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VERIFICATION BY VIRTUAL GAUGE USING A STATISTICAL CRITERION

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Abstract: For the most part, metrology software is currently based on the measurement of distances or angles between geometrical elements. If this method of verification is well adapted to geometrical specification without a virtual state modifier, this is not appropriate for specification based on envelop zone such as in maximal matter condition, for example, for the ISO2692 standard. Usually the least squares best fit method is used to estimate derived surfaces, but the statistical information contained in the acquired coordinates remains under-used. The aim of this paper is to present a new approach for the verification of a part, based on a virtual gauge and using a statistical criterion.

Keywords: measurement, verification, virtual gauge, interference probability map.

1. INTRODUCTION

The guiding principle of most verification processes is an assembly test between a set of points or its derived surface and a tolerance zone built from geometrical specifications. Hence the geometrical verification is articulated around three important points: the measurement process, part characterization and geometrical specification interpretation.

The measurement process has been the source of numerous research projects for the last decade. However, it will be assumed that the verification process will start with one or a set of real surfaces measurements taken on a classical coordinate measuring machine (CMM). Current metrology software is based on the measurement of distances or angles between geometrical elements. Usually, the least squares best fit method is used to associate a perfect feature to the acquired coordinates. However, the statistical information contained in the set of points is generally not put to use.

In this paper, a geometrical verification by virtual gauge using a statistical criterion is presented. The statistical information contained in a set of points representing the real surface, permits through uncertainty propagation to define the probability density of the position of the matter around the mean associated surface. Through uncertainty propagation method, the probability density function of the position of the matter around the mean associated surface can be defined by the statistical information contained in a set of points. On the other hand, a gauge model offers a closed integration domain which is perfectly adapted for the calculation of interference probability.

2. THE ISSUE OF GEOMETRICAL VERIFICATION

As mentioned in the introduction, the geometrical verifications issue could be summarized in three points (figure 1). First, the measurement of the real surfaces (here done on with CMM) will provide a set of digitized points. Secondly, the part characterization is achieved through a best fitting to obtain derived features or integral associated surfaces. Finally, the geometrical specifications and the building of tolerance zones (TZ) must be analyzed in order to perform a conformance test between the best fitted surfaces or the set of measured points, and the tolerance zones.

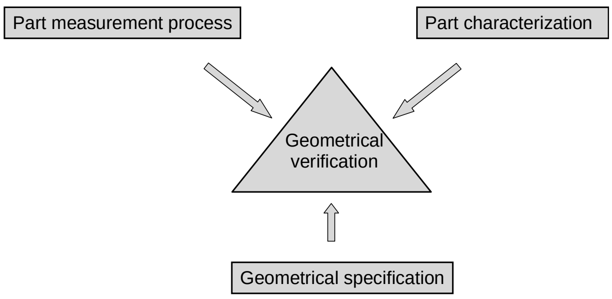

Fig.1. Geometrical verification issue

2.1. Part measurement process and part characterization

Several research projects have been carried out on measurement processes. First kind of work has been performed on the improvement of the use of measurement instruments [1]. This improvement could be made with more efficient methods of calibration [2]; other work proposes methods to decrease measurement errors [3]. Then work has been done sampling strategy [4].

Nowadays measurements by CMM are widely used in manufacturing industries. The data supplied by CMM a set of acquired coordinates. To estimate an associated surface representing the real part surface, a best fitting must be done. This association is made by optimizing a criterion like least squares, infinite norm or likelihood functions [5] [6]. Part characterization can be sorted into two categories: a best fit of a complex surface including classical shape error, or best fit of a simple feature and an estimation of statistical data. A method based on a likelihood function proposes a non-linear optimization association [7].

Nevertheless, it is well known in measurement that the sampling of the real surface and hence the best fitting, imply that the estimation of the surface is provided with uncertainties.

2.2. Geometrical Dimensioning & Tolerancing

A geometrical specification describes a set of geometrical conditions which must be met by a set of real surfaces composing the workpiece. The geometrical product specification system (GPS) provides a set of efficient tools for univocal specification descriptions and verification processes [8]. These tools are based on the concept of tolerance zone, i.e. a domain of the 3D space related or not to a datum reference surface and where the real surface must be contained. However if international standards ISO provide accurate definitions for geometrical specifications, it is necessary to perform a mathematical translation of this requirement adapted to the measurement data. Aiming toward coherence between geometrical specifying and verifying, several mathematical-based models of specifications have been proposed. One approach involves a classification of symmetry groups and its impact on functional feature taxonomy, datum definition and parameterization [9]. As tolerance zone can be seen as geometrical boundaries, it is possible to describe the permissible geometrical variations of the parts by virtual gauges with internal mobility [10]. Still based on geometrical limit, the concept of Virtual Boundary Requirement (VBR), generalized with Maximum Material Condition (MMC) and Least Material Condition (LMC), is now widely used.

