XVII IMEKO World Congress
Metrology in the 3rd Millennium
June 22–27, 2003, Dubrovnik, Croatia

TIME DOMAIN SPECTRAL ANALYSIS, NEW APPROACH FOR HIGH ACCURACY

N.Hlupic, D.Petrinovic, J.Butorac

Faculty of Elecrical Engineering and Computing, Zagreb, Croatia

Abstract – Time Domain Spectral Analysis is presented in this paper as a new, original approach to the analysis of signal spectrum (having samples of the measured signal). The uniqueness of this method is in avoiding of any kind of Fourier transform, which showed to be of limited accuracy when the samples' errors (measurement imperfections) are taken into account, even applying windows and various approximations. The majority of calculations are based on our previously developed method for high accuracy measurement of a single sine wave. The basic idea of this method is to extract all time domain samples of a particular wave from samples of the whole signal, and then to calculate parameters of that component by applying the method for measurement of a single sine wave. This approach has been tested with extended and rigorous computer simulations (explained further in the text) and partially in real laboratory measurements. It is demonstrated that estimated accuracy of parameters of the fundamental wave is about 4 ppm, even though the samples' errors are of the order of 20 ppm. The estimated accuracy of the higher harmonics is better than 500 ppm (regardless of their number, and the same for amplitude and phase).

Keywords: spectral analysis, high accuracy

1. INTRODUCTION

In spite of all efforts made in last few decades, high accuracy analysis of spectrum, under real measurement conditions (non-neglectable distortions, imperfect signals and sampling devices), is still not as highly accurate as we would like it to be. The best achievable accuracy of fundamental wave is about 100 ppm, and errors of parameters of harmonics are of order of few tenths percent [1]. There are several common problems in all widely accepted methods for spectral analysis. First of all, frequency of the fundamental wave (i.e. ratio of its and sampling frequency) is never exactly known, because none of the real measurement systems (signal sources and sampling devices) can perfectly follow or reproduce time intervals needed by measurement procedure. That fact leads to the scaling of the frequency axis [2,3] and the exact location of the fundamental wave in the signal spectrum is not known. Therefore, we are not able to determine the parameters of the fundamental harmonic with sufficient accuracy. This forces us to use various approximations to calculate frequency of the fundamental wave as accurately as possible, but with such an approach, we are obviously limited already in the first step. Next problem arises when we start to estimate the remaining part of the spectrum. Frequency of each harmonic is an integer multiple of the fundamental wave frequency, so if we have an error in fundamental frequency, it is going to be k times larger for k^{th} harmonic (k = 2, 3, …). This means that we shall calculate parameters of higher harmonics at rather wrong positions on the frequency axis, resulting in poorer and poorer estimation as we proceed to the higher frequencies. Moreover, most of the methods based on such approach usually subtract contributions of all previously estimated signal components from FT of the whole signal before calculation of parameters of a single sine wave. Since the estimation of the lower harmonics is performed on the wrong frequencies, the residual signal after subtraction still has significant energy around the signal component under calculation, reducing the estimation accuracy of the higher harmonics. Although suffering from the mentioned shortages, methods based on such procedure enable relatively good results (better than any windowed DFT – see [1]) for signals with fundamental wave much larger (more than 1000 times) than all other harmonics. However, if the higher harmonics are not that small, or disturbances are strong, accuracy will be unsatisfactory for more precise measurements[1].

Since the accuracy of the conventional estimation methods was not sufficient for our applications, in the last few years we have developed a new method for high accuracy spectral analysis - Time Domain Spectral Analysis (TDSA).

2. PHILOSOPHY OF TDSA

The signal we were particularly interested to estimate was a periodic signal with several harmonically related components, and the main task of the algorithm was to determine the parameters of these components as accurately as possible. For the signal consisting of a single sine wave the problem is much simpler and was solved previously. We have developed a time-domain calculation procedure for high accuracy measurement of a single sine wave that was

[1] Large harmonics will significantly reduce accuracy of calculation of fundamental wave, while they themselves will be easier to determine. In contrary, low harmonics will not affect fundamental wave too much, but they themselves will be hardly detectable.

named *sine-wave calculator* or abbreviated SWC [4-6]. The proposed Time Domain Spectral Analysis method is based on repetitive usage of SWC, which is applied to all harmonic components of a periodic signal individually. In contrast to the conventional estimation techniques that are based on Fourier transform and the spectral samples of the signal, TDSA is performed in time domain, directly on the samples of the particular signal component. The only considerable problem in application of TDSA is how to isolate signal components form each other such that they do not interfere mutually during calculations. The outputs of TDSA algorithm are all parameters (amplitude, frequency and phase) of all harmonic signal components, i.e., frequency domain representation of the signal. Since the frequency domain representation is obtained applying time domain calculations, the name "Time Domain Spectral Analysis" perfectly describes this concept.

