

NEW CONCEPT OF FORCE REALIZATION BASED ON MACROSCOPIC QUANTUM EFFECT

*Jae-Hyuk Choi*¹, *Mahn-Soo Choi*², *Min-Seok Kim*³, *Yon-Kyu Park*⁴, *Jin-Hoon Sohn*⁵

¹ Division of Physical Metrology, KRISS, Daejeon, Republic of Korea, jhchoi@kriss.re.kr

² Department of Physics, Korea University, Seoul, Republic of Korea, choims@korea.ac.kr

³ Division of Physical Metrology, KRISS, Daejeon, Republic of Korea, minsk@kriss.re.kr

⁴ Division of Physical Metrology, KRISS, Daejeon, Republic of Korea, ykpark@kriss.re.kr

⁵ Department of Psychology, Chungnam National University, Daejeon, Republic of Korea, jhsohn@cnu.ac.kr

Abstract: We present a new concept of force realization, flux quantum-based mechanism in the sub-pico-Newton range. By controlling the number of flux quantum in a superconducting ring, a force can be generated as an integer multiple of a constant force step. For a 50 nm-thick Nb annulus with the inner and outer radii of 5 μm and 10 μm , respectively, and the field gradient of 10 T/m the force step is estimated to be 184 fN. Realization procedure, feasibility, and the stability against thermal fluctuations are also addressed.

Keywords: force realization, quantum-based, sub-pico-Newton.

1. INTRODUCTION

Due to remarkable improvement in force sensitivity, force measurement has become a useful and essential probe for the leading-edge nano/bio-research,[1] which covers protein folding studies, high-density data-storage, nanoscale imaging, nanoindentation and so on. The force detection range keeps being expanded, for example, down to atto-Newton (10^{-18} N) level in magnetic resonance force microscopy.[1] Such an ultimate resolution is enough to read a single electron spin.

Unfortunately, however, no direct SI-traceable force realization has been established even at sub-Newton level. Prevailing dead-weight method, which adopts individual artifacts or weights, is no longer valid below micro-Newton level.[1] Recently, National Institute of Science and Technology recently launched a Microforce Realization and Measurement Project,[1] with the goal of realizing force below 10^{-5} N in high precision. Using a coaxial capacitor design, they create an electrostatic force between two electrodes in maintaining a constant voltage. Related electrical units are traced to their standards based on Josephson and quantum Hall effects. At the nano-Newton or pico-Newton level, however, no force realization for standard has been suggested despite needs in precise measurement. It is essential, for instance, to testify fundamental forces such as Casimir force, non-Newtonian gravitation,[5] etc.

Also, it casts a striking contrast to the case of electrical units such as voltage, that (to our best knowledge) no attempt has been tried to directly use quantum phenomena in realizing a mechanical force. Here, we present a concept of quantum-based force realization utilizing a macroscopic quantum phenomenon known as magnetic flux quantization in a superconducting annulus. Magnetic force exerted on flux quanta can be increased or decreased by a constant step, which is estimated to be sub-pico-Newton level. Moreover, those force-generating states are stable and robust against thermal fluctuation at liquid helium temperature. Therefore, the quantum acts as a role of an individual weight in this ultra-small force region. This weight, however, is not an artifact, but a SI-traceable and quantum-mechanically equivalent object with exactly same weight or force.

2. BASIC PRINCIPLES

Figure 1 shows a schematic of quantum-based force realization. A tens-micron-sized superconducting annulus or a superconducting quantum interference device (SQUID) is mounted on an ultrasoft micro-cantilever. Below a superconducting transition temperature of the superconducting material, magnetic fluxoid through the annulus or SQUID is quantized, and the resultant magnetic moment has a component with constant steps, the number of which depends on the quantum number. The step size is determined by fundamental constants such as electron

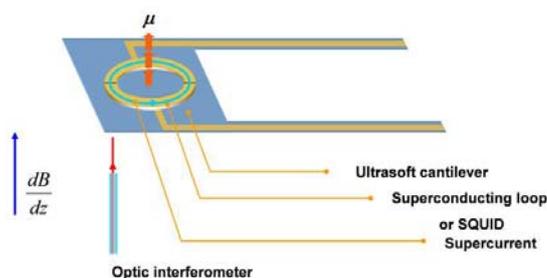


Fig. 1. A schematic of force realization based on flux quanta in superconducting annulus.

charge and the length quantities of the annulus. In a calibrated magnetic field gradient, dB/dz , a force is created on the superconducting annulus, and therefore on the micro-cantilever. Through a designed procedure of magnetic field and temperature, we can leave trapped flux quanta as many as we want in zero external field, while the field gradient is not zero. The displacement of the cantilever is monitored by optic interferometer or lever and can be used in calibrating its spring constant.