2.3. Conformance test

The conformance test is a test which will validate that the specified surface of the part meets the geometrical requirement. The pre-condition of a conformance test is the characterization of the part and the interpretation of the geometrical specifications. In practical terms, the conformance test will determine if the estimation of the derived feature or the set of points can be contained in the tolerance zone. Most measurement software, in agreement with normalized specifications (ISO standard), is based on the measurement of distances or angles between geometrical elements. These geometrical quantities can be described as the following types:

- point/point distance,
- point/plane distance,
- point/line distance,
- line/line angle,
- line/plane angle,
- plane/plane angle.

This will result in a set of inequations representing the tolerance zone, which must be satisfied. With the current industrial landscape, the competition in products manufacturing is becoming stiffer and stiffer due to the globalization and subcontractors management. This implies the fact that product manufacturing must always must continue to accelerate with a higher level of quality. Hence verification should be fast and reliable. As said earlier, tolerance zone can be seen as a space boundary, especially when modifiers of the envelope condition, maximum or least material condition, are specified. Consequently, gauges are actually a good solution to this issue. They are one of the most reliable tools for geometrical verifications. They can directly validate the largest part of assembly and functional requirements. There are two kinds of gauges: virtual gauges and physical gauges. Physical gauges have been used for a century. They permit extremely fast and reliable validation of functional requirements, such as assembly requirements. They are involved in specification verifying [11] [12] but also in machine tools and CMM [2] calibration. A virtual gauge is a set of ideal virtual features. It permits a comparison between the real part and a model of the nominal design. The goal of virtual gauge is to be defined as soon as possible in the conception process of a product, the product's verifier gauge and then to compare it to the obtained real part. With the improvement of CAD software and CMM, virtual gauges are showing all their potential.

3. CLASSICAL & STATISTICAL POINTS OF VIEW

If the concept of physical and virtual gauge is still the same, i.e., testing if the matter is completely inside/outside a spatial boundary, there is, however, different ways to perform it. The first method is to work directly on the real part with a material gauge. The second works on the set of digitized points without any kind of best fitting or on a derived feature element given by the best fit method.

3.1. Verification by virtual gauge without best fit

The aim of this method is to find the position of the virtual gauge where it will include all the set of points (figure 2.). Of course, this position shall be in agreement with the degrees of freedom given by the geometrical specifications.

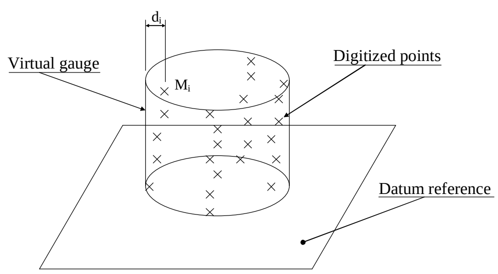

Fig.2. Cloud of point verification

From a mathematical point of view, an algebraic distance d_i will be calculated between each digitized point M_i and the surface of the virtual gauge. The conformance test will be to check if the whole distances d_i are positive or negative, i.e. if the points are inside or outside the virtual gauge, according to geometrical specifications. The main advantage of this kind of conformance test is to avoid geometrical construction and hence to avoid uncertainties propagation.

3.2. Verification with associated feature

The principle behind this method is to perform an assembly test between an estimated and/or constructed feature and the virtual gauge.

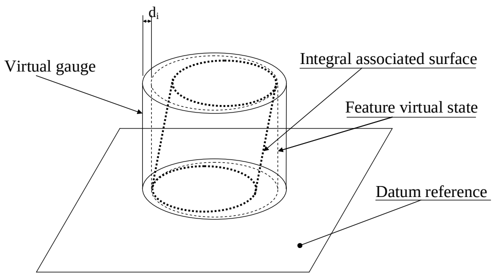

Fig.3. Verification with virtual state of the matter

In order to perform the conformance test, a surface representing the virtual state of the feature is constructed. The estimation of the virtual state results from the addition of the perpendicularity error, and the estimate dimension of the extreme fit. Then, to be accepted, the virtual state dimension must be inferior or superior to the virtual gauge dimension, i.e. it must be inside or outside the matter according to the geometrical specifications.

The virtual gauge could also have a fitter behavior [13]. In this case, an extreme fitting will be done with the whole set of digitized points. This extreme fitting of the set of features is done in a particular order defined by the geometrical specification. Then the same test as above will be applied between the fitter gauge and the limiter gauge obtained from the geometrical specifications.

