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A PRACTICAL MODEL FOR MESOPIC PHOTOMETRY

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Abstract: This article reports the results of a multinational research project investigating visual performance in half-light (mesopic) conditions. From the experimental investigations using reaction time, detection threshold and recognition contrast threshold techniques, the results were used to develop a system for practical mesopic photometry, which provides an acceptably good fit to the experimental data and is also suitable for practical implementation by the lighting industry. A major feature of the model is that it can be implemented in terms of photopic and scotopic luminance measurements, measurements that can easily be made in practice.

Keywords: mesopic photometry, visual performance.

1. INTRODUCTION

This paper discusses the development of a practical model for mesopic photometry undertaken as part of a European project concerned with mesopic visual performance (Mesopic Optimisation of Visual Efficiency: MOVE) with application to night-time driving [1]. The project was concerned mainly with the perception of broadband stimuli in the presence of broadband background radiances at mesopic luminance levels, that is luminances between approximately 0.001 cd m^{-2} and 10 cd m^{-2}. For targets and background with various spectral distributions and luminance levels, experiments where conducted to answer the following questions 1) can it be seen? (detection/contrast threshold experiments), 2) how quickly can it be seen? (reaction time experiments), and 3) can it be recognised? (recognition threshold experiments). In total, experiments involving over 100 subjects were performed. While answering these questions were the direct aims of the experiments, a more general aim was to provide data on which candidate models for a practical system of photometry valid in mesopic conditions could be tested and refined.

2. PHOTOMETRY

The response of the human eye to light stimuli can described in terms of luminous efficiency curves that give the responsiveness of the eye as a function of the wavelength of the stimulus. There are two internationally-recognised such functions $V(\lambda)$ and $V'(\lambda)$ that describe, respectively, the response in bright light (photopic) and dark (scotopic) conditions [2]; see Fig. 1. Both curves are approximately Gaussian, normalised to have a peak value of one. The photopic curve has its peak at 555 nm and is greater than 0.5 for wavelengths between 510 nm and 610 nm, while for the scotopic curve the corresponding wavelengths are 507 nm, 454 nm and 549 nm, respectively. The scotopic curve gives more weight to the blue end of the spectrum, relative to the photopic curve. Physiologically, the photopic curve describes the behaviour of the cones (under a fixed set of conditions) while the scotopic that of the rods. In half-light (mesopic) conditions, both cones and rods operate and the visual response is generally more difficult to characterise.

A third function $V_{10}(\lambda)$ [2] was derived from colour matches for a centrally-viewed field 10^o in diameter and is used to model responses for tasks involving large targets, i.e., foveal (central) and peripheral vision. It is similar to $V(\lambda)$ at the red end of the spectrum but indicates greater sensitivity in the blue region. This function is also graphed in Fig. 1.

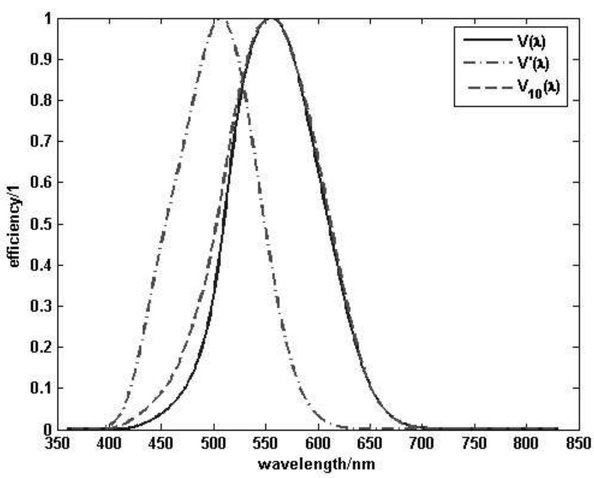

Figure 1. Efficiency functions $V(\lambda)$, $V'(\lambda)$ **and** $V_{10}(\lambda)$.