Implementation of this approach nevertheless requires theory of discrete time signal processing, especially theory of digital filters, because we have to have extremely selective filters to extract samples of a particular wave from samples of the whole signal. Fortunately, selectivity is something where digital filters are unsurpassable, and even more powerful than we need. Of course, strictly following the idea of TDSA one would need many filters with different passbands (different central frequencies), that is, one filter for each spectral component, what would be rather awkward in applications. Therefore, in reality we make use of well known *frequency shifting* or *multiplication property* of Fourier transform [2], which states that FT of a signal multiplied by $e^{j\omega_0 t}$ is FT of the original signal shifted to the left by ω_0, or symbolically

$$\text{if} \qquad x(t) \xleftrightarrow{FT} X(j\omega) \ ,$$

$$\text{then} \qquad e^{j\omega_0 t}x(t) \xleftrightarrow{FT} X(j(\omega-\omega_0)) \ . \qquad (1)$$

Exploiting (1), we can have only one filter for extraction of samples of all components of the signal. We just have to multiply the signal with appropriate factor $e^{j\omega_0 t}$ before filtration to shift the spectrum on such position, that frequency of the interesting component enters the passband of the filter. After filtration we need to shift the signal spectrum back to the original position in order to perform SWC. Both complex multiplications (by $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$) and (complex) filtration will make a real signal complex valued, but according to the theory [2], we need to retain only real part of the resulting sequence, so that we again deal with a real number samples. Due to significant simplification in implementation of the basic TDSA idea, such procedure deserves a special identifier, and as one could expect, we named it "Frequency Shifting - Time Domain Spectral Analysis" (FS-TDSA).

2.1. How does it really work – algorithm of TDSA

Clearness and simplicity of TDSA philosophy can be very fallacious. TDSA itself is just a concept, approach to the problem, but it is only "brain without hands". Applying its algorithm we shall get information we need, but only if the frequency of the fundamental wave is known (accurately enough). Without that information, we can not know where to set the central frequency of the filter (i.e. how much should we shift the spectrum to place the desired component into the filter's passband for FS-TDSA), and TDSA itself would be as helpless as all the other methods. However, TDSA works in conjunction with SWC, its "hands", and it just have to ensure appropriate conditions for SWC to enable it to work properly. That is why TDSA demands some redundancy[1] in the number of samples of the whole signal. This redundancy has two positive effects. Firstly, it causes spreading of the neighbouring spectral components apart from each other, reducing mutual influence of tails (side lobes) of the neighbouring window spectrums [3]. Secondly, it provides us with larger total number of samples, enabling filtration with higher order filters (higher selectivity), without reducing the number of usable samples after filtration too much (SWC will still be able to reach its ordinary accuracy [4]). Once SWC gets sufficient number of pure samples of the fundamental wave, highly accurate determination of fundamental frequency is just a routine [7]. As result of this cooperation between TDSA and SWC, algorithm of FS-TDSA (most convenient version of TDSA, at least by our opinion) comprises following several steps:

1) Redundant sampling of the signal (more about it in the next section).
2) Rough estimation ω_0' of the fundamental frequency ω_0 (based on the known parameters of the measurement system and measured signal).
3) Multiplying signal samples by $e^{j\omega_0' t}$ (shifting of the spectrum left by ω_0').
4) Lowpass filtering to extract samples of the fundamental wave.
5) Multiplying filtered samples by $e^{-j\omega_0' t}$ (shifting of the spectrum to the right by ω_0').
6) Forwarding real part of the resulting samples to SWC to obtain amplitude, frequency and phase of the fundamental wave. Here we get ω_0 highly accurate.
7) Neutralisation of gain and phase effects of thefilter.

At this point we have all information about the fundamental wave, and calculation of the rest of the spectrum is performed repeating steps 8…12) for each harmonic.