3. MAGNETIC MOMENT OF A SUPERCONDUCTING RING

For the accurate force realization, it is essential first to evaluate the magnetic moment of the superconducting annulus. We consider a superconducting annulus of inner and outer radii, a and b , respectively, and thickness d in a uniform external magnetic field $\mathbf{B}_{\text{ext}} = B_{\text{ext}}\mathbf{z}$. In response to the external magnetic field or the fluxoid trapped in the hole, currents \mathbf{J} are induced in the annulus and the field is modified. The net magnetic vector potential $\mathbf{A} = \mathbf{A}_{\text{ext}} + \mathbf{A}_{\text{ind}}$ consists of the external part \mathbf{A}_{ext} and the induced part \mathbf{A}_{ind} . \mathbf{J} and \mathbf{A}_{ind} are related by the Biot-Savart law

$$\mathbf{A}_{\text{ind}} = \int d^3\mathbf{r}' \frac{\mu_0 \mathbf{J}(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} \quad (1)$$

where μ_0 is the magnetic permeability. On the other hand, within a superconductor, the current is governed by the London equation

$$\mathbf{J} = -\frac{1}{\mu_0 \lambda_L^2} \left(\mathbf{A} + \frac{\Phi_0}{2\pi} \nabla \theta \right) \quad (2)$$

where θ is the phase of the superconducting order parameter, λ_L is the London penetration depth, and $\Phi_0 \equiv h/2|e| \approx 2.07 \times 10^{-15}$ weber is the flux quantum. Equations (1) and (2) give an integral equation for \mathbf{J} , from which one can get the magnetic moment. Given the geometry of the superconductor, one can greatly simplify the integral equation by following [6].

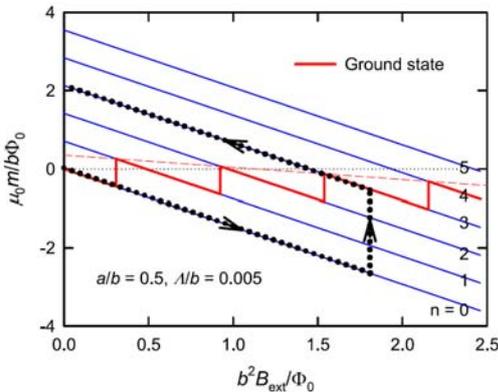


Fig. 2. Total magnetic moment as a function of B_{ext} for n trapped fluxoids (solid blue lines), $a/b = 0.5$, and $\Lambda/b = 0.005$. The thick (red) line is for the ground state configuration. The dashed (red) line is the guide for eyes connecting the peaks of ground-state magnetic moment. The thick dotted (black) line indicates the experimental procedure (see the text).

Since the current flows only in the thin film ($-d/2 \leq z \leq d/2$, $d \ll \lambda_L$), it is customary to define a sheet current such that $\mathbf{J}(\mathbf{x}, \mathbf{y}, z) \approx \mathbf{J}_{\text{sheet}}(\mathbf{x}, \mathbf{y})\delta(z)$ and to represent the physical quantities in the polar coordinates (r, φ, z) so that $\mathbf{J}_{\text{sheet}}(\mathbf{x}, \mathbf{y}) = J_{\text{sheet}}(r)\boldsymbol{\varphi}$, $\mathbf{A}(\mathbf{x}, \mathbf{y}, 0) = [A_{\text{ext}}(r) + A_{\text{ind}}(r)]\boldsymbol{\varphi}$, and $\nabla\theta(\mathbf{x}, \mathbf{y}, 0) = (n/r)\boldsymbol{\varphi}$, where the integer n is the number of fluxes trapped in the hole. Then the London equation (2) reads as

$$\mu_0 J_{\text{sheet}} = -\frac{1}{\Lambda} \left[\frac{n\Phi_0}{2\pi r} + A_{\text{ext}}(r) + A_{\text{ind}}(r) \right] \quad (3)$$

where $\Lambda \equiv \lambda_L/d$ is the effective penetration depth and the Biot-Savart law (1) is rewritten as

$$A_{\text{ind}}(r) = \int_a^b dr' Q(r, r') \mu_0 J_{\text{sheet}}(r') \quad (4)$$

where

$$Q(r, r') = \int_a^b \frac{d\varphi}{2\pi} \frac{\cos\varphi}{\sqrt{1 - 2(r/r')\cos\varphi + (r/r')^2}}. \quad (5)$$

