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EXPLORING THE SAMPLING RATE REQUIREMENTS FOR BEHAVIOURAL AMPLIFIER MODELLING

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Abstract: In this paper it is shown that for the purpose of nonlinear power amplifier behavioural modelling, the sampling rate can be set to the Nyquist rate of the input signal, rather than to the Nyquist rate of the output signal by making use of Zhu's generalized sampling theorem. This claim is supported by measurements on a basestation power amplifier. The findings are that the model error obtained when the output signal is sampled at the Nyquist rate of the input signal is approximately 1.5 dB higher than when the sampling rate is set to the Nyquist rate of the output signal. However, if a sampling rate of twice the Nyquist rate of the input signal is used, which is still typically, much lower than the Nyquist rate of the output signal, the degradation is only 0.2 dB. These are important findings that will substantially ease the requirements on ADCs used in measurement set-ups used for amplifier modelling.

Keywords: Amplifier Modelling, Generalized Sampling Theorem, Undersampling.

1. INTRODUCTION

A dominant trend in the telecommunication industry for the last years has been the introduction of wideband digitally modulation systems. This trend has had profound effects on the requirements on the radio frequency (RF) power amplifiers (PAs). In contrast to older systems, such as GSM, it must now amplify a signal which has a fast changing envelope, a high peak-to-average power ratio and a bandwidth of several megahertz. This has spawned an interest in behavioural power amplifier modelling with the aim to better understand and predict the behaviour of power amplifiers when subjected to digitally modulated wideband signals.

In this paper the requirements on the sampling rate for behavioural modelling of power amplifiers are investigated. Such behavioural models are most often extracted using measurement set-ups like the one depicted in Fig. 1. It consists of a vector signal generator (VSG) and a vector signal analyzer (VSA). The VSG generates the signal and up-converts it to the desired RF while the VSA down-converts and samples the input and output signals of the power amplifier [1].

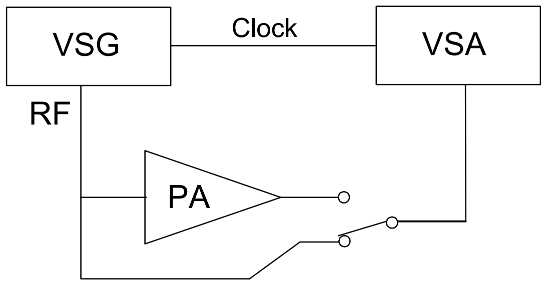

Fig. 1. Typical measurement system, basically consisting of a VSA and a VSG, for measurements intended for behavioural power amplifier modelling.

In Fig. 2, the measured power spectral density of typical measured input and output signals of a base station power amplifier are shown when the signal is a WCDMA signal used in the third-generation (3G) telecommunication system with the same name. The WCDMA signal has a bandwidth of approximately 4 MHz. The x-axis is relative to the carrier frequency of the system, which in this case is 2.14 GHz and the y-axis is normalised power. The observed asymmetry between the upper and lower sidebands for the output signal is due to "memory effects" in the amplifier. This asymmetry is most often caused by the terminating impedances at the baseband or difference frequencies [2].

An approach often taken when setting the sampling rate is to set it to twice the modulation bandwidth of the output signal, called the Nyquist rate of the output signal. If a signal is sampled using a lower sampling rate than its Nyquist rate, it is said to be undersampled. In Fig. 2 this means that the choice of sampling rate, after down-conversion, must be based on the entire 30 MHz of RF signal bandwidth. As will be shown here, this is a conservative approach, which has some serious drawbacks such as the high requirements it put on the analogue-to-digital converter (ADC). A higher sampling rate requirement implicates that an ADC with a lower dynamic range has to be chosen. Hence, to set the sampling rate as low as possible, and thereby gaining dynamic range, would be a method to increase the overall performance of the measurement system, and, thus, also for improving the amplifier modelling.

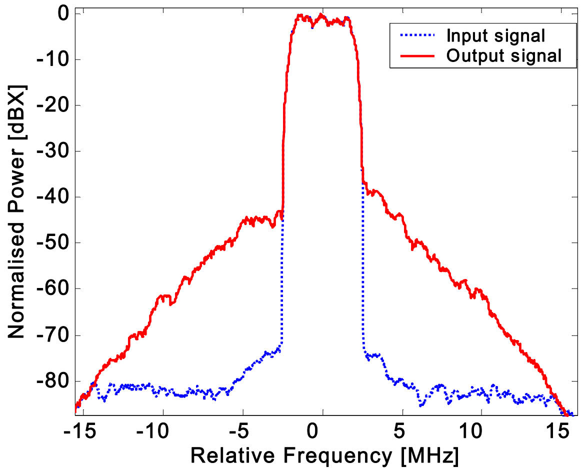

Fig. 2. Power spectral density of the input (blue) and output (red) signals of a nonlinear power amplifier using a WCDMA signal.