Of the candidate models for mesopic efficiency curves, a model similar to that described by Rea *et al*. [3] was developed, in which the mesopic efficiency curve $V_m(\lambda, x)$ is defined as a normalised convex combination

$$V_m(\lambda, x) = \left[xV(\lambda) + (1-x)V'(\lambda)\right]/M(x)$$

of the photopic and scotopic curves, where $M(x)$ is such that $V_m(\lambda, x)$ attains a maximum value of 1. An efficiency function can be used to determine the luminance of a light source. If $E(\lambda)$ is the spectral radiance distribution of a light source, the *mesopic I-value* is given by

$$I_m(E,x) = \int_{-\infty}^{\infty} E(\lambda)V_m(\lambda,x)d\lambda ,$$

and the *mesopic luminance* $L_m(E,x)$ is given by

$$L_m(E,x) = K(x)I_m(E,x) ,$$

where $K(x) = 683/V_m(\lambda_0,x)$, with $\lambda_0 = 555$ nm, is a normalising constant used to relate the luminance (in cd m^{-2}) to the radiance (in W sr^{-1} m^{-2}).

The main advantage of this type of model is that given *x*, the mesopic I-value $I_m(E,x) = I_m(x,L_p,L_s)$ and the mesopic luminance $L_m(E,x) = L_m(x,L_p,L_s)$ can be determined from measurements of the photopic and scotopic luminances $L_p = L_m(E,1)$ and $L_s = L_m(E,0)$ respectively:

$$L_m(x,L_p,L_s) = \frac{xL_p + (1-x)L_s V'(\lambda_0)}{x + (1-x)V'(\lambda_0)} ;$$

no additional spectral information about the light source is required. If $L_p = L_s = L$, then $L_m(x,L_p,L_s) = L$ also (for any *x*). If $L_p > L_s$, then $L_m(x,L_p,L_s) \ge L_s$, with equality only if *x* = 0; if $L_p < L_s$, then $L_m(x,L_p,L_s) \ge L_p$, with equality only if *x* = 1.

While this practical mesopic model gives only an approximate description of the dependence of visual response as a function of wavelength in mesopic conditions, for the purposes of assessing the adequacy of lighting conditions for tasks associated with night-time driving (and similar activities), the model is a significant improvement over that based on the photopic or scotopic models alone. In other words, the ability to perform tasks at mesopic lighting levels scales significantly better with mesopic luminance than with either photopic or scotopic luminance.

3. DETERMINING BEST-FIT MODELS TO DATA

Let E_s and E_b be the spectral radiance distributions associated with a stimulus and background, respectively, and $V_m(\lambda,x)$ a model for mesopic efficiency curve. The *modelled mesopic contrast* is given by

$$C(E_s,E_b,x) = \frac{I_m(E_s,x) - I_m(E_b,x)}{I_m(E_b,x)} ,$$

i.e., the difference between the modelled mesopic luminances (or I-values) of the stimulus and background divided by that of the background.

3.1 Contrast threshold experiments

In contrast threshold experiments carried out as part of the MOVE project, the intensity of the target stimulus is increased (decreased) until the stimulus becomes (in)visible. At the threshold the photopic contrast is measured. Let E_i, $i = 1,\ldots,m$, be the spectral distributions associated with a set of stimulus targets, each normalised so that $\max_\lambda E_i(\lambda) = 1$ and let E_b be that associated with the background, similarly normalised. From a measurement of the photopic luminance associated with background, the scale factor b_b associated with the background spectral distribution can be estimated. Similarly, from the measurements of the photopic contrasts at threshold, it is possible to estimated the scale factor b_i associated with the stimulus spectrum. From this information, it is possible to estimate the modelled mesopic contrast $C_i(x) = C(b_iE_i, b_bE_b, x)$ at threshold. Suppose that for fixed experimental conditions (e.g., adaptation background and eccentricity of target presentation), the threshold values b_i correspond to the same contrast as determined by the mesopic efficiency function that pertains to those conditions. Then, if that efficiency function is described by $V_m(\lambda,x)$, there must be a value of the parameter *x* and a fixed contrast C_0 such that $C_0 = C_i(x)$, subject to random effects. Hence, estimates of *x* and C_0 can be found by solving

$$\min_{x,C_0} \sum w_i^2 (C_0 - |C_i(x)|)^2 ,$$

where the weights w_i are chosen to reflect the uncertainties in the $C_i(x)$ arising from the uncertainty associated with the measurements of b_i and b_b. (The absolute value $|C_i(x)|$ is required since we assume that the contrast threshold depends only on the magnitude of the contrast.)