From the London equation (3), one identifies two sources: one from the London fluxoid [7] $D_1(r) \equiv n\Phi_0/2\pi r$ and the other from the external field $D_2(r) \equiv A_{\text{ext}}(r)$. Accordingly, we decompose the induced quantities, J_{sheet} and A_{ind} , into contributions from D_1 and D_2 , respectively: $J_{\text{sheet}} = J_1 + J_2$ and $A_{\text{ind}} = A_1 + A_2$. Combining the two equations (3) and (4), one arrives at the integral equations for J_1 and J_2

$$\int_a^b dr' Q(r, r') \mu_0 J_\nu(r') + \Lambda \mu_0 J_\nu(r) = -D_\nu(r) \quad (6)$$

respectively ($\nu = 1, 2$) [6, 8]. The nice feature of this form is that $a \leq r \leq b$, which facilitates pretty much the numerical solution of the equation. Once J_ν ($\nu = 1, 2$) are at hand, the magnetic moment $m = m_1 + m_2$ is given by

$$m_\nu = \pi \int_a^b dr r^2 J_\nu(r) \quad (7)$$

We have followed [6] to discretize the integral equation (6) and get J_ν and m_ν numerically.

Figure 2 shows typical behavior of the total magnetic moment m of the superconducting annulus as a function of B_{ext} . It is clearly seen that at a given B_{ext} , the magnetic moment is quantized depending on the number of trapped fluxes. At a given value of B_{ext} , the allowed quantized value of the magnetic moment corresponds to a metastable state of the system (the thin blue lines in Fig. 2). The magnetic moment corresponding to the most stable state (the thick red line in Fig. 2) is determined by the Gibbs free energy [6]

$$G_n(B_{\text{ext}}) = -\frac{1}{2} m B_{\text{ext}} + \frac{1}{2} n \Phi_0 I \quad (8)$$

where $I = I_1 + I_2$ with

$$I_\nu = \int_a^b dr J_\nu(r) \quad (9)$$

is the total current along the annulus. The ground-state magnetic moment is a periodic function of B_{ext} with period B_0 . Figure 3 plots B_0 versus a/b .

4. ESTIMATION OF MAGNETIC MOMENT, FORCE STEP, AND FORCE LIMIT

To estimate the magnitude of the magnetic moment, let us consider an Nb annulus of inner and outer radii, $a = 5 \mu\text{m}$ and $b = 10 \mu\text{m}$, respectively, with thickness of 50 nm. The Nb material was chosen because its superconducting transition temperature, 9.25 K,[9] is readily accessible in cryogenics and its thin film fabrication is well established to have provided good quality of thin-film circuits such as SQUID. Its London penetration depth λ_L and coherence length ξ are 50 nm and 39 nm, respectively.[9] The choice of radii is reasonable considering the typical dimension of microfabricated cantilevers with a low spring constant ($k = 10^{-4}$ to 10^{-5} N/m).[10] These parameters fall on the case of $a/b = 0.5$ and $\lambda_L/b = 0.005$.

Then, a magnetic moment step, m_Q , by adding a single flux quantum, is numerically estimated as $1.116 m_b$, where the magnetic moment unit $m_b \equiv \Phi_0 b / \mu_0$ is 1.65×10^{-14} A·m². The force on the superconducting annulus is given by

$$F = m_Q \frac{dB_{\text{ext}}}{dz} \quad (10)$$

and assuming a modest field gradient, dB_{ext}/dz , of 10 T/m,[11] a corresponding force step is estimated to be 1.84×10^{-13} N or 184 fN, which causes ≈ 2 nm static displacement of a cantilever with $k = 10^{-4}$ N/m. In principle, nano- and pico-Newton level can be accessed by stacking these steps.

