

MASS CALIBRATION DESIGNS FOR COMPARATORS LOADING 3 WEIGHTS USING A WEIGHT HANDLER

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Abstract: Mass comparators have been available for weighing a magazine of weights automatically. The comparators however allow loading of only 3 weights at a time. Therefore new calibration designs for those cases are to be sought, having least amount of uncertainty. Least squares methods with restrictions are used in finding mass calibration designs. Elementary weighings are formed and efficient designs are searched. The comparison is to be made between the searched ones and calibration designs supplied from the manufacturer.

Keywords: mass, calibration, designs.

1. MASS CALIBRATION DESIGNS

The efficiency of the least squares method depends on the choice of calibration designs [1,2]. Mass calibration designs, or design matrix X , are searched for those providing with least amount of uncertainty from all possible calibration designs. It was shown that the matrix C introduced by Cameron et al. [1], which is conceptually close to $(X^* X^*)^{-1}$, plays an important role because the variances of the estimates of the solution is proportional to the matrix C .

Prior to the search, elementary measurements are formed, which could form all possible combinations. An elementary measurements table for comparators loading 3 weights at a time is tabulated in Table 1.

Table 1. Elementary measurements for 10-10-5-5-2-2-1-1 series

	10	10	5	5	2	2	1	1
1	+	-						
2	+		-	-				
3		+	-	-				
4			+	-				
5			+		-	-	-	
6			+		-	-		-
7				+	-	-	-	
8				+	-	-		-
9					+	-		-
10					+		-	-
11						+	-	-
12							+	-

With the elementary measurements no orthogonal designs were found. One of the weighing designs

recommended from a comparator manufacturer is $G' = \{1,1,1, 1,1,0, 0,1,1, 1,1,1\}$. In G' notation which has 12 digits, each one digit number is the number of weighing in the respective row of the Table 1. Here we use the sum C_{ij} for the normalized sum of the absolute values of all the elements in the inverse matrices of the normal equation.

The sum C_{ij} for the recommended design is 0.846. In the search we have found that a new design of $G' = \{1,2,0, 1,1,0, 0,1,1, 1,1,1\}$ having the sum C_{ij} of 0.696, which reduces the uncertainty or improves the characteristics of orthogonality in calibration design.

Elementary measurements for 10-5-5-2-2-1-1 series are listed in Table 2.

Table 2. Elementary measurements for 10-5-5-2-2-1-1 series

	10	5	5	2	2	1	1
1	+	-	-				
2		+	-				
3		+		-	-	-	
4		+		-	-		-
5			+	-	-	-	
6			+	-	-		-
7				+	-		
8				+		-	-
9					+	-	-
10						+	-

Designs were found to give minimized sum C_{ij} for the elementary measurements of Table 2, whose weighing numbers are from 12 to 18. These designs are listed in Table 3.

Table 3. Efficient designs for 10-5-5-2-2-1-1 series

Number of weighings	Sum C_{ij}	G'
12	0.333	3,2,0, 1,1,0, 1,1,1, 2
13	0.298	3,2,1, 0,0,1, 2,1,1, 2
14	0.277	3,3,1, 0,0,1, 2,1,1, 2
15	0.252	4,3,1, 0,0,1, 2,1,1, 2
16	0.242	4,4,0, 0,1,1, 2,1,1, 2
17	0.236	4,2,1, 1,1,1, 3,1,1, 2
18	0.220	5,5,1, 0,0,1, 2,1,1, 2

2. CONSIDERATION OF CASCADE TYPE DESIGN

We could not find any orthogonal designs from the elementary measurements of Table 1 and Table 2. Therefore we tried to have a cascade type design consisting of two orthogonal designs. In other words a series of 10-5-5-2-2-1-1 could be considered as a combination of a series of 10-5-5 and a series of 5-2-2-1-1. The series of 10-5-5 has elementary measurements of (+ - -) and ((0 + -). From these elementary measurements we could find orthogonal designs of such as {1,1},{2,2} and {3,3}. The series of 5-2-2-1-1 has elementary measurements (+ - - 0), (+ - - 0 -), (0 + - 0 0), (0 + 0 - -), (0 0 + - -) and (0 0 0 + -), which give an orthogonal design of $G'=\{1,1,2, 1,1,2\}$.

The elementary measurements corresponding to the cascade type designs could be listed as in Table 4.

Table 4. Elementary measurements for corresponding type

	10	5	5	2	2	1	1
1	+	-	-				
2		+	-				
3			+	-	-	-	
4			+	-	-		-
5				+	-		
6				+		-	-
7					+	-	-
8						+	-

The above elementary measurements give efficient designs such as $G'=\{4,4,1, 1,2,1, 1,2\}$, whose C_{ij} 's are listed as in Table 5.

Table 5. C_{ij} values of $G'=\{4,4,1, 1,2,1, 1,2\}$

10	5	5	2	2	1	1
	0.119	0.006	0.025	0.025	0.024	0.001
	0.006	0.119	0.025	0.025	0.001	0.024
	0.025	0.025	0.210	0.010	0.005	0.005
	0.025	0.025	0.010	0.210	0.005	0.005
	0.024	0.001	0.005	0.005	0.205	0.000
	0.001	0.024	0.005	0.005	0.000	0.205

Here we could show the C_{ij} 's for the cascade designs as in Table 5.

Table 6. C_{ij} values of $G'=\{4,4\}$, $G'=\{1,1,2, 1,1,2\}$

10	5	5		5	2	2	1	1
0	0	0		0	0	0	0	0
0	0.125	0		0	0.2	0	0	0
0	0	0.125		0	0	0.2	0	0
0	0	0		0	0	0	0.2	0
	0	0		0	0	0	0	0.2

In the cascade design the second orthogonal designs should be considered the effect of uncertainty coming from the first orthogonal design. Considering this original variance in cascade design we could have the adjusted C_{ij} 's of weight 2 and 1 as 0.2062 and 0.2016.

To see the difference between the cascade design and the corresponding design, numerical simulation was performed.

In the simulation white noise was imposed with a comparator of 5 g capacity and 0.1 microgram readability, where its standard deviations were 0.4, 0.25, 0.15 microgram in respective ranges of 2~5 g, 1~2 g, 0~1 g. 20 determinations were made to give standard deviations as shown in the Table 7.

Table 7. Standard deviations of the two designs

design		5	5	2	2	1	1
1		0.063	0.065	0.079	0.071	0.088	0.088
2		0.078	0.063	0.103	0.096	0.060	0.080

In the Table 'design 1' indicates the design of Table 5 and 'design 2' indicates the cascade design of Table 6. As shown in the Table 7 we could find any significant improvement from using the cascade design.

3. CONCLUSION

Efficient designs were found for a comparator loading no more than 3 weights, where no orthogonal designs were found. Conceptually we devised a cascade design consisting of two orthogonal designs. However we could not find any significant improvement from the cascade design.

REFERENCES

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