Fig. 2 shows the photopic and mesopic contrasts for sets of contrast detection threshold data obtained during the MOVE project using broadband stimuli and photopic luminances of 0.01 cd m^{-2}, 0.1 cd m^{-2} and 1.0 cd m^{-2}. On the left of each set of measurements are the photopic contrasts calculated using $V(\lambda)$ while on the right are the modelled mesopic contrasts determined using an optimal convex combination $V_m(\lambda,x)$ of $V(\lambda)$ and $V'(\lambda)$. Especially for the lower luminances, the modelled mesopic contrasts show much less variation relative to the photopic contrasts, indicating that the modelled mesopic efficiency function more adequately explains the visual response than does $V(\lambda)$.

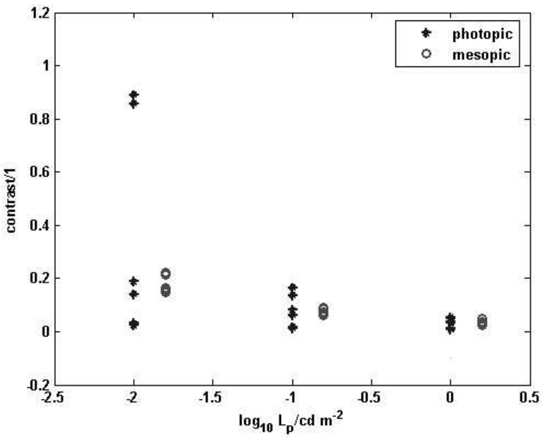

Figure 2. Photopic and mesopic contrasts (determined using an optimal convex combination of *V*(λ) and *V′*(λ) for sets of contrast threshold data for backgrounds with photopic luminances of 0.01 cd m^{-2}, 0.1 cd m^{-2} and 1.0 cd m^{-2}.

3.2 Reaction time experiments

Reaction times to the presentation of a target stimulus can be modelled as a function of contrast of the target with respect to the background. It is expected (and research has shown, e.g., He *et al.* [4]) that the larger the contrast, the smaller the reaction time. As the contrast increases, the reaction times reduce to a lower limit R_0. As the contrast decreases to a lower limit C_0, the reaction times become longer. This behaviour suggests a model of the form:

$$R(C, \mathbf{b}) = R_0 + \frac{B}{\| C | - C_0 |}, \qquad \mathbf{b} = (R_0, B, C_0)^{\mathrm{T}},$$

where B presents the rate of change of reaction time with respect change in contrast. Below the threshold C_0, the target is not detected; as the background luminance decreases we expect the threshold contrast to increase.

In a variable reaction time experiment, a number of stimuli are presented with corresponding spectral power distributions $E_i(\lambda)$, $i = 1, \ldots, m$, for a fixed background with spectral power distribution $E_b(\lambda)$. The mean reaction times R_i for each stimulus are recorded and their associated standard uncertainties $u_i = u(R_i)$ evaluated. Assuming that reaction times follow the behaviour described above, for a mesopic efficiency function $V_m(\lambda, x)$, the model prediction for the ith reaction time is:

$$R_i(x, \mathbf{b}) = R_0 + \frac{B}{\| C(E_i, E_b, x) | - C_0 |},$$

where $C(E_i, E_b, x)$ is the modelled mesopic contrast. If $w_i = 1/u_i$, estimates of the model parameters x and $\mathbf{b}$ are found by minimising

$$\min_{x,\mathbf{b}} \sum w_i^2 (R_i - R_i(x, \mathbf{b}))^2 .$$

(The parameters x specify the shape of the efficiency curve while the parameters **b** specify the shape of the reaction time curve.) The model can be expanded to take into account the fact that the uncertainty in the measured spectral power distributions is generally not negligible, due to the low power levels involved.

Fig. 3 shows the reaction time curve (model values) determined from finding the optimal convex combination $V_m(\lambda, x)$ fit to reaction time data gathered, as part of the MOVE project, corresponding to background photopic luminances of 0.3 cd m^{-2}. The error bars represent the standard uncertainties associated with the averaged reaction time measurements.