Practically, a maximum possible force, or the maximum number of steps, is limited by the critical current of the superconducting material, at which its superconducting state cannot be sustained. For the Nb annulus with above geometry, the force limit is roughly estimated to be ≈ 40 pN from the approximate equation $F_{\text{max}} = I_c[(a + b)/2]^2 (dB/dz)$ and the critical current density of Nb, $j_c = 9 \times 10^{10}$ A/m² at $T = 6$ K.[12]

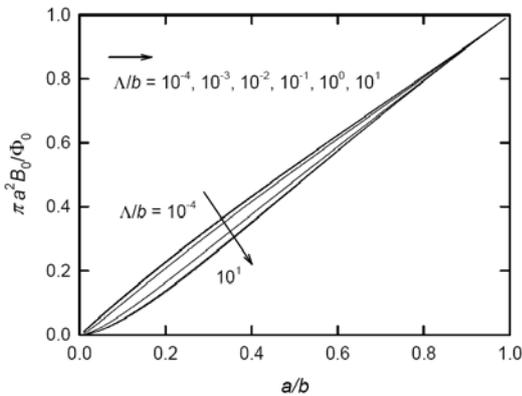


Fig. 3. B_0 as a function of a/b for different values of Λ/b

5. FORCE REALIZATION PROCEDURE

Realization of only a quantum-induced force requires a specially designed procedure, because the magnetic moment of a superconducting annulus includes a field-induced component linearly proportional B_{ext} as seen in Fig. 2. The magnetic field at the annulus from the z-gradient magnet, the background, and so on is not simple to null satisfactorily, considering the small value of B_0 (≈ 0.13 Oe for the parameters above), and would generate a non-zero force offset even in zero flux quantum state. Therefore, we need to extract selectively the flux quantum contribution.

In Fig. 2, the difference of magnetic moment, for example, between $n = 0$ and $n = 3$ states is always constant, three-times as large as m_Q at any fixed B_{ext} , and a corresponding change can be realized by transition between them in principle. Uncontrolled transitions, such as quantum or thermal tunneling (to be discussed later), are practically negligible at operation temperatures, ≈ 4 K, due to high energy barriers between meta-stable states.

We suggest a force-realization and cantilever calibration procedure as follows: (1) After measuring a zero-reference position of a cantilever in $n = n_i$ state at $B_{\text{ext}} = 0$ and $T \ll T_c$ [20] turn on a uniform external field up to a target value roughly where $n = n_f = n_i + \Delta n$ is a global minimum of Gibbs free energy. But, the system still stays at $n = n_i$ state due to energy barriers. (2) Then, increase temperature above T_c and decrease it back to $T \ll T_c$, which would send the system to the global minimum state, $n = n_f$. (3) Decrease and turn off the uniform external field and measure the displacement of the cantilever. The net change of magnetic moment, $\Delta n \times m_Q$, is only related to flux quanta and so is that of a force. From the number change of flux quantum (Δn), annulus dimension, penetration depth, and field gradient, the change in force can be evaluated and used in determining the spring constant of the cantilever in combination with its displacement.

The starting condition $B_{\text{ext}} = 0$ can be lifted; our procedure is still valid even for $B_{\text{ext}} \neq 0$ as long as it is the same at the beginning and the end. This is a great benefit because background magnetic field is hard to shield and usually disturbs delicate magnetic experiments.

6. IN-SITU CALIBRATION OF FIELD GRADIENT

For precise force realization, one has to be able to calibrate the magnetic field gradient dB_{ext}/dz as accurate as possible. Experimentally, it is tricky to know the exact value at the local position of the annulus on a micro-cantilever, unless the field gradient is strictly uniform in a large volume and well pre-calibrated in micro-scale.

Here we argue that we can calibrate it in-situ by means of the characteristic properties of the magnetic moment of the superconducting annulus. The magnetization $m(B_{\text{ext}})$ in Eq. (9) can be written into the form

$$m_1 = m_1(n) + \chi B_{\text{ext}}, \quad \chi \equiv \frac{m_2(B_{\text{ext}})}{B_{\text{ext}}} \quad (11)$$

where it should be noticed that $m_1(n)$ and $m_2(B_{\text{ext}})$ are linearly proportional to n and B_{ext} , respectively. The field is expanded as

$$B_{\text{ext}}(z) = B_{\text{ext},0} + \frac{dB_{\text{ext}}}{dz} z \quad (12)$$

From Eqs. (9), (10), and (11), the force $F(z)$ at the position z is given by

$$F(z) = [m_1(1)n + \chi B_{\text{ext}}] \frac{dB_{\text{ext}}}{dz} + \chi \left(\frac{dB_{\text{ext}}}{dz} \right)^2 z \quad (13)$$