3.3. Statistical point of view

However, all the previous kinds of verification do not take into account measurement uncertainties. In this case, each point is considered as being a point from the real surface. Actually it is well known fact that every measurement is tainted with uncertainties, and hence the acquired coordinates are a random sample of the real points of the surface.

In the next section, a method taking into account this statistical aspect is used.

4. STATISTICAL PART CHARACTERIZATION

As shown in the previous section, statistical parameters are unused in most of classical geometrical verification processes. With the method to be described in this section, a three-dimensional scalar field representing the probabilities of being inside the matter can be expressed.

The real surface to be verified is composed of an infinity of points which could be seen as the statistical population to be characterized. The main hypothesis of this method is that the set of digitized points is considered as a statistical sample of the real surface population. The first step of the statistical characterization process is the best fitting of a derived feature and a set of intrinsic parameters with the set of acquired coordinates. In fact, the derived feature is expressed by a random vector representing the mathematical expectation of the surface parameters. To complete the best fitting, the maximization of the likelihood method will be used.

Two sets of values are expressed:

- The first moment order representing the mean value of the derived feature and intrinsic parameter values.

- The second moment representing the variance and covariance of the random vector parameters and intrinsic parameters. These values will be provided on the form of a covariance matrix.

The second step is the uncertainties propagation to the complete associated feature. This feature can be constructed from the first moment order values, i.e. the mean values obtained at the best fitting. The aim of this step is to calculate the uncertainties directly on the complete associated surface. This is done via the propagation of a covariance matrix which is achieved with the following formula (1):

$$\sum\left(\overrightarrow{OM}\right) = J\left(\overrightarrow{OM}\right)\cdot\Sigma\left(\vec{p}\right)\cdot J\left(\overrightarrow{OM}\right)^T \Leftrightarrow \begin{aligned} \sigma_{\vec{n}}^2\left(\overrightarrow{OM}\right) &= \overrightarrow{n^T}\cdot\sum\left(\overrightarrow{OC}\right)\cdot\vec{n} \\ &+ \lambda^2\cdot\overrightarrow{n^T}\cdot\sum\left(\vec{v}\right)\cdot\vec{n} \\ &+ 2\lambda\cdot\overrightarrow{n^T}\cdot\sum\left(\overrightarrow{OC},\vec{v}\right)\cdot\vec{n} \\ &+ \sigma^2(\text{residues}) \end{aligned} \qquad (1)$$

Where J represents the Jacobean operator, Σ the covariance matrix and λ the distance to the barycenter.

Next, the variance of the best fit residues should be added. In practice, the variance at the current point M belonging to the complete associated feature along the feature normal has been calculated in the equation (1).

This will bring about two important results:

- For a fixed risk α, it is possible to express the statistical envelop containing the whole real surface of the part (figure 4).

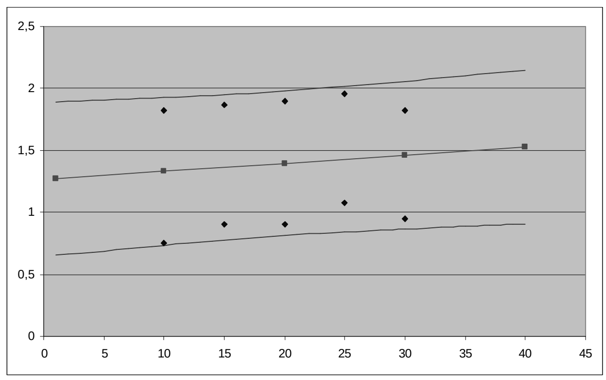

Fig.4. Statistical limit of the matter

- It is possible to calculate the probability at any point to be inside the matter. This can be done by an integration of the normal law along the normal of the complete associated feature. The utilization of this result in a verification process will be developed in the next section.

5. STUDYING A SIMPLE CASE

In order to highlight this statistical approach, a simple case will be studied. In the next two subsections, the case of a geometrical verification problem of a bore in a parallelepiped will be considered. Two different kinds of specifications will be seen: firstly, envelop requirement specifications, and then, orientation specification using the maximum material condition.

5.1. Envelop zone specification:

As shown in figure 5, the case of a bore geometrically specified by an envelop requirement is taken into consideration. According to standards (ISO 8015 - 1985), the principle of envelop requirement implies the nominally cylindrical real surface of the bore must be outside a perfect cylinder at the state of maximal material (figure 6.). In the case under consideration the envelop diameter is 30.93 mm (figure 6). Moreover, every distance between two points in opposition must be inferior to the maximum allowed distance 30.97 mm, in this case (ISO 14 660-1). However, only the constraint of the envelop zone will be examined in this sample.