In a fixed reaction time experiment carried out as part of the MOVE project, the intensity of the stimulus is adjusted so as to elicit a fixed reaction time. Assuming that common reaction times are generated by common mesopic contrasts, estimates of x can be found using the same approach as for a contrast threshold experiment. Fig. 4 shows the photopic and mesopic contrasts calculated using the optimal convex combination $V_m(\lambda, x)$ determined from fixed reaction time data. The figure shows that the mesopic model brings the calculated contrasts much closer to being constant.

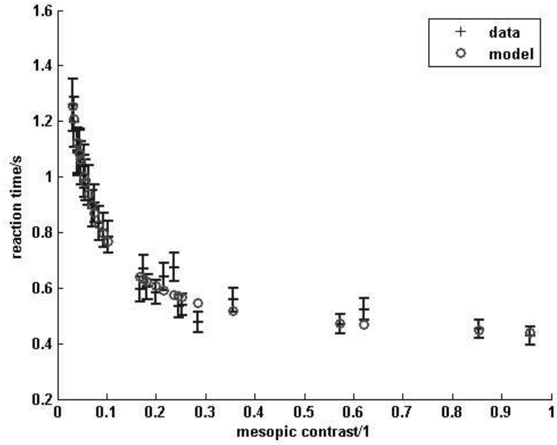

Figure 3. The reaction time curve (model values) determined from finding the best convex combination $V_m(\lambda, x)$ fit to reaction time data corresponding to background photopic luminances of 0.3 cd m^{-2}. The error bars represent the standard uncertainties associated with the reaction time measurements.

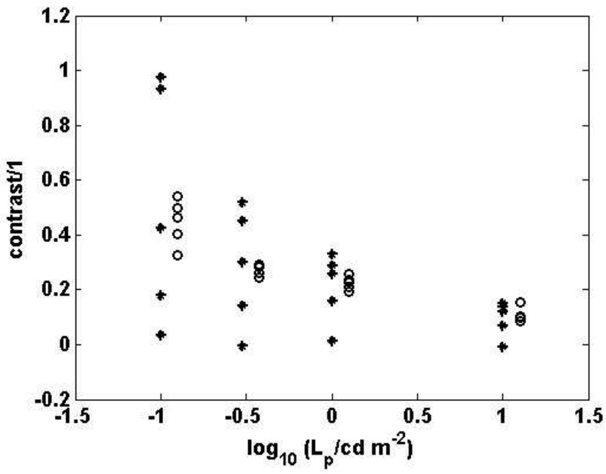

Figure 4. Photopic '*' and optimal mesopic contrasts 'o' for a fixed reaction time experiment. If the model provided a perfect match to the results then the mesopic contrasts at each level would be equal. The residual scatter is consistent with the noise in the measurements.

3.3 Recognition Threshold Experiments

In the MOVE project recognition threshold experiment, the intensity of a Landolt C target stimulus is increased (decreased) until it becomes (im)possible to determine the orientation of the target. At the threshold, the photopic contrast is recorded. These experiments are therefore very similar to the contrast threshold experiments and are modelled in a similar way. Fig. 5 shows the photopic and mesopic contrasts calculated for recognition threshold experiments and shows that the mesopic model results in a reduced scatter in the calculated contrasts. The raw data in this cases is particularly noisy so that the reduction in the scatter is more modest than for the contrast threshold experiments illustrated in Fig. 2 or the fixed reaction time experiments illustrated in Fig. 4.

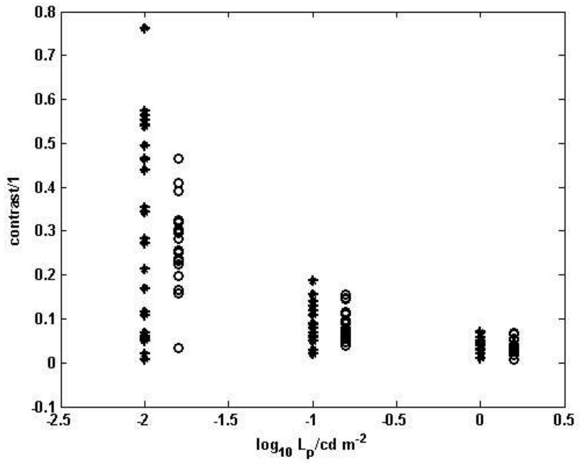

Figure 5. Comparison of mesopic contrast calculated for recognition threshold experiments using the practical model for mesopic photometry. '*' = photopic contrast; 'o' = mesopic contrast. The raw data has a large amount of inherent scatter which is reflected in the variation of the mesopic contrasts.