8) Multiplying signal samples by $e^{jk\omega_0 t}$, where ω_0 is the fundamental frequency obtained in step 6) in radians and k is the ordinal number of the harmonic (k = 2, 3, …).
9) Lowpass filtering to extract samples of the k^{th} harmonic.
10) Multiplying filtered samples by $e^{-jk\omega_0 t}$.
11) Forwarding real part of the resulting samples to SWC (amplitude, frequency and phase of the k^{th} harmonic).
12) Neutralisation of gain and phase effects of the filter.

3. STATE OF THE ART - TDSA IN COMPUTER SIMULATION TESTS

Due to its theoretical clearness and simplicity, TDSA is very suitable for automated measurements, and can be very reliably tested by computer simulations. Now we are going to explain in details our simulation that was used during

[1] Here *redundancy* does not mean *oversampling* (sampling with higher frequency than needed), but simply sampling of more signal periods than usually (than theoretically required).

development of TDSA and for testing its performance. All computer programs were written in MS Visual Basic under MS Excel 97 (calculations are done in double precision), but for some critical routines the main program calls Matlab® computing engine (e.g., generation of the filter coefficients, convolution and complex multiplication). To ensure precise repeatability of our simulation, we shall provide most important Matlab commands explicitly.

Simulation starts with generation of samples, taking into account the signal structure and measurement imperfections. If the signal consists of m components (m+1 with DC one), every instantaneous (and ideal) sample of it will have value equal to the sum of instantaneous values of all components, that is, value U_i of i^{th} sample of the whole signal will be given by

$$U_i = A_0 + \sum_{k=1}^{m} U_{ki} , \qquad (2)$$

where U_{ki} is value of k^{th} harmonic in the i^{th} moment of sampling (i^{th} sample of the k^{th} harmonic) and A_0 is DC level. In reality, of course, we can not achieve instantaneous sampling, and we must take into account the integration[1] time needed by sampling device for measurement of a single signal value. Because of that integration, samples obtained in the real measurement will form the so called mirror signal [4]. This mirror signal can be thought of as sequence of ideal samples of some other signal composed of components A_{mk} (mirror components of the original components A_k), and written analogous to (2) as

$$U_{mi} = A_{m0} + \sum_{k=1}^{m} U_{mki} , \qquad (3)$$

where A_{m0} is DC level of the mirror signal ($A_{m0} = A_0$) and U_{mki} is value of the k^{th} mirror harmonic in the i^{th} moment of sampling (i^{th} sample of the k^{th} mirror harmonic). It is well known [4] that these mirror components are in firm mathematical relationship with original ones. Depending on the sampling parameters, that their relationship is defined by formulae

$$U_{mki} = A_{mk} \sin(\omega_{mk} i + \varphi_{mk}) \quad ; \qquad (4)$$

$$A_{mk} = \frac{A_k}{k\pi} \frac{T}{T_m(1+\Delta t_m)} \sin \frac{k\pi T_m(1+\Delta t_m)}{T} \quad ,$$

$$\omega_{m1} = 2\pi \frac{T_S(1+\Delta t_S)}{T} \quad , \quad \omega_{mk} = k\omega_{m1} \quad ,$$

$$\varphi_{mk} = k2\pi \frac{T_o + T_d}{T} + k\pi \frac{T_m(1+\Delta t_m)}{T} + \varphi_k \; .$$

Meanings of the most important symbols are clear from previous text, and in (4) we have useed the same symbols with the same meanings as in [4], so the reader should consult that reference for more details. Here we can just briefly repeat that T is period of the original fundamental wave, T_S is the sampling period ($1/f_S$), Δt_s its relative error ($\Delta T_S/T_S$), T_m is the integration interval, Δt_m its relative error ($\Delta T_m/T_m$), and T_o and T_d are certain offset times in the measurement system (see [4]). Formulae (3) and (4) enable very precise calculation (generation) of samples that would be obtained if the signal given by (2) was sampled with systematic errors Δt_s, Δt_m, T_o and T_d in the sampling process. However, if we know systematic errors (and we can determine them with neglectable uncertainty [4-6]), there will be no error in conversion of parameters of the mirror signal to the parameters of the original one applying (4). This means that, for simulation purposes, it is sufficient to calculate the spectrum of the mirror signal and to examine its accuracy in order to evaluate the accuracy of calculation procedure[2], so we are going to do like that in this text[3]. Another thing we have to do to simulate the actual measurement setup is to incorporate (simulate) random errors of the samples into values we obtain by (3). In our simulation we do that assuming uniform distribution[4] of errors, adding to each sample obtained by (3) appropriate random number inside some adjustable boundaries. Specifically, if the maximum possible random error of any sample is dU (usually given in ppm, relatively to the measurement range of the sampling device), we generate a random number between -1 and 1 (uniformly distributed) and multiply it by dU. Then we add this product to each sample calculated according to (3), that is, $U_{mi} \rightarrow U_{mi}$ + RND · dU. For example, if sample value was U_{mi} = 1, dU set to 10 ppm and RND = -0,46 , the final sample value considered to be the result of real sampling would be $1\text{-}0{,}46\cdot 10^{-6}$ = 0,9999954. We believe that this is trustworthy way for introducing random errors in computer simulation and it enables us to easily force our method to deal with samples that are surely much worse than those that can be expected in real high accuracy laboratory measurements (we simply change dU to, let us say, 50 ppm).

