XVIII IMEKO WORLD CONGRESS
Metrology for a Sustainable Development
September, 17 – 22, 2006, Rio de Janeiro, Brazil

UNCERTAINTY ESTIMATION OF MECHANICAL ASSAYS BY ISO-GUM 95 AND MONTE-CARLO SIMULATION – CASE STUDY: TENSILE STRENGTH, TORQUE AND BRINELL HARDNESS MEASUREMENTS

Paulo R. G. Couto[1], *Jailton C. Damasceno*[2], *Renata M. H .Borges*[3]

[1] Mechanical Metrology Division – Inmetro, Rio de Janeiro, Brazil, prcouto@inmetro.gov.br
[2] Materials Metrology Division – Inmetro, Rio de Janeiro, Brazil, jcdamasceno@inmetro.gov.br
[3] Chemical Metrology Division – Inmetro, Rio de Janeiro, Brazil, rmborges@inmetro.gov.br

Abstract: The limitations presented by the ISO-GUM 95 methodology can by bypassed by applying Monte-Carlo simulation on measurement uncertainty estimation. In this work, uncertainty results from tensile strength, torque and Brinell hardness measurements evaluated using ISO-GUM 95 recommendations, Monte-Carlo simulation and the EURACHEM numerical simulated method are compared and discussed.

Keywords: Uncertainty, ISO-GUM 95, Monte-Carlo.

1. INTRODUCTION

Measurement results are used as control parameters in various applications, such as [1]: a) estimation of losses in the manufacture and commercialization of a product, b) inspection of products in relation to their specifications or limits determined by a quality standard, c) support for many medical and judicial decisions, d) support for the evaluation of the parameters that define a project in order to reduce the losses determined by a standard of quality, e) support for the conduction and conclusion of scientific research projects, establishing limitations or assumptions of a model, f) mutual recognition between metrological systems as an important tool for the world-wide market, g) definition and optimization of the quality of a product. In the great majority of these applications, measurement results must be always available for comparison, and for that reason, a unique model for uncertainty estimation should be used.

World-wide commercialization and the interchange between national and international institutions demand the adoption of a universal procedure for measurement uncertainty estimation, in attendance mainly to the world-wide market. In this way, the “Guide to the Expression of Uncertainty in Measurement”, more known as ISO-GUM 95 [2], was elaborated by the International Organization for Standarization in order to establish an harmonized methodology for uncertainty estimation. Moreover, other established international documents based on ISO-GUM 95 can be used, as for example the EURACHEM/CITAC “Quantifying Uncertainty in Analytical Measurement” [3] in the field of chemistry.

2. METHODS

2.1. The ISO-GUM 95

Evaluation of measurement uncertainty by the ISO-GUM 95 recommendations can be summarized in the following steps: (1) Definition of the quantity being measured – the mensurand; (2) definition of the cause-effect diagram for the mensurand; (3) estimation of the standard uncertainties of the main sources; (4) determination of the sensitivity coefficients of the mensurand for each main source; (5) calculation of the components of uncertainty for each main source; (6) combination of the components; (7) calculation of the effective degrees of freedom of the standard combined uncertainty and (8) calculation of the expanded uncertainty.

However the ISO-GUM 95 approach exhibits some limitations [4], like: (1) Model linearization: The principle of error propagation applied to obtain the standard combined uncertainty truncates the Taylor’s series expansion in first order terms. This is a linear approximation that in some cases could need terms of higher order. (2) Assumption of normality of the mensurand (z): In common practice, the distribution of the result is taken as normal and consequently, expanded uncertainty $U(z)$ is calculated as the product of the coverage factor k and the combined uncertainty $u(z)$. Thus $k = 2$ is a very commonly declared value, which corresponds to a level of significance of approximately 95% (95.45% in fact). (3) Calculation of the effective degrees of freedom: if the distribution of z is approximated to a Student’s distribution, the coverage factor k is taken as the tabulated Student’s t-value for a given significance level and the effective degrees of freedom calculated by the Welch-Satterthwaite equation. In the general case (including correlation terms), the analytical evaluation of the effective degrees of freedom is still an unsolved problem, type B uncertainties, generally contributing with infinite degrees of freedom.

2.2. The EURACHEM numerical simulated method

The EURACHEM/CITAC document [3], based on ISO-GUM 95, presents two other very interesting ways to estimate measurement uncertainties where the calculation of sensitivity coefficients of the mensurand is not necessary. In one of then, after definition of the standard uncertainties, a numerical simulation methodology is used to calculate values for the mensurand by adding each source value and its uncertainty individually. The variation of the mensurand is then calculated for each source of influence. The combined standard uncertainty is estimated like in the ISO-GUM 95. The other methodology presented by the EURACHEM is the estimation of the relative combined standard uncertainty of the mensurand by combining the relative standard uncertainties of the sources of influence.