3.4 Model validation

The analysis of the measurement results showed that the convex combination $V_m(\lambda, x)$ of $V(\lambda)$ and $V'(\lambda)$ performed reasonably well in explaining the variation in the data. However, it may be that other functions explain the data better. In order to assess the effectiveness of $V_m(\lambda, x)$, other semi-empirical models were also tried including a Gaussian model defined by two parameters, its mean and standard deviation. More general models including radial basis functions were also tried. In almost all cases, the more general models did not perform significantly better than the practical model $V_m(\lambda, x)$. For example, Fig. 6 shows the best convex combination $V_m(\lambda, x)$ and fits of a Gaussian to fixed reaction time data corresponding to background photopic luminances of 1.0 cd m^{-2} and 0.1 cd m^{-2}. The figure shows a) relatively close agreement between the convex combination and the Gaussian fits and b) the shift in peak sensitivity to the blue end of the spectrum as the background luminance is reduced from 1.0 cd m^{-2} to 0.1 cd m^{-2}. The Gaussian model is defined by two parameters while the convex combination is defined by only one. The fact that the Gaussian fits are close to the convex combination fits suggests that the more flexible model does not provide any significant advantage. This provides evidence to support the simpler model.

The measurement data was based mainly on indirect, psychophysical measurements, such as reaction times, which involve the visual system as a whole (and other psychophysical elements) rather than a direct measurement of the photoreceptor responses. The reaction times are believed to depend on the visual efficiency curves through aggregate quantities such as contrasts so that fine detail in an efficiency curve is unlikely to manifest itself in the measurement data. (On the other hand, the results of measurement experiments can be realistically translated to tasks such as night-time driving, a primary interest of the project.) On the basis of inferences about the visual efficiency curves that can be usefully made from this type of data, the practical model is concise and gives an adequate explanation of the responses.

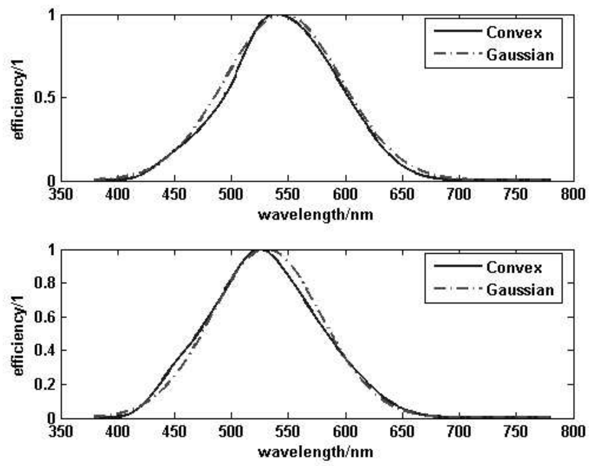

Figure 6. Best convex combination $V_m(\lambda, x)$ and Gaussian fits to fixed reaction time data corresponding to background photopic luminances of 1.0 cd m^{-2} (top) and 0.1 cd m^{-2} (bottom).

4. DETERMINING THE *x* PARAMETER

The practical model assumes that the mesopic regime $V_m(\lambda, x)$ that pertains to a particular set of lighting conditions depends only on the photopic and scotopic luminances, L_p and L_s, respectively, i.e., x can be defined as a function of L_p and L_s. From the MOVE experiments, an approximately linear relationship of the form

$$x = a + b \log_{10} I_m(x, L_p, L_s) \tag{1}$$

was observed with a = 1.49 and b = 0.282. This relationship has been derived from calculating the x_i that best explains the *i*th set of measurement data and then calculating the I-value $I_i = I_m(x_i)$ associated with the background. The parameters a and b have then been estimated by fitting a straight line to data points $(x_i, \log_{10} I_i)$.

Eq. (1) implicitly defines $x = x(L_p, L_s)$ as a function of L_p and L_s and a simple iterative scheme [5] can be used to determine the unique x that satisfies the equation given L_p and L_s. Tab. 1 gives example calculated mesopic luminances L_m for selected values of photopic luminance L_p and scotopic/photopic ratios L_s / L_p; the unit for luminances is cd m^{-2}. As the photopic luminance decreases, the greater the influence of the scotopic luminance on the calculated mesopic luminance. Tab. 2 gives the corresponding x-values that solve Eq. (1) for the selected values of L_p and L_s / L_p.