Then it follows that

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \frac{\Delta k}{k} = \frac{\chi}{2k} \left(\frac{dB_{\text{ext}}}{dz} \right)^2 \quad (14)$$

where $\omega(k)$ is the natural vibration frequency (Hook's coefficient) of the cantilever and $\Delta\omega(\Delta k)$ is the shift in $\omega(k)$ due to the field gradient. Let $\Delta z = z_{\text{eq}}(n) - z_{\text{eq}}(n=0)$, i.e., the shift in the equilibrium position z_{eq} for nonzero flux n . Since the force $F(z_{\text{eq}})$ at $z = z_{\text{eq}}$ in Eq. (12) should be at equilibrium with $(k + \Delta k) z_{\text{eq}}$, i.e.,

$$m_1 n \frac{dB_{\text{ext}}}{dz} = k \Delta z \quad (15)$$

one arrives at the relation

$$\frac{dB_{\text{ext}}}{dz} = \frac{2n}{\Delta z} \frac{m_1(1)}{\chi} \frac{\Delta\omega}{\omega} \quad (16)$$

We now note that in Eq. (16), the field gradient dB_{ext}/dz has been expressed only in known quantities of quantities that can be determined to high precision: The relative frequency shift $\Delta\omega/\omega$ is typically probed with high precision. As shown in Fig. 4, the ratio $m_1(1)/\chi$ in Eq. (16) does not depend strongly on the geometry and can be easily determined accurately. We therefore conclude that Eq. (16) allows us to calibrate the field gradient in-situ very precisely.

For $k = 10^{-4}$ N/m and other parameters adopted above for a force step estimation, a relative frequency shift is estimated to be 2×10^{-3} from the Eq. (14), and becomes smaller for lower spring constant. The relative shift is much larger than the sharpness of the resonance peak or $1/Q$, with a typical quality factor $Q = 10^4 \square 10^5$ for single-crystalline

Table 1. Expected values of $m_1(1)$, F_Q , χ , F_{OFFSET} , $\Delta\omega/\omega$, and relative material-dependence $(dm_1(1)/d\Lambda)(m_1(1)/\Lambda)^{-1}$ for various annulus geometries, a and b . F_{OFFSET} , is an offset of static force due to background magnetic field of 0.5 Oe. For evaluation, $\Lambda = 50$ nm, $dB_{\text{ext}}/dz = 10$ T/m, $k = 10^{-4}$ N/m were assumed.

a [μm]	b [μm]	$m_1(1)/(\Phi_0 b/\mu_0)$	F_Q [pN]	$\chi/(b^3/\mu_0)$	F_{OFFSET} [pN]	$\Delta\omega/\omega$ [$\times 10^{-3}$]	$(dm_1(1)/d\Lambda)(m_1(1)/\Lambda)^{-1}$
2.5	10	1.207	0.1992	-2.520	-1.0053	-1.003	-0.0289
5	10	1.116	0.1841	-2.508	-1.0004	-0.998	-0.0342
7.5	10	0.9365	0.1545	-2.398	-0.9565	-0.954	-0.0485
5	20	1.207	0.3983	-2.520	-8.042	-8.021	-0.0289
10	20	1.116	0.3683	-2.508	-8.003	-7.982	-0.0342

Si cantilevers,[10] and can be measured to high precision.

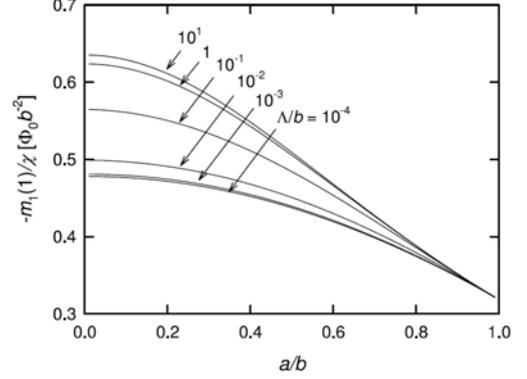


Fig. 4. $m_1(1)/\chi$ as a function of a/b for different values of Λ/b

7. DISCUSSION

For several sets of annulus geometries, i.e. a and b , the expected values of various quantities related to feasibility are tabulated in Table 1, which provides us insight about the choices of geometry. A larger force step can be obtained by larger a and smaller b , and the b -dependence is most dominant. However, larger b is not always desirable, because it causes rapidly increasing effect of background, as an offset force, F_{OFFSET} , due to a background magnetic field shows. On the other hand, larger b results in a greater shift of cantilever resonance frequency ω , which enables us to calibrate the field gradient more precisely.