Precise explanation of our simulation demands two more details: shifting of the spectrum and filtration. Shifting of the spectrum is performed by complex multiplication, typically executed by Matlab command similar to

```
leftsmp=samples.*conj(exp(j*ω0*(0:N))),
```

where `samples` is vector of original samples, ω_0 frequency of the fundamental wave and N total number of samples. Filtering is lowpass, because periodic (circular) nature of discrete frequency domain makes passband filtration after shifting of the spectrum senseless [3], or at least extremely hard for implementation. For purpose of TDSA, frequency response of the filter in passband is not crucial, because all imperfections can be compensated for. This means that we

[1] For this method to be applicable and computer simulation valid, sampling device must have A/D converter that, in some manner, integrates AC input signal (measures signal's mean value during integration interval).

[2] This computer simulation models sampling by HP3458A DMM or similar device, and therefore we deal with the mirror signal. However, TDSA is generally applicable algorithm that can highly accurate calculate the spectrum of any sequence of samples, regardless of how they originated (i.e. regardless of whether Δt_m, T_o and T_d exist in the samples' values or not).

[3] According to [4], we need two samplings with $T_{m1} \neq T_{m2}$, so we have two sets of mirror parameters. In real measurements, it is generally better to calculate original parameters from mirror ones obtained with longer T_m [4], but in simulation, both sets of parameters will be equally accurate. For convenience, we are nevertheless going to evaluate accuracy of mirror parameters obtained with longer T_m.

[4] Accuracy of digital instruments (in our application, sampling devices) is usually specified assuming uniform distribution of errors inside certain boundaries.

can use several filter types and it is matter of personal inclination of surveyor which one is to be used. We prefer FIR equiripple filter and we calculate its coefficients by Matlab command "remez". Specifically, in this simulation we execute[1]

```
filt=remez(160,[0 5 45 500]/500,
            [1 1 0 0]),
```

and the resulting filter magnitude response is shown in the Figure 1.

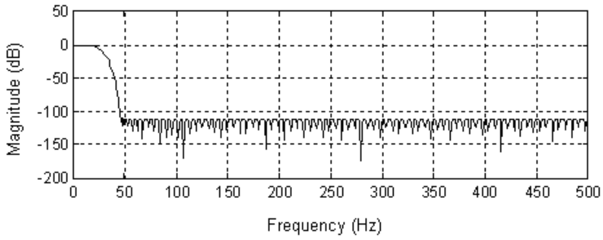

Figure 1: Magnitude response of the FIR filter used for extraction of samples of a particular component.

This should by enough information for everyone to repeat our results, so let us finally test the FS-TDSA. We shall calculate the spectrum of a signal composed of nine sine waves and DC level with parameters given in Table 1 (units of amplitude are omitted as irrelevant, but values are supposed to be in Volts).