The model to determine mesopic luminances was tested on its ability to predict performance of subjects using a simulator reacting to visual events while simulating driving. For a given background with known photopic and scotopic luminances, the *x*-value for $V_m(\lambda, x)$ was calculated and then used to determine the mesopic contrasts associated with the visual events. It was found that the reaction times plotted against mesopic contrasts followed a reaction time curve similar to that in Fig. 3. These results indicated that the behaviour of the subjects could be explained by the specified mesopic efficiency curve.

Tab. 1 Mesopic luminances L_m (cd m^{-2}) for selected values of photopic luminance L_p and scotopic/photopic ratios L_s / L_p.

L_s / L_p	L_p =0.01	L_p = 0.1	L_p =1.0	L_p =10.0
0.25	0.0025	0.0644	0.8742	9.9133
0.50	0.0050	0.0778	0.9181	9.9430
1.00	0.0100	0.1000	1.0000	10.0000
1.50	0.0141	0.1190	1.0756	10.0542
2.00	0.0176	0.1378	1.1462	10.1058
2.50	0.0208	0.1521	1.2129	10.1502

Tab. 2 Calculated *x*-values for selected values of photopic luminance L_p and scotopic/photopic ratios L_s / L_p.

L_s / L_p	L_p =0.01	L_p = 0.1	L_p =1.0	L_p =10.0
0.25	0.00	0.31	0.67	0.97
0.50	0.00	0.33	0.67	0.97
1.00	0.02	0.37	0.68	0.97
1.50	0.08	0.40	0.69	0.97
2.00	0.11	0.41	0.70	0.97
2.50	0.14	0.43	0.71	0.97

4.1 Maximum entropy estimates of spectral radiant power distributions

For the case of the practical mesopic efficiency modelled as a convex combination $V_m(\lambda, x)$ of $V(\lambda)$ and $V'(\lambda)$ the mesopic luminance can be determined from the photopic and mesopic luminances. For other models this will not be the case and information about the background spectral radiant power distribution $E(\lambda)$ will be required. In this section we show how a small number of luminance measurements can be used to estimate $E(\lambda)$. We assume we have access to luminances L_k, measured using a small number of known efficiency functions V_k, $k = 1,\ldots,n_K$. Since there are infinitely many spectral distribution that could give rise to the same set of luminances the problem of determining $E(\lambda)$ as it stands is ill-posed. In order to formulate a well-posed problem it is necessary to introduce some measure of merit and select the distribution that maximises the merit function. To make the computation more tractable, the distribution function can be discretised $E_j = E(\lambda_j)$ at a fixed number of wavelengths λ_j, $j = 1,\ldots,n$. One approach is to the concept of information entropy [6] as a basis for the merit function. For discrete probability distributions with probability masses p_j, $0 \le p_j \le 1$, $j = 1,\ldots,n$, the information entropy is

$$-\sum_{j=1}^{n} p_j \log p_j .$$

The information entropy can be thought of as the amount of uncertainty associated with the distribution. Writing $p_j = E_j / S$, $S = \sum_j E_j$, then p_j can be regarded as a discrete probability distribution. In this way, for a fixed discretisation, we can associate with a spectral distribution a unique probability distribution. In the absence of other information, the spectral distribution that maximises the entropy of the corresponding probability distribution is one with $E_1 = \ldots = E_n$. Given luminances L_k, we look for the spectral distribution that maximises the entropy of the associated probability distribution subject to the constraints that the associated probability distribution comes from a spectral distribution that has the correct luminances, i.e.,

$$L_k = K_k \sum_{j=1}^{n} E_j V_k(\lambda_j), \quad k = 1,\ldots,n_K .$$

Here, K_k is the normalising constant used to calculate luminances using V_k and the discretisation. This maximum entropy solution maximises the uncertainty of the probability distribution subject to the information encoded in the luminance values and represents the fairest estimate of the spectral distribution on the basis of the evidence to hand.

Table 3. Calculated photopic and scotopic I-values for three spectral distributions.