The dependence of a realized force on superconducting material can be quantified as the normalized derivative, $(dm_1(1)/d\Lambda)/(m_1(1)/\Lambda)$: the dependence of a magnetic moment step $m_1(1)$ on the penetration depth. As seen in the table, it is very small while $\Lambda \ll b$, and therefore the transfer of uncertainty in Λ is much limited.

The static displacement Δz of the cantilever can be precisely measured using, for example, an optic interferometer, which is known to have the rms noise less than 0.01 nm in 1 kHz bandwidth at low frequencies.[13] However, thermal vibration amplitude, Δz_{th} , may not be small in comparison with a static displacement, Δz_Q , due to a single flux quantum. The ratio, $\Delta z_{\text{th}}/\Delta z_Q$, is given by

$$\frac{\Delta z_{\text{th}}}{\Delta z_Q} = \frac{\sqrt{kk_B T}}{F_Q} \quad (17)$$

15	20	0.9365	0.3090	-2.398	-7.652	-7.632	-0.0485
----	----	--------	--------	--------	--------	--------	---------

where k_B and F_Q are the Boltzmann constant and the force step due to a flux quantum, respectively, and is about 0.41 for $k = 10^{-4}$ N/m and $T = 4.2$ K, becoming smaller for decreasing k and T . But, it may not cause a serious problem because the thermal vibration happens most dominantly at a resonance frequency (a few kHz), especially for high-Q cantilever, not disturbing the measurement of static component. In addition, for large multiples of F_Q the relative error due to interferometer and thermal noises gets proportionally smaller.

Instead of a superconducting annulus, a superconducting quantum interference device (SQUID) with leads could be mounted on the micro-cantilever. Then, instead of thermal cycling, a bias current through SQUID can be used to permit additional flux quanta, with a merit that the whole procedure is done at a fixed temperature. Moreover, each entrance of quantum can be monitored in real time from the current-voltage characteristics. For its benefit to be used, an accurate calculation of magnetic moment for the SQUID case is awaited.

Finally, we briefly remark on the effects of thermal fluctuations. In principle, the fluxoids trapped in the annulus may undergo relaxation processes and their number changes. At our working temperature ($\square 4$ K), the relaxation is dominated by the thermal activation, and governed by the law $\gamma_0 \exp(-\Delta U/k_B T)$, where ΔU is the energy barrier against the relevant process. For very narrow annulus with $b - a \square \Lambda$, the underlying mechanism is the phase slip [14, 15, 16, 17], and the rate is order of several transitions per second [18, 19]. For a wide annulus with $b - a \square \Lambda$, the relaxation comes from crossing of the vortices across the annulus of superconducting thin film. Unlike the narrow limit, there is no detailed study of flux-state transitions in this case, neither experimental nor theoretical. Naively, the energy barrier can be estimated by the condensation energy cost inside the vortex core $\Delta U \square H_c^2 \xi^2 d / 8\pi$, where H_c is the thermodynamic critical field. For Nb, $H_c \square 0.1$ T, and $\Delta U \square 10^5$ K.[9] It implies that the effects of the flux relaxation can be safely ignored in our scheme using wide enough annulus.

8. CONCLUSION

A new concept of force realization based on flux quantum was suggested, which may be potentially used as a standard force in the sub-pico-Newton range. A stepwise force can be generated from the annulus on a micro-cantilever in magnetic field gradient and the step size depends only on fundamental constants and the length dimensions of the annulus beside the field gradient. For a 50 nm-thick Nb 5 annulus with inner and outer radii of 5 μ m and 10 μ m, respectively, a force can be created up to $\square 40$ pN by a step of 184 fN, assuming the field gradient of 10 T/m. This method requires low operating temperature for superconducting state to be maintained. Application is open to precision improvement of force detection in quantum regime such as Casimir force, non-Newtonian gravitation, magnetic resonance force microscopy, and so on.

ACKNOWLEDGMENTS

M.-S.C. was supported by the SRC/ERC (R11-2000-071), the KRF Grant (KRF-2005-070-C00055), and the SK Fund.