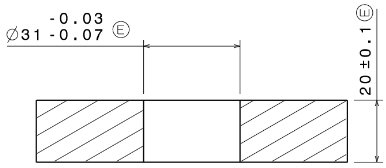

Fig.5. Nominal design and geometrical tolerance

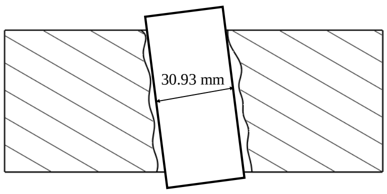

Fig.6.Conformance test

Basically, the virtual gauge will be a perfect cylinder with 6 degrees of freedom. Thus, the optimization of the gauge position will be gained on the 3 rotations and the 3 translations.

The first step of the verification process is the measurement of the real part. For this case, it has been carried out on a classic CMM. In order to show the influence of the number of digitized points on uncertainties, two sets of coordinate points have been acquired on the same bore: first with 16 points and then with 32 points. Acquired coordinates are provided by the CMM programs in a classic ASCII file of points.

The second part is the best fitting which is done with the maximization of the likelihood function. This step will provide the mean associated surface parameters (position and intrinsic dimension) and their covariance matrix.

Thirdly, the virtual gauge is made according to the constraints of the geometrical specification. Generally, the initial position and the dimension of an elementary virtual gauge should be raised from 3D part's design. An offset operation from nominal geometry is performed. For complex geometrical specification, a global virtual gauge could be made with an assembly of a set of elementary virtual gauges. According to the virtual gauge freedom degrees, a matrix representing the set of allowed solid displacements is generated for each elementary gauge. It will represent degrees of freedom defined in the geometrical specification. This matrix will be used in the optimization problem. As demonstrated in section 4, it is possible to express, for each point of the 3D space, the probability of being inside the matter. This will be used for the conformance test. Assuming the fact that the virtual gauge is a closed surface easily formulated by a parametric equation, a meshing of the gauge should be carried out. Thus, a probability of interference can be calculated for each node of the mesh. The result is given in a matrix permitting via linear interpolation the calculation of interference probability according to the curvilinear coordinate of the gauge.

Fourthly, the optimization of the virtual gauge position is performed aiming to minimize the highest interference probability.

Finally, the drawing of an interference probability map (IPM) is generated for each elementary virtual gauge for the optimal position. Using a grey scale, an IPM represents the interference probability according to the curvilinear coordinate of the gauge. This should be seen as an unfolded texture of the gauge.

Before seeing the statistical geometrical verification result, a classical verification process based on least square and shape default has been made. In the case of the 16 points, a diameter of 30.987 mm and a shape default of 0.021mm have been found. Hence the estimation of the diameter of the smallest envelop tangent to the matter is 30.987-0.021=30.966 mm, which is large enough to accept the part. In the case of the 32 points, a diameter of 30.988 mm and a shape default of 0.018mm have been found. Thus the part should be declared good with an estimation of the diameter of the smallest envelop tangent to the matter of 30.970 mm. It must be noted that these results do not take into account the measurement uncertainties.

In the case where the part is characterized from the set of 16 points, the following IPM is obtained after optimization (figure 7.):

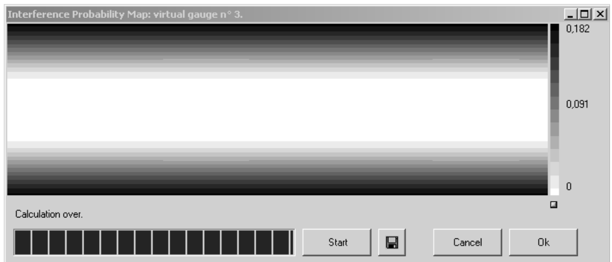

Fig.7.Interference probability map for 16 points

This IPM shows three different zones. The centered one is the well-known zone of the part; here the probability of interference between real surface and virtual gauge is nearly null (below 2%). This zone is located around the barycenter of the digitized point. Next, there are the two extremity zones where the knowledge of the part is the worst (from 2% to 18.2% at edges); here uncertainty on the mean surface increases proportionally with the distance to the barycenter of acquired coordinates.

An obvious fact is the symmetry around the plane perpendicular to the cylinder axis. This shows that the result of gauge position optimization is that the gauge axis is collinear to the mean associated surface axis. This alignment seems foreseeable due to the envelop requirement verification leaving all degrees of freedom on gauge position.