I-value	E_B	E_G	E_R
$V(\lambda)$	6.3	15.2	12.4
$V'(\lambda)$	27.8	10.8	0.4

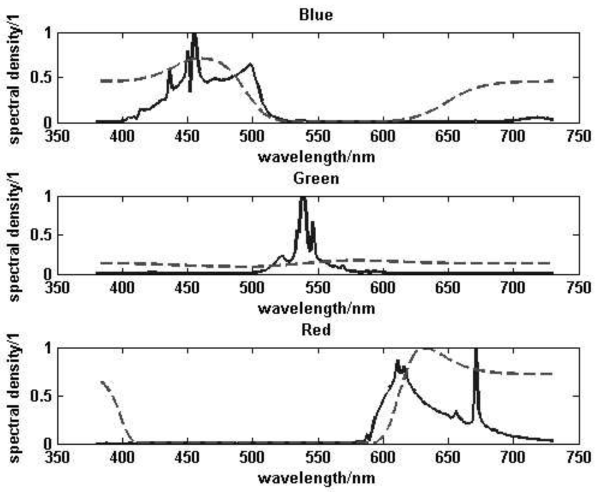

Figure 7. Normalised spectral distributions E_B (blue), E_G (green) and E_R (red) (top, middle and bottom, solid line), and their maximum entropy estimates (dashed line).

Fig. 7 shows the maximum entropy estimates of three LED spectra, E_B (blue), E_G (green) and E_R (red) given the calculated values of photopic and scotopic I-values given in Tab. 3. It is seen that from the knowledge of just two luminances, the estimate of the spectral distribution calculated from maximum entropy principles captures a substantial amount of the qualitative behaviour of the distributions.

A maximum entropy approach can incorporate additional information. Suppose, for example, that it is known that a finite number n_Q of light sources are present with corresponding normalised spectral distributions $E_q(\lambda)$, $q = 1,\ldots,n_Q$. Modelling the spectral distribution of the combined light sources as

$$E\lambda) = \sum_{q=1}^{n_Q} b_q E_q(\lambda), \quad b_q \geq 0,$$

we can find estimates of the coefficients b_q by maximising the information entropy. With spectral distributions E_B, E_G and E_R as shown in Fig. 7, Fig. 8 graphs $E = E_B + 2E_G + 3E_R$ and $E = 3E_B + 2E_G + E_R$ along with their maximum entropy estimates determined from the corresponding photopic and scotopic luminances. In this case the maximum entropy estimates provide good approximations to the spectral distributions. The computation is straightforward as it involves only three optimisation parameters.

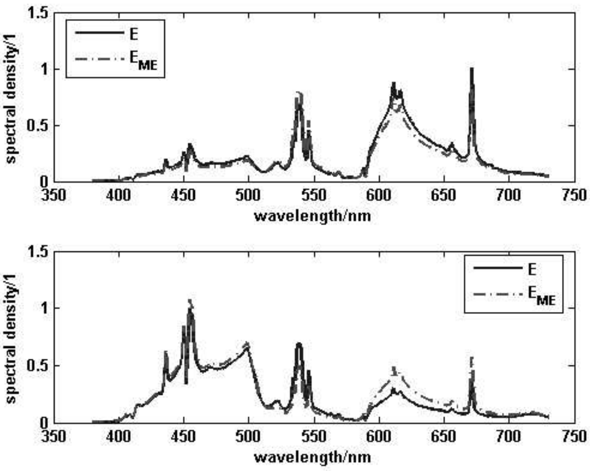

Figure 8. Spectral distributions $E = E_B + 2E_G + 3E_R$ (top) and $E = 3E_B + 2E_G + E_R$ (bottom) along with their maximum entropy estimates determined from the corresponding photopic and scotopic luminances.

5. CONCLUDING REMARKS

In this paper, we have described a practical model for mesopic photometry based on a combination, determined by a single parameter *x*, of the photopic and scotopic efficiency functions. From measurement data gathered over a range of experiments and using over 100 observers, we have determined how the parameter *x* describing the mesopic regime can be determined as a function of the photopic and scotopic luminances. This enables the mesopic luminance to be calculated, from which an assessment of the suitability of the lighting conditions for various tasks can be made. We have also shown how mesopic luminances for other models can be estimated from photopic and scotopic luminances.

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