Table 1: Structure of the original (sampled) signal (A_k , φ_k) and its mirror signal (A_{mk} , φ_{mk}) for T_{m2}.

k	A_k	φ_k / rad	A_{mk}	φ_{mk} / rad
0	0,3	-	0,3000000	-
1	$220\sqrt{2}$	0	264,1356301	0,9748677
2	A_3 / 2	-0,1	0,7412670	1,8497353
3	A_1 / 100	1,4	0,2290321	-1,9585823
4	A_5 / 2	2,1	0,0548441	2,8578780
5	A_1 / 500	-2,3	0,1259887	-0,5672543
6	A_7 / 2	-1,1	0,0111831	1,6076133
7	A_1 / 1000	0,2	0,0234755	0,7408883
8	A_9 / 2	-0,3	0,0099583	1,2157560
9	A_1 / 2000	1,6	0,0107439	-2,1925617

Original fundamental frequency will be f_1 = 50 Hz, sampling frequency f_S = 1000 Hz, relative error df of ratio f_1/f_S will be 0,05, Δt_m = 0,001 (irrelevant), T_o = T_d = 0 (irrelevant), T_{m1}/T = 0,13 , T_{m2}/T = 0,31 (for details about T_{m1} and T_{m2} see [4]), number of samples per period of the fundamental wave n = 20, number of fundamental periods that will be sampled np = 32 and dU = 20 ppm. With such sampling parameters, the mirror signal for T_{m2} will be as given in the two rightmost columns of Table 1. These are all much worse parameters (conditions) than we have in a real sampling, especially df and dU, but as we shall see shortly, they will not affect accuracy of FS-TDSA. Our spectral components are 50 Hz apart from each other, and Figure 1 shows that both neighbouring waves to the one that we want to extract, will be damped by about million times. That means we may count on pretty clean samples for SWC, i.e., we can expect SWC to work with its nominal accuracy. According to the sampling parameters, the distance between two consecutive spectral components will be np = 32 times larger than half width of the main lobe of window spectrum [3]. Although rectangular window has relatively large side lobes, large distance should sufficiently reduce their influence on the neighbouring components. Now we have everything we need for application of FS-TDSA procedure and typical result[2] we obtain with these parameters is given in the Table 2.

Table 2: Typical result of FS-TDSA (δ denotes relative error).

k	A_{mk}	δA_{mk}	φ_{mk} / rad	$\delta\varphi_{mk}$
0	0,2999419	$-1{,}94\cdot10^{-4}$	-	-
1	264,1364193	$2{,}99\cdot10^{-6}$	0,9748676	$-9{,}09\cdot10^{-8}$
2	0,7410944	$-2{,}33\cdot10^{-4}$	1,8506620	$5{,}01\cdot10^{-4}$
3	0,2290050	$-1{,}18\cdot10^{-4}$	-1,9561528	$-1{,}24\cdot10^{-3}$
4	0,0551882	$6{,}27\cdot10^{-3}$	2,8540824	$-1{,}33\cdot10^{-3}$
5	0,1255242	$-3{,}69\cdot10^{-3}$	-0,5711781	$6{,}92\cdot10^{-3}$
6	0,0116486	$4{,}16\cdot10^{-2}$	1,5922486	$-9{,}56\cdot10^{-3}$
7	0,0237349	$1{,}10\cdot10^{-2}$	0,7456783	$6{,}47\cdot10^{-3}$
8	0,0099257	$-3{,}27\cdot10^{-3}$	1,2695848	$4{,}43\cdot10^{-2}$
9	0,0149018	$3{,}87\cdot10^{-1}$	-2,1824539	$-4{,}61\cdot10^{-1}$

Error of the fundamental wave parameters[3] is hardly 3 ppm, and accuracy of all (except ninth) harmonics[4] is of the order of magnitude of few percents or less. Please recall that every sample could have maximal random error of 20 ppm! The largest inaccuracy of the ninth harmonic is consequence of its very low amplitude in relation to the amplitude of the fundamental wave. Their ratio A_{m9} / A_{m1} = $4{,}07\cdot10^{-5}$ is very close to the calculation noise level when performing many calculations, as for instance during shifting of the spectrum. This is obvious if we notice the growing trend of errors as we advance to the higher harmonics, which have lower and lower amplitudes.

For suspicious readers, who still do not believe in TDSA, let us make another experiment. We shall set all harmonics (amplitudes) to be 1% of the fundamental wave (very polluted signal), to make their amplitudes considerably larger then the contributions of the remaining leakage after filtration and the calculation noise (phases and all other parameters will be the same as in the previous simulation). Simulation results are given in the Table 3. Notice firstly

[1] We use Matlab 5.3 and reader should consult Matlab Signal Processing Toolbox Manual for details about algorithm and syntax of the function "remez". Briefly, 160 is the filter order, 0 to 5 is filter's passband, 45 to 500 filter's stopband (5 to 45 transition or "don't care" band), and last four parameters (1,1,0,0) are filter's gains at boundary frequencies.