The undersampling technique also has some advantages over alternative techniques, such as the one presented in [3], in that it only requires one measurement of the output signal to be taken. The technique in [3], on the other hand, which make use of multiple narrowband measurements of the output signal to reconstruct a wideband signal, requires a number of measurements to be taken, which consumes time and introduces errors.

A second use of undersampling is for simulation purposes as described in [4]. By running the simulation at the lowest possible sampling rate, simulation time can be saved.

The foundation of the method presented here is Zhu's generalized sampling theorem (ZGST) [5], which essentially says that under some conditions, the output signal of a nonlinear system can be reconstructed by sampling it at the Nyquist rate of the input signal rather than the Nyquist rate of the output signal. The required conditions are essentially that the input and output signals are band-limited and that the nonlinear system performs a one-to-one mapping of the input signal to the output signal. These conditions are normally met in practice for real world power amplifiers. Similar claims have subsequently been made in [6] and [7]. The concept was extended to the general case of Volterra systems [8] in [9] and applied to power amplifiers and digital predistortion in [4] and [10] in order to linearise an amplifier.

In the specific case shown in Fig. 2 the ZGST and its extensions says that the sampling rate could be set based on the 4 MHz RF bandwidth of the input signal rather than based on the 30 MHz RF bandwidth of the output signal. Hence, the sampling rate could be reduced by a factor of more than seven.

Previous work on power amplifier applications mostly focuses on theoretical and simulation results. In this paper, the findings in e.g. [4] and [10], in addition to a brief theory introduction, are supported by measurements on a real 3G basestation power amplifier.

2. BEHAVIOURAL AMPLIFIER MODELLING

The behavioural amplifier model that will be used here to evaluate the ZGST is the tapped delay line model, also denoted the parallel Hammerstein model. It was first proposed in [11] and has been widely used both for amplifier modelling and digital predistortion purposes, see e.g. [12] and [13]. The parallel Hammerstein model is a complex envelope model. Consequently, the signals will be given on their complex envelope form. The complex envelope $z(t)$ of an RF signal $s(t) = r(t)\cos(2\pi F_c t + \varphi(t))$, where $r(t)$, $\varphi(t)$ are the envelope and phase of the signal, respectively, and F_c is the carrier frequency in Hz, is given by [14]

$$z(t) = (s(t) + j\tilde{s}(t))e^{-j2\pi F_c t} = r(t)e^{j\varphi(t)}, \tag{1}$$

where $\tilde{s}(t)$ is the Hilbert transform of $s(t)$. The structure of the parallel Hammerstein model is shown in Fig. 3, where $x(nT)$ and $y(nT)$ are the discrete-time complex envelopes of the input and output signals of the power amplifier in Fig. 1, respectively.

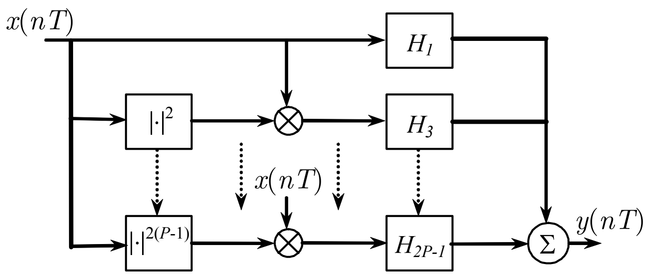

Fig. 3. Structure of the parallel Hammerstein model. Each odd nonlinear order is filtered by an individual filter. The outputs of these filters are then added together to form the final output signal of the model.

The output signal of the parallel Hammerstein model is, on discrete-time complex envelope form, given by [13]

$$y(nT) = \sum_{p=1}^{P} H_{2p-1}\left(q^{-1}\right)|x(nT)|^{2(p-1)}x(nT) = \sum_{p=1}^{P}\sum_{m=0}^{M} b_{m,2p-1}|x((n-m)T)|^{2(p-1)}x((n-m)T). \tag{2}$$

The filters $H_i(q^{-1})$ are complex-valued linear finite impulse response (FIR) filters of length M, with complex-valued coefficients $b_{i,j}$, q^{-1} is the delay operator, T is the sampling period when the output signal is sampled at its

Nyquist rate and $n = 0, 1, ..., N\text{-}1$, where N is the number of samples taken. Hence, a parallel Hammerstein model can be said to be of the nonlinear order $2P$-1 and have a memory depth of M samples.