REFERENCES

- [1] J. R. Pratt, D. T. Smith, D. B. Newell, J. A. Kramar, and E. R. Williams, "Progress toward Systeme International d'Unites traceable force metrology for nanomechanics," *J. Mater. Res.*, vol. 19, pp. 366-379, 2004; and references therein.
- [2] D. Rugar, R. Budakian, H. J. Mamin, and B. W. Chul, "Single spin detection by magnetic resonance force microscopy," *Nature*, vol. 430, pp. 329, 2004.
- [3] R. G. Knobel and A. N. Cleland, "Nanometre-scale displacement sensing using a single electron transistor," *Nature*, vol. 424, pp. 291 (2003).
- [4] X. M. H. Huang, C. A. Zorman, M. Mehregany, and M. L. Roukes, "Nanoelectromechanical systems: Nanodevice motion at microwave frequencies," *Nature*, vol. 421, pp. 496, 2003.
- [5] J. Chiaverini, S. J. Smullin, A. A. Geraci, D. M. Weld, and A. Kapitulnik, "New Experimental Constraints on Non-Newtonian Forces below 100 μ m," *Phys. Rev. Lett.*, vol. 90, pp. 151101, 2003.
- [6] E. H. Brandt and J. R. Clem, "Superconducting thin rings with finite penetration depth," *Phys. Rev. B*, vol. 69, pp. 184 509, 2004.
- [7] F. London, *Superfluids*, vol. 2 (John Wiley & Sons, New York, 1954).
- [8] A. L. Fetter, "Flux penetration in a thin superconducting disk," *Phys. Rev. B*, vol. 22 (3), pp. 1200, 1980.
- [9] C. P. Pool, *Handbook of Superconductivity* (Academic Press, San Diego, 2000).
- [10] H. J. Mamin and D. Rugar, "Sub-attoneutron force detection at millikelvin temperatures," *Appl. Phys. Lett.*, vol. 79, pp. 3358, 2001; T. D. Stowe, K. Yasumura, T. W. Kenny, D. Botkin, K. Wago, and D. Rugar, "Attonewton force detection using ultrathin silicon cantilevers," *Appl. Phys. Lett.*, vol. 71, pp. 288, 1997; C. Miller and J. T. Markert, "Quantitative determination of the adiabatic condition using force-detected nuclear magnetic resonance," *Phys. Rev. B*, vol. 72, pp. 224402, 2005.
- [11] This value is comparable to the typical gradient by commercial superconducting or permanent gradient magnets (2 ~ 20 T/m).
- [12] M. E. Geers, M. B. S. Hesselberth, J. Aarts, and A. A. Golubov, "Depairing currents in the superconductor/ferromagnet proximity system Nb/Fe,"

Phys. Rev. B, vol. 64, pp. 094506, 2001.

- [13] D. Rugar, H. Mamin, and P. Guethner, Appl. Phys. Lett., “Improved fiber-optic interferometer for atomic force microscopy,” vol. 55, pp. 2588, 1989.
- [14] J. S. Langer and V. Ambegaokar, “Intrinsic Resistive Transition in Narrow Superconducting Channels,” Phys. Rev., vol. 164, pp. 498, 1967.
- [15] D. E. McCumber and B. I. Halperin, “Time Scale of Intrinsic Resistive Fluctuations in Thin Superconducting Wires,” Phys. Rev. B, vol. 1, pp. 1054, 1970.
- [16] E. M. Horane, J. I. Castro, G. C. Buscaglia, & A. L’opez, “Transition between different quantum states in a mesoscopic system: The superconducting ring,” Phys. Rev. B, vol. 53, pp. 9296, 1996.
- [17] D. Y. Vodolazov and F. M. Peeters, “Dynamic transitions between metastable states in a superconducting ring,” Phys. Rev. B, vol. 66, pp. 054537, 2002.
- [18] J. E. Lukens and J. M. Goodkind, “Flux Fluctuations and Critical Currents in a Superconducting Ring Near T_c ,” Phys. Rev. Lett., vol. 20, pp. 1363, 1968.
- [19] S. Pedersen, G. R. Kofod, J. C. Hollingbery, C. B. Sørensen, and P. E. Lindelof, “Dilation of the giant vortex state in a mesoscopic superconducting loop,” Phys. Rev. B, vol. 64, pp. 104522, 2001.
- [20] The z-gradient magnet is assumed to be adjusted so that the field at the position of the annulus vanishes.