[2] Strictly speaking, every execution of the program will yield different result, because each time we use different set of random numbers for simulation of random samples' errors. Of course, variations are very small and noticable mostly in the values of harmonics, and not of the fundamental wave. Exactly the same result can be repeated only by setting the seed of the random number generator to the same value each time we start the program.

[3] The phase accuracy of the fundamental wave was 0,1 ppm for this simulation run, but normaly the accuracy of the phase of the fundamental wave is the same as for its amplitude.

[4] In all tables, last (tenth) harmonic is omitted, because its frequency is theoretically exactly half of the sampling frequency and therefore it can not be reliably represented by its samples.

that the accuracy of the fundamental wave is practically the same! In all conventional methods, that large harmonics would greatly reduce the accuracy of calculation of the fundamental wave parameters, especially in conjunction with random samples' errors of maximum 20 ppm and large inaccuracy of the sampling frequency.

Table 3: Typical result of FS-TDSA when amplitudes of all harmonics are 1% of the fundamental wave (δ denotes relative error).

K	A_{mk}	δA_{mk}	φ_{mk} / rad	$\delta\varphi_{mk}$
0	0,3000547	$1{,}82\cdot10^{-4}$	-	-
1	264,1365521	$3{,}49\cdot10^{-6}$	0,9748674	$-2{,}99\cdot10^{-7}$
2	1,4823662	$-1{,}13\cdot10^{-4}$	1,8497502	$8{,}03\cdot10^{-6}$
3	0,2292008	$7{,}37\cdot10^{-4}$	-1,9579864	$-3{,}04\cdot10^{-4}$
4	0,5480477	$-7{,}17\cdot10^{-4}$	2,8579255	$1{,}66\cdot10^{-5}$
5	0,6293845	$-8{,}88\cdot10^{-4}$	-0,5678909	$1{,}12\cdot10^{-3}$
6	0,2238463	$8{,}27\cdot10^{-4}$	1,6082017	$3{,}66\cdot10^{-4}$
7	0,2347181	$-1{,}57\cdot10^{-4}$	0,7394904	$-1{,}89\cdot10^{-3}$
8	0,4009014	$6{,}45\cdot10^{-3}$	1,2171594	$1{,}15\cdot10^{-3}$
9	0,2141563	$-3{,}36\cdot10^{-3}$	-2,1918549	$-3{,}22\cdot10^{-4}$

Comparing errors of harmonics in Table 2 and Table 3, we can see that errors in the second experiment are generally lower by one order of magnitude, and that they are mutually very similar. This proves our assumption about causes of relatively large errors of ninth harmonic parameters. Also, pay attention to the errors of phases. In both experiments these were practically the same (or better to say, equally low) as the errors of amplitudes. That is another property of TDSA, which is not common to the other methods and can be of great importance in certain applications.

Finally, to satisfy one's curiosity, let us push things to the extreme. We shall set all signal components to be of the same amplitudes A = 100 (phases and all other parameters will be the same as before), just to examine how TDSA can serve us when we need to measure a signal that is composed of equally large and important components, or when we need to measure one particular signal component not larger than any other one in the spectrum. For illustration of these extremely inconvenient conditions for spectral analysis, we provide a waveform of the mirror signal (samples) as shown in the Figure 2.

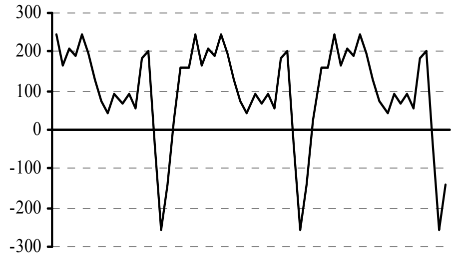

Figure 2: Waveform (three periods) of the mirror signal when all original components are of the same amplitude A = 100. Result of FS-TDSA spectral analysis of this signal is in Table 4.

Again, the simulation results demonstrate the high accuracy offered by the FS-TDSA algorithm, even for such inconvenient conditions. Due to the limited space, we provide only relative errors of parameters of the mirror signal for longer Tm (Table 4).