In the case of undersampling of the output signal, (2) is replaced by

$$y(LnT) = \sum_{p=1}^{P}\sum_{m=0}^{M} b_{m,2p-1}\,|x((Ln-m)T)|^{2(p-1)}\,x((Ln-m)T), \quad (3)$$

where L is the undersampling factor, i.e. $L = 1$ means Nyquist sampling of the output signal.

One of the key strengths of the parallel Hammerstein model is that it, i.e. (2) and (3) can be formulated as a linear regression, that is as a linear function in the unknowns $b_{i,j}$. Thus, its parameters can be found using the powerful standard techniques that exists for such problems [15], [16].

When forming this set of equations from (3) it is observed that the output signal is sampled at $1/L$ times its Nyquist rate, i.e. at $1/LT$ [Hz], while the input signal is sampled at the Nyquist rate of the output signal, i.e. at $1/T$ [Hz].

The parameters of the model, $b_{i,j}$, can be found directly from the undersampled output data in (3) and it is not necessary to explicitly reconstruct the output signal to its Nyquist sampling rate prior to the system identification.

The input signal on the other hand has to be available at the Nyquist sampling rate of the output signal, which is easily accomplished using upsampling.

3. RESULTS AND DISCUSSION

Measurements were taken on a power amplifier intended for the 3G system WCDMA which has an analogue bandwidth of approximately 4 MHz. Three different complex sampling rates where used when measuring the output signal of the power amplifier, 40.8 MHz ($L = 1$, no aliasing), 8.16 MHz ($L = 5$, aliasing), 4.08 MHz ($L = 10$, aliasing). With complex sampling rate is here meant the sampling rate of the complex-valued input and output signals, i.e. the sampling rate is e.g. 40.8 MHz for both the real and imaginary parts of the signals. Thus, given the Nyquist theorem, a complex sampling rate of 40.8 MHz, in the sense used here, is enough to sample a signal with a bandwidth of 40.8 MHz in theory. In practice the complex sampling rate must be set somewhat above the Nyquist rate for the signal at hand.

The power spectral density of the input and output signals were shown in Fig. 2 for a complex sampling rate of 40.8 MHz.

The parameters of the parallel Hammerstein model, $b_{i,j}$ in (3), were fitted to the data taken at the three different complex sampling rates using standard techniques similar to the technique described in [13]. The performance was evaluated with respect to the out-of-band distortion, i.e. the spectral widening of the signal caused by the nonlinear amplifier seen in Fig. 2, using the adjacent channel error power ratio (ACEPR) figure-of-merit defined in [17] as

$$ACEPR = \frac{\int_{adj} |E(f)|^2\,df}{\int_{ch} |Y(f)|^2\,df}, \quad (4)$$

where $E(f)$ and $Y(f)$ are the time discrete Fourier transforms (TDFTs) of $e(n) = y(n) - x(n)$ and $y(n)$, respectively. The channel (*ch*) and adjacent channels (*adj*) in (4) are 3.84 MHz wide and spaced 5 MHz apart. The result is shown in Fig. 4.

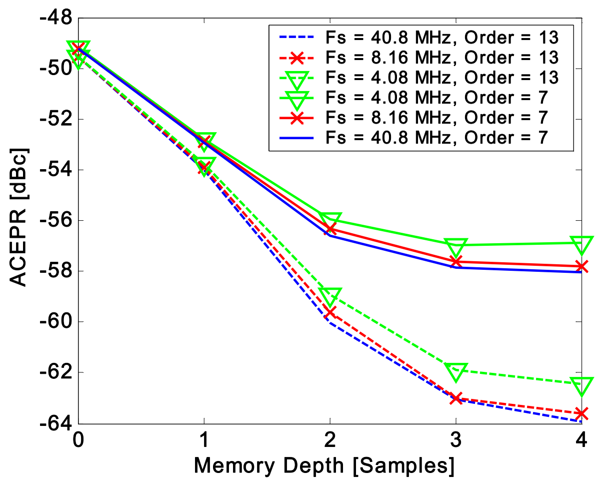

Fig. 4. Performance of the parallel Hammerstein model for order 7 and 13 as a function of the memory depth of the input signal for three different complex sampling rates (Fs) 40.8 MHz (blue), 8.16 MHZ (red) and 4.08 MHz (green).