2.3. Monte-Carlo simulation

In order to overcome the limitations of ISO-GUM 95, the Monte-Carlo simulation approach can be applied to the evaluation of measurement uncertainties [4-6]. This methodology is a numerical procedure that uses the generation of random numbers to simulate the values of the uncertainty sources, combining distributions instead of statistically propagating errors. The Monte-Carlo simulation is therefore a generalization of the law of propagation of uncertainties, providing uncertainty evaluations that are more valid than those provided by the use of the law of propagation of uncertainties in circumstances where the conditions for the application of that law are not fulfilled. These simulations can be easily used due to the growing popularity of high-speed personal computers.

Evaluation of measurement uncertainty by the Monte-Carlo simulation method can be carried out through the following steps: (1) Establishment of the model equation for the mensurand in function of the individual parameters of influence (definition of the mensurand); (2) Selection of the significant sources of uncertainty (estimation of uncertainties); (3) Identification of the probability density functions corresponding to each source of uncertainty selected; (4) Selection of the number M of Monte-Carlo trials; (5) Simulation of M samples of each source of uncertainty, considered as a random variable with a probability density function; (6) Calculation of the M results by applying the equation that was defined for the mensurand.

In this work, Monte Carlo simulations were executed by programming Microsoft Excel® to generate pseudo-random probabilities for the distributions of the involved quantities. In this way, possible random values are generated for each quantity, according to their distribution functions. Uniform pseudo-random numbers were generated using the Hill-Wichmann algorithm [7]. In the case of normal distributions, an algorithm for the polar form of the Box-Muller transformation was used [8]. These algorithms were implemented in Excel® by adding new macro functions. Both are recommended by the ISO-GUM supplement on numerical simulations as suitable for metrology calculations [6].

3. RESULTS AND DISCUSSION

ISO-GUM 95, the EURACHEM numerical simulation and Monte-Carlo simulation approaches for measurement uncertainty estimation are compared for the mechanical assays: tensile strength, torque and Brinell hardness measurements.

Tensile strength σ is determined by Eq. (1).

$$\sigma = \left(\frac{4F}{\pi D^2} \right) \tag{1}$$

Where F is the applied force at the maximum point of the stress-strain curve and D is the cross-sectional diameter of the specimen.

Torque measurement by a standard torque meter is based in Eq. (2).

$$T = m \cdot g \cdot L \tag{2}$$

Where m is the mass; g is the local gravity acceleration and L is the arm length of the torque meter.

Brinell hardness is determined by Eq. (3) as recommended by ASTM 10.

$$HB = \frac{0.204F}{\pi D(D - \sqrt{D^2 - d^2})} [N/mm^2] \tag{3}$$

Where F is the applied force (in Newtons), D is the indenter diameter (in mm) and d is the indentation diameter (in mm).

Table 1 shows the values of the parameters, the considered uncertainties and their distributions for each case.

Table 1. Parameters, uncertainties and distributions used in the tensile strength, torque and Brinell hardness measurement uncertainty evaluations.

Quantity	Value	Distribution	Uncertainty
Tensile strength			
Force (kgf)			
- Type A	2930.125	Normal	4.048
- Type B	0	Normal	5.658
Diameter (mm)	8.04	Normal	0.01
Gravity accel. (m/s²)	9.787487	Normal	0.00000005
Torque			
Mass (kg)	34.05700	Normal	0.00003
Gravity accel. (m/s²)	9.787487	Normal	0.00000005
Arm length (m)	1.500000	Normal	0.000046
Brinell hardness			
Force (N)	29400	Normal	294
Indenter diameter (mm)	10.000	Normal	0.005
Indentation diameter – type A (mm)	3.000	Normal	0.035

In the case of Monte-Carlo simulations, values of tensile strength, torque and Brinell hardness were calculated for each iteration according to Eqs. 1, 2 and 3 and evaluated statistically in the end of each simulation. A total of M = 50,000 iterations were used for each example.

Table 2 shows the statistical parameters obtained for all the simulations. The standard deviation represents the combined uncertainty of the estimated mean value, while the fisher skewness is the symmetry of the distribution function obtained in each case. If the skewness value is sufficiently near to zero, the distribution is considered to be symmetrical and the expanded uncertainty can be obtained by calculating half the difference between the confidence interval extremes. The coverage factor is then obtained by dividing the expanded uncertainty by the combined uncertainty.

Table 3 shows the results of estimated measurement uncertainties obtained for the three mechanical assays following the ISO-GUM 95 approach, the numerical simulation method and by using Monte-Carlo simulation. As can be observed, very similar values for the combined uncertainties were found by using the three approaches in the three measurement cases.

In the cases of tensile strength and torque measurements, very symmetrical distributions were obtained by the Monte-Carlo approach, indicated by the low skewness values (0.0047 and –0.0021, respectively). Expanded uncertainties for these two assays were calculated by using the confidence interval extremes (as described before) and the values that were found are in strong agreement with those obtained by the EURACHEM and ISO-GUM 95 approaches (see Table 3). The Monte-Carlo simulation used for the Brinell hardness measurement uncertainty estimation produced a relatively non-symmetrical distribution function (skewness = 0.1010) and an expanded uncertainty could not be calculated using this method. In this case, the result is presented only as an interval with 95% of confidence: [393.90 to 436.35].