Revolution symmetry is notable. This is a particular case. Indeed the covariance matrix is symmetric and defined positively. With propagation calculation using the Jacobian operator, the theoretical form of a statistical envelop is a hyperboloid with an elliptic base. At the moment only first and second moment order are used for part characterization. Moreover, for the best fit, the hypothesis that every point has the same standard deviation is made forward. Therefore, this symmetry is actually showing that the position of digitized points is symmetric.

In the case where the part is characterized from the set of 32 points, the following IPM is obtained after optimization (figure 8.):

Fig.8. Interference probability map for 32 points

The maximal interference probability is 2.6% at the edge. This great decrease of uncertainties is due to the increase in point numbers. However, shape default also decreases from 0.021 mm with 16 points to 0.018 mm.

An important remark is the highlighting of the influence of orientation parameters on uncertainty due to the lever arm effect depending on the distance to barycenter of the set of points. If it is not so important in envelop verification, this will take its entire place in perpendicularity verification.

4.2. Perpendicularity specification:

In this subsection, the case of a perpendicularity requirement between a bore and a plane is considered (figure 9.). The plane A is taken as datum reference surface. Once again the case of 16 and 32 points will be studied.

According to geometrical specification standards, the datum reference surface is defined as the perfect plane extreme fitting to the real surface, nominally plane A, and minimizing the highest gap. To be accepted the nominally cylindrical real surface of the bore must be outside a perfect cylinder perpendicular to plane A with a diameter of 30.93 mm (ISO 2692 - 1988) (figure 10.). The difference with the first case studied is the constraint put on the virtual gauge. In this case, degrees of freedom are the two translations leaving the reference plane A globally invariant.

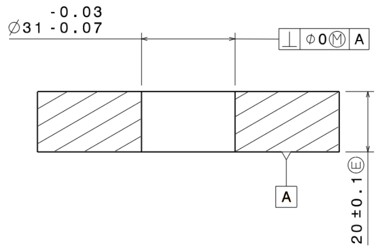

Fig.9. Nominal design and geometrical tolerance

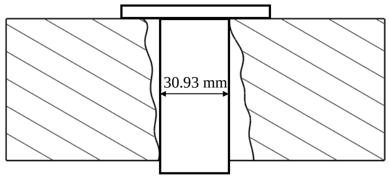

Fig.10.Conformance test

The surfaces characterization is achieved with two previous sets of digitized points. The datum reference surface is built on a cloud of 16 points. Although it is wrong, this surface will be admitted without uncertainty. Using the classical least square and shape default method, the following results are obtained:

- In the 16 points case the measured diameter is 30.987 mm, the shape default is 0.021 mm and the perpendicular default is 0.029 mm. Hence the diameter of the perfect cylinder representing the state of the maximum of matter is of 30.937 mm. So the part should be accepted and has a clearance of 7 µm.
- In the 32 points case the measured diameter is 30.988 mm, the shape default is 0.018 mm and the perpendicular default is 0.028 mm. Hence the diameter of the perfect cylinder representing the state of the maximum of matter is of 30.942 mm. So the part should be accepted and has a clearance of 12 µm.

In the case where the part is characterized from the set of 16 points, the following IPM is obtained after optimization (figure 11.):

Fig.11. Interference probability map for 16 points

With this IPM, the two critical zones of the verified cylinder can be spotted. In these two zones, the maximal probability for the part to be nonconform is of 32.8%. This high risk is mainly due on the one hand to the uncertainties on cylinder direction and to shape error and on the other hand to the mean value showing a high orientation error.

In the case where the part is characterized from the set of 32 points, the following IPM is obtained after optimization (figure 12.):

Fig.12. Interference probability map for 32 points

With a high number of digitized points, uncertainties on direction are highly decreased. Here the probability of interference decreases to 15.8% at critical zone.

6. CONCLUSION

This paper introduced a new method of part characterization based on statistical estimation of the real surface of the part, on the one hand, and, on the other hand, a new point of view in geometrical specification verification with the interference probability map. For characterization, the use of first and second order statistical moments permits the expression of the probability at any point of the space to be inside the matter. Thus, coupled with the concept of virtual gauge, it is possible to have an accurate statistical point of view of risk of interference between real surface and tolerance zone. The representation of the interference probability map permits a fast estimation of the efficiency of the measurement process planning and hence to correct it for critical cases.

However, the verification of complex parts composed of numerous elementary virtual gauges and a set of freedom degrees implies complex geometrical formulation. Moreover, due to the type of functions to optimize, which are continuous but not derivable (non-smooth optimization), the optimization problem must be solved with unconventional algorithms. A solution could be found in metric tensors and the perturbation matrix. This will be the object of future works.

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