Clearly, there is a non-significant degradation in the performance of less than 0.2 dB, when the complex sampling rate is changed from 40.8 MHz to 8.16 MHz, both when the parallel Hammerstein model is of order seven and when it is of order 13. A degradation of only about 1.5 dB is observed when the complex sampling rate is further reduced to 4.08 MHz. This degradation, especially the degradation that is seen when the complex sampling rate is changed from 40.8 MHz to 8.16 MHz is certainly acceptable, or could even be neglected. It is also noted that the degradation is similar regardless of if the parallel Hammerstein model is of order 7 or 13 and that the degradation increases with increased memory depth.

These results do also suggest that measurements could be taken on output signals from the amplifier with input signals as wide as almost 20 MHz using a complex sampling rate of 40.8 MHz with little or no loss of performance.

4. CONCLUSION

In this paper it has been shown using real measurements on a power amplifier that a sampling rate of the output signal that is considerably lower than the Nyquist rate can be used for amplifier modelling purposes with little or no negative effects for the behavioural model identification process. These findings have important implications since it means that an ADC with lower sampling rate, and thus, more bits and higher dynamic range, can be used.

Further studies should be done in order to quantify to which extent anti-aliasing filtering will affect the model identification process. Initial simulations indicate that a quite substantial amount of anti-aliasing filtering could be tolerated, but to which extent is likely to be heavily dependent on the system at hand. Systems with a short memory depth are probably the best candidates.

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REFERENCES

[1] D. Wisell, "A Baseband Time Domain Measurement System for Dynamic Characterisation of Power Amplifiers with High Dynamic Range over Large Bandwidths," in *IEEE Instrum. Meas. Tech. Conf. Dig.*, Vail, Colorado, 2003, pp. 1177-1180.

[2] N. B. Carvalho and J. C. Pedro, "A Comprehensive Explanation of Distortion Sideband Asymmetries," IEEE Trans. Microwave Theory Tech., vol. 50, pp. 2090-2101, 2002.

[3] D. Wisell, D. Rönnow, and P. Händel, "A Bandwidth Extension Technique for Dynamic Characterization of Power Amplifiers," in *IEEE Instrum. Meas. Tech. Conf. 2006*, Sorrento, Italy, 2006.

[4] P. Singerl and H. Koeppl, "Volterra Kernel Interpolation for System Modeling and Predistortion Purposes," in *International Symposium on Signals, Circuits and Systems Conf. Dig.*, 2005, pp. 251-54.

[5] Y.-M. Zhu, "Generalized Sampling Theorem," *IEEE Trans. Circuits and Syst.*, vol. 39, pp. 587-588, 1992.

[6] J. Tsimbinos, "Identification and Compensation of Nonlinear Distortion," Ph. D. Thesis, *Institute for Telecommunications Research*, University of South Australia, 1995.

[7] T. Wang and T. J. Brazil, "Volterra-Mapping-Based Behavioral Modeling of Nonlinear Circuits and Systems for High Frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. 51, pp. 1433-1440, 2003.

[8] M. Schetzen, *The Volterra & Wiener Theories of Nonlinear Systems*. New York: Wiley, 1989.

[9] W. A. Frank, "Sampling Requirements for Volterra System Identification," *IEEE Signal Proc. Lett.*, vol. 3, pp. 266-268, 1996.

[10] P. Singerl and H. Koeppl, "A Low-Rate Identification Method for Digital Predistorters Based on Volterra Kernel Interpolation " in *48th Midwest Symposium on Circuits and Systems Conf. Dig.*, 2005, pp. 1533-36.

[11] M. S. Heutmaker, E. Wu, and J. R. Welch, "Envelope Distortion Models with Memory Improve the Prediction of Spectral Regrowth for Some RF Amplifiers," in *48th ARFTG Conf. Dig.*, Clearwater, FL, USA, 1996, pp. 10-15.

[12] H. Ku and J. S. Kenney, "Behavioral Modeling of Nonlinear RF Power Amplifiers Considering Memory Effects," *IEEE Trans. Microwave Theory Tech.*, vol. 51, pp. 2495-2504, 2003.

[13] M. Isaksson, D. Wisell, and D. Rönnow, "A Comparative Analysis of Behavioral Models for RF Power Amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. 54, pp. 348-359, 2006.

[14] J. Proakis, *Digital Communications*: McGraw-Hill, 1995.

[15] T. Söderström and P. Stoica, *System Identification*: Prentice Hall, 1989.

[16] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, vol. I. Upper Saddle River: Prentice-Hall, 1993.

[17] M. Isaksson, D. Wisell, and D. Rönnow, "Nonlinear Behavioral Modeling of Power Amplifiers Using Radial-Basis Function Neural Networks," in *IEEE MTT-S Int. Symp. Dig.*, Los Angeles, 2005, pp. 1967-70.