Table 4: Result of FS-TDSA analysis of the signal shown in the Figure 2 (relative errors of mirror parameters - longer T_m).

k	0	1	2	3	4
δA_{mk}	$-2{,}8\cdot10^{-7}$	$3{,}15\cdot10^{-6}$	$-4{,}8\cdot10^{-6}$	$-7{,}6\cdot10^{-6}$	$8{,}96\cdot10^{-7}$
$\delta\varphi_{mk}$	-	$-2{,}9\cdot10^{-6}$	$1{,}72\cdot10^{-6}$	$-8{,}7\cdot10^{-6}$	$7{,}54\cdot10^{-6}$

Table 4: continuation

k	5	6	7	8	9
δA_{mk}	$-6{,}8\cdot10^{-5}$	$1{,}82\cdot10^{-6}$	$-5\cdot10^{-5}$	$-2\cdot10^{-4}$	$-3{,}4\cdot10^{-6}$
$\delta\varphi_{mk}$	$-1{,}6\cdot10^{-5}$	$1{,}2\cdot10^{-5}$	$1{,}54\cdot10^{-5}$	$4{,}75\cdot10^{-8}$	$3\cdot10^{-5}$

4. RESULTS OF REAL LABORATORY MEASUREMENT

To eliminate the last doubt in TDSA accuracy, we tried to measure the spectrum of a calibrator output signal. All measurements were done in Primary Electromagnetic Laboratory (PEL) of the Faculty of Electrical Engineering and Computing in Zagreb. As the signal source we used HP 5700A AC calibrator, and sampling device was HP3458A Digital Multimeter. Unfortunately, results for all higher harmonics were very unreliable, because they scattered randomly out of any control. However, this can not be understood as TDSA failure, but rather as consequence of measurement system imperfection. Calibrator output signal is so pure sinusoid that all harmonics are under detection level (calculation noise). Even if harmonics in the source signal had been repeatable, measurement (sampling) errors would have expressed themselves as random fluctuations in the spectrum, low enough that they can not affect fundamental wave, but too large for measurement of low amplitude higher harmonics. As we can not generate a signal composed of adjustable and time invariant harmonics, we can not test TDSA completely.

As a partial prove of TDSA accuracy we provide results obtained for fundamental wave in two experiments. In the first one we have just measured calibrator output nine times successively (Figure 3). Calibrator output voltage was set to 7 V_{rms}, HP3458A was set on DC mode, range 10 V, and integration intervals were the same as in the previous computer simulations.

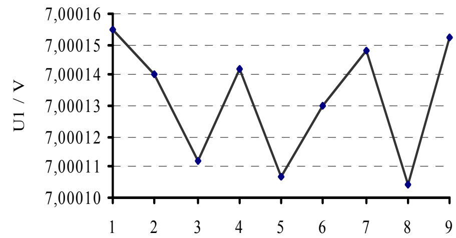

Figure 3: Result of nine successive measurements of HP 5700A calibrator output voltage (fundamental wave rms value) applying FS-TDSA.

Mean value of these results is U_{rms} = 7,000132 V, difference between maximal and minimal value is 51 μV or 7,29 ppm relatively to U_{rms}, difference $U_{max} - U_{rms}$ = 23 μV (3,29 ppm), difference $U_{min} - U_{rms}$ = -28 μV (-4 ppm) and standard deviation is 19,9 μV or relatively 2,84 ppm. As expected, repeatability of these values is very good, and is in supplement to our belief that TDSA is highly accurate method. As the second argument to this statement we provide Table 5. It contains result of another experiment in which we examined TDSA sensitivity (resolution) when measuring fundamental wave amplitude (rms value). With the same settings as in the first experiment, we measured calibrator output five times successfully (column one). Then we changed calibrator voltage setting to 7,00007 V or +10 ppm relatively (column two). Finally, we changed calibrator voltage to +20 ppm (7,00014 V - third column). If TDSA measured accurately (with high resolution), at least we might have expected in such experiment was good tracking among measurement results and the measured voltage.

Table 5: Examination of TDSA resolution when measuring the fundamental wave amplitude (rms value).

	U_{ref} / V	+10 ppm	+20 ppm
	7,000256	7,000170	7,000250
	7,000251	7,000275	7,000204
	7,000082	7,000344	7,000356
	7,000151	7,000225	7,000507
	7,000186	7,000250	7,000264
U_{rms} / V	7,0001852	7,0002528	7,0003162

The mean value of the estimated voltage is given in the last row of the Table 5 averaged across all five measurements for each of the three input signal voltages. Reference voltage is the mean value of the first column U_{ref}. After we changed voltage to 10 ppm larger value, the mean value of the measurement results (second column) was 9,7 ppm larger. After change to +20 ppm, the mean measured value changed to +18,7 ppm (third column). Obviously, TDSA followed measured voltage very reliably, so we may conclude that it measured with very high resolution. Slight deviations can be consequence of the calibrator setting error too, so we can not ascribe them to TDSA itself without experiments with better equipment.