Table 2. Statistical parameters obtained for the Monte-Carlo simulations for tensile strength, torque and Brinell hardness measurement uncertainty evaluations.

Statistical parameters	Tensile strength (MPa)	Torque (N.m)	Brinell hardness (N/mm²)
Mean	564.879	499.9987	414.65
Standard deviation	1.947	0.0155	10.82
Fisher skewness	0.0047	-0.0021	0.1010
Confidence interval for 95%	[561.077 to 568.706]	[499.9682 to 500.0291]	[393.90 to 436.35]

Table 3. Comparison between ISO-GUM 95, EURACHEM numerical simulation and Monte-Carlo simulation approaches for tensile strength, torque and Brinell hardness measurement uncertainty evaluations.

	ISO-GUM	EURACHEM	M-Carlo
Tensile strength (MPa)			
Estimate	564.879	564.879	564.879
Combined uncertainty	1.943	1.941	1.947
Coverage factor	2	2	1.96
Expanded uncertainty	3.885	3.881	3.815
Torque (N.m)			
Estimate	499.9987	499.9987	499.9987
Combined uncertainty	0.0154	0.0154	0.0155
Coverage factor	2	2	1.97
Expanded uncertainty	0.0308	0.0308	0.0304
Brinell hardness (N/mm²)			
Estimate	414.47	414.47	414.65
Combined uncertainty	10.83	10.67	10.82
Coverage factor	2.57	2.57	--
Expanded uncertainty	27.84	27.43	--

In order to investigate the causes of this non-symmetrical behavior, a total of 5 Monte-Carlo simulations were performed for the Brinell hardness example by varying the uncertainty value due to the indentation diameter (*d*) from 0.000035 to 0.35. Figure 1 shows the skewness values found for each simulation in function of the indentation diameter uncertainty. As can be noted, the obtained skewness increases as *d* uncertainty increases. In other words, relatively higher uncertainty values of indentation diameter contribute to the augment the asymmetrical behavior of the final Brinell hardness distribution.

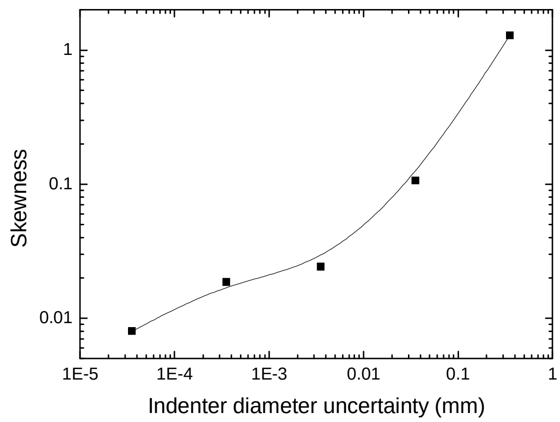

Fig. 1. Skewness values of the final distribution function obtained by Monte-Carlo simulations for the Brinell hardness measurement uncertainty example in function of the indenter diameter uncertainty. The line is only a guide to the eye.

3. CONCLUSIONS

Three approaches of measurement uncertainty estimation were applied to three examples of mechanical assays: tensile strength, torque and Brinell hardness measurements. Very similar results of combined uncertainties values were obtained for the ISO-GUM 95, EURACHEM numerical simulation and Monte-Carlo simulation approaches, with minimum differences. The three studied methodologies can be applied individually to estimate measurement uncertainties of these mechanical assays, producing analogous results.

In the case of the Brinell hardness uncertainty estimation, a type A source of uncertainty (which is dominant) related to the repetitions of the indentation diameter measurement, contributes to cause a symmetry dislocation in the final Brinell hardness distribution.

ACKNOWLEDGEMENTS

The authors would like to acknowledge CNPq / PROMETRO and FINEP.

REFERENCES

[1] P. R.. G. Couto, J. S. Oliveira, L. R. Cinelli, "Algumas considerações sobre as metodologias de cálculo da estimativa da incerteza de medição citadas no ISO-GUM 95", Metrosul 2002 – Curitiba – PR.

[2] "Guide to the Expression of Uncertainty in Measurement", ISO, Switzerland, 1995.

[3] EURACHEM/CITAC "Quantifying Uncertainty in

Analytical Measurement” – Second Edition – 2000.

[4] M. Ángeles Herrador and A. Gustavo González, “Evaluation of measurement uncertainty in analytical assays by means of Monte-Carlo Simulation”, Talanta, 64 (2004) pp. 415-422, April 2004.

[5] R. Y. Rubistein, “Simulation and the Monte-Carlo Methods”, John Wiley and Sons, New York, 1981.

[6] ISO/DGuide 99998 – “Guide to the Expression of Uncertainty in Measurement (GUM) – Supplement 1: Numerical methods for the propagation of distributions”.

[7] Hill, I.D., Wichmann, B.A., “An efficient and portable pseudo-random number generator”, *Appl. Statist.*, vol. 31, 1982, pp. 188-190.

[8] Box, G.E.P., Muller, M.E., “A note on the generation of random normal deviates”, *Annals Math. Statist.*, vol. 29, 1958, pp. 610-611.