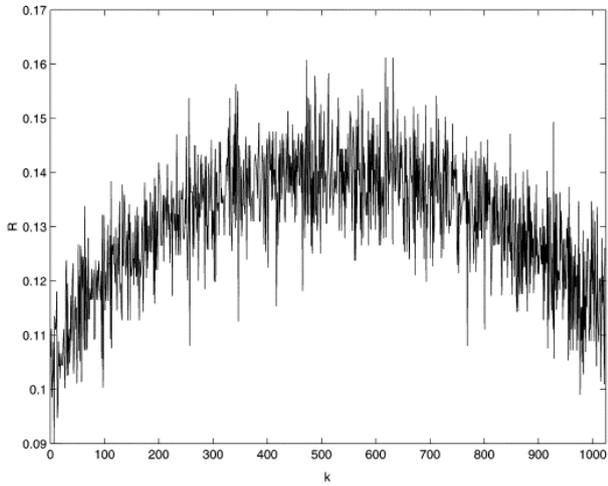


Fig. 11 – Representation of the correlation coefficient between adjacent transition voltages.

Consequently the standard deviation of the estimate of the code bin widths is always smaller than twice the standard deviation of the transition voltages estimate:

$$\sigma_{L_k}^2 = \sigma_{\hat{U}_k}^2 + \sigma_{\hat{U}_{k+1}}^2 - 2\sigma_{\hat{U}_k \hat{U}_{k+1}} \leq 2\sigma_{\hat{U}_k}^2. \quad (15)$$

From (9) we have:

$$\sigma_{L_k} \approx \frac{\pi}{M} \sqrt{2 \cdot \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{\pi\sqrt{\pi}}\right)\right)}. \quad (16)$$

4.4 Experimental Validation of the Standard Deviation

In Fig. 12 we present the data for the standard deviation of the estimates of code bin width. A good agreement between experimental (vertical bars) and theoretical values is patent.

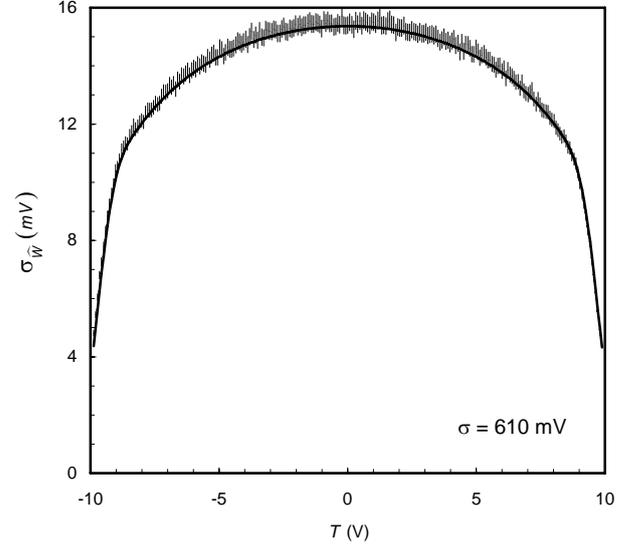


Fig. 12 – Representation of the standard deviation of the estimated of code bin width for different transition voltages in the presence of additive noise. The solid line represents the theoretical values and the vertical bars the 99.9% confidence interval for the experimental determined standard deviation.

The difference between experimental and theoretical values of the code bin widths standard deviation estimate is represented in Fig. 13.

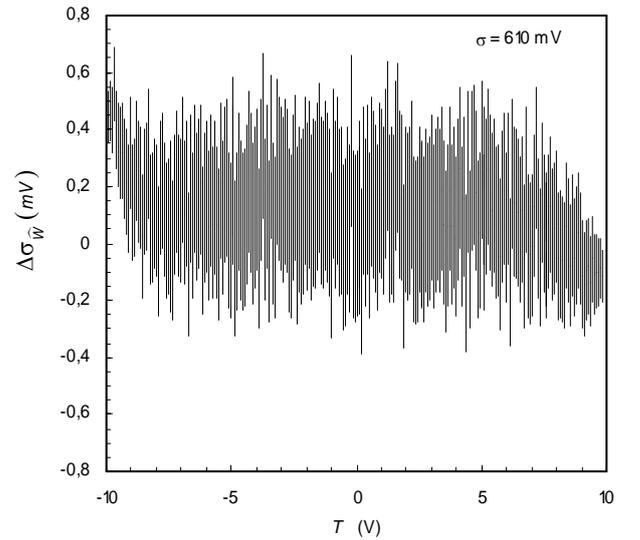


Fig. 13 – Representation of the difference between theoretical and experimental values of the standard deviation of estimates of code bin width for different transition voltages in the presence of additive noise. The vertical bars represent the 99.9% confidence interval.

4.5 Uncertainty

Combining the two uncertainty sources using

$$u_{\hat{L}} = \sqrt{e_L^2 + \sigma_L^2}, \quad (17)$$

leads, using (14) and (16), to

$$u_{\hat{L}} = \sqrt{(0.65\hat{L})^2 + 2\frac{\pi^2}{M^2} \max\left(\frac{1}{4}, M \cdot \min\left(\frac{1}{4}, \frac{\sigma_n}{\pi\sqrt{\pi}}\right)\right)}. \quad (18)$$

Taking into account (6) we get

$$u_{\hat{W}} = \sqrt{\frac{\hat{W}^2}{A^2} u_A^2 + 0.65^2 \hat{W}^2 + \frac{2\pi^2}{M^2} \max\left(\frac{A^2}{4}, M \cdot \min\left(\frac{A^2}{4}, \frac{A\sigma}{\pi\sqrt{\pi}}\right)\right)}. \quad (19)$$

This expression can be used to determine the standard uncertainty of the ADC code bin width estimates. Note that it does not depend on the stimulus signal offset uncertainty.

5 EXPANDED UNCERTAINTY

From the combined uncertainty, it is possible to determine a confidence interval for the ADC transition voltages and code bin widths using a coverage factor, K_u , of choice. The use of a value of 2 corresponds to a confidence level of approximately 95% since we can consider the estimators as being normally distributed.

Therefore we have

$$\begin{aligned} \hat{T}_k - K_u \cdot u_{\hat{T}_k} &\leq T_k \leq \hat{T}_k + K_u \cdot u_{\hat{T}_k}, \\ \hat{W}_k - K_u \cdot u_{\hat{W}_k} &\leq W_k \leq \hat{W}_k + K_u \cdot u_{\hat{W}_k}, \end{aligned} \quad (20)$$

In which $u_{\hat{T}_k}$ is given by (13) and $u_{\hat{W}_k}$ is given by (19).

6 CONCLUSIONS

The specification of confidence intervals for the parameters of an ADC estimated with any method and in particular with the Histogram Method, is paramount in computing the uncertainty interval of any measurement made with systems that use the ADC and consequently is fundamental in quantifying the quality of the measurements made.

We presented here two expressions that allow the computation of the uncertainty intervals of the ADC transition voltages and code bin widths. The sources of uncertainty considered here were the presence of additive noise and the uncertainty of the sinusoidal stimulus signal amplitude and offset.

ACKNOWLEDGMENTS

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