5. CONCLUSION

We believe that presented simulations and last real measurements proved high accuracy of TDSA. In the normal circumstances, accuracy of parameters of the fundamental wave will be about few ppm, and of parameters of higher harmonics about few hundred ppm (few percent, if their amplitudes are very low in comparison to the amplitude of the fundamental wave). Accuracy probably can be even better (but not much) if we apply some other window rather than the rectangular one to the sample sequence. That is something we are going to examine in the near future.

However, there are certain shortages regarding TDSA, and as the first one, we can mention the requirement for relatively long sampling interval and short-term (periodical) time-invariant signal. As explained previously in the text, TDSA works in conjunction with SWC, which requires certain minimum number of signal samples in order to achieve its nominal accuracy. Samples forwarded to SWC are product of relatively high order filtration, and it is well known that the first and the last M samples (M is the filter order) are useless after convolution, at least for SWC which is very sensitive to the waveform of the signal. That is why we must ensure noticeable redundancy in the total number of samples, causing longer sampling intervals. For reliable estimations of the 50 Hz fundamental wave, the signal must be stationary for around 30 s. As the second objection we can stress practically impossible hardware implementation of SWC, i.e. TDSA, what prevents its usage in high speed applications. Although mostly not important in laboratory measurements, that might be crucial for more commercial applications.

Spectral analysis will probably remain an open question and challenge for the science, but we believe that TDSA, especially its frequency shifting version (FS-TDSA), satisfies most of the present needs and enables new advances in various fields of science and applications where accurate spectral analysis is required. Its simplicity and straightforward algorithm makes it suitable for a wide range of usage, and provides several benefits. Perhaps the most significant one, in comparison to all conventional methods, is the equal accuracy of amplitudes and phases of signal components, what enables surveyor deep insight in the signal properties and hidden information that he would probably miss applying some other measurement method. That is why we propose TDSA as a new solution for the old problem.

6. REFERENCES

[1] V.K.Jain, W.L.Colling, D.C.Davis: High-Accuracy Analog Measurements via Interpolated FFT, *IEEE Trans. on Instr. and Meas.*, Vol. IM-28, No. 2, June 1979

[2] A.V.Oppenheim, A.S.Willsky, S.H.Nawab: Signals & Systems, *Prentice Hall*, 1996

[3] A.V.Oppenheim, R.W.Schafer, J.R.Buck: Discrete-Time Signal Processing, *Prentice Hall*, 1998

[4] N.Hlupic: A sampling method for high accuracy AC measurement, IMEKO TC-4 International Symposium, Zagreb 2002, Vol. I, p. 55-59

[5] N.Hlupic, J.Butorac : A sampling method for high accuracy phase difference measurement, IMEKO 2001, Lisbon, Vol. I, p. 60-63

[6] N.Hlupic : Compensation of integration interval inaccuracy in sampling methods for AC measurements, IMEKO 2001, Lisbon,Vol. II, p. 365-367

[7] N.Hlupic, J.Butorac, M.Kresic : Improved Frequency Measurement by means of DMM Applying a Sampling Method, (unpublished – internal work)

(1) Ph.D. N. Hlupic, Dept. of Electrical Engineering Fundamentals and Measurement, Faculty of Elecrical Engineering and Computing, Unska 3, Zagreb 10000, Croatia, tel. +385-1-6129-650, fax. +385-1-6129-616, E-mail: nikica.hlupic@fer.hr

(2) Ass. Prof. D. Petrinovic, Ph.D., Dept. of Electronic Systems and Signal Processing, Faculty of Elecrical Engineering and Computing, Unska 3, Zagreb 10000, Croatia, tel. +385-1-6129-968, fax. +385-1-6129-652, E-mail: davor.petrinovic@fer.hr

(3) Full Prof. J. Butorac, Ph.D., Dept. of Electrical Engineering Fundamentals and Measurement, Faculty of Elecrical Engineering and Computing, Unska 3, Zagreb 10000, Croatia, tel. +385-1-6129-632, fax. +385-1-6129-616, E-mail: josip.butorac@fer.hr