

# ERROR LIMITS AND MEASUREMENT UNCERTAINTY IN LEGAL METROLOGY

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*Abstract: Quality statements for calibrated measuring instruments give the systematic error and the uncertainty of measurement at the moment of calibration. In contrast to this, the statement of conformity for verified measuring instruments indicates whether or not the error limits on verification specified by law are exceeded. The conformity assessment is based on the result of a previous calibration. The calibration uncertainty thus becomes an uncertainty of conformity decision. There is, therefore, a certain risk for the manufacturer that acceptable instruments might be rejected. The probability of this is calculated from the uncertainty of measurement and the spread of the errors of measurement of the measuring instrument population to be tested. For the instrument user the maximum permissible error in service applies. There is, therefore, practically no risk, in the sense that no measured value – even allowing for the measurement uncertainty – will be outside this tolerance band.*

*Keywords: Legal metrology, error limits, uncertainty, verification, conformity, risk*

## 1 INTRODUCTION

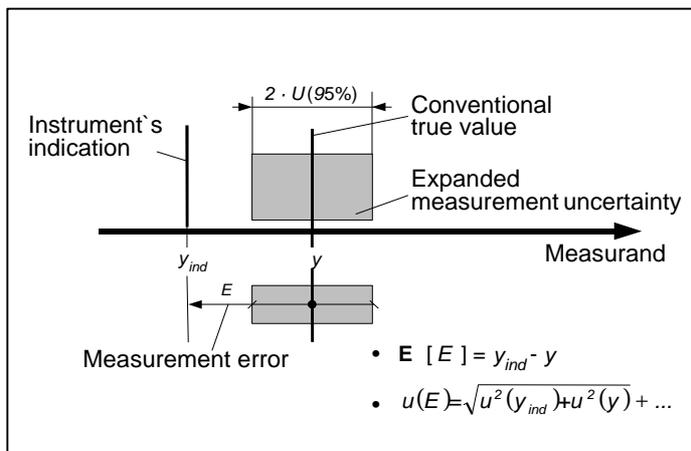
The accuracy of measuring instruments must be in keeping with the intended use. In compliance with the ISO 9000 and EN 45000 series of standards, traceability of measuring and test equipment to SI units must be guaranteed by an unbroken chain of comparison measurements to allow statements on their metrological quality to be made. The most important measures for correct measurement and measuring instruments are:

- **in industrial metrology:** regular calibration of the measuring instruments according to the implemented quality systems
- **in legal metrology:** periodic verification or conformity testing of the measuring instruments that are subject of legal regulations.

## 2 MEASURING INSTRUMENTS IN INDUSTRIAL METROLOGY

In industrial metrology calibrated instruments are normally used for measurements and tests which influence product quality. Quality statements for calibrated measuring instruments give both, the relationship between values of a quantity indicated by the instrument and the corresponding values realised by standards,

and the respective measurement uncertainties. Fig. 1 shows how a (single) calibration result is typically presented. The uncertainty of measurement is determined in compliance with unified rules [1]. In metrology it must be stated for a coverage probability of 95 %. Its value, together with the calibration result, is valid for the moment of calibration and the relevant calibration conditions. If a recently calibrated measuring instrument is used under the same conditions as during the calibration, the calibration uncertainty  $u_{cal}$  enters into the total uncertainty of measurement  $u_{meas}$  as an (independent) contribution:



**Fig. 1:** Typical presentation of a (single) calibration result

rement  $u_{meas}$  as an (independent) contribution:

$$u_{\text{meas}}^2 = u_{\text{cal}}^2 + \sum_i u_i^2 \quad (1)$$

where  $u_i$  are the other contributions to  $u_{\text{meas}}$ .

When the calibrated instrument is used in a different environment the measurement uncertainty determined by the calibration laboratory will often be exceeded if the instrument is susceptible to environmental influences. A problem can also arise if the instrument's performance is degraded after prolonged use. The user of the calibrated object must, therefore, consider all of these problems on the basis of his technical knowledge.

### 3 MEASURING INSTRUMENTS IN LEGAL METROLOGY

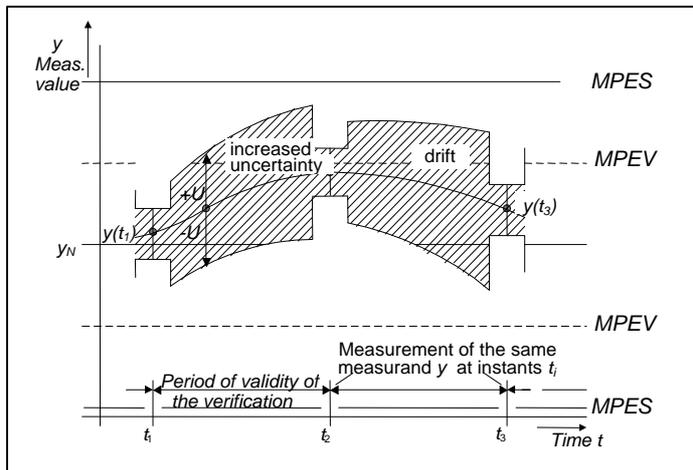
#### 3.1 Conformity verification

Conformity verification of measuring instruments is a special method of testing covered by legal regulations. The initial elements of verification are:

- a qualitative test, which is effectively an inspection,
- a quantitative test which is almost the same as a calibration.

The results of these two tests are then evaluated to ensure that the legal requirements are being met (see sections 3.2 and 3.3). Provided that this assessment of compliance leads to the instrument being accepted, a verification mark should be fixed to it and a verification certificate may be issued [2].

#### 3.2 Maximum permissible errors on verification and in service



**Fig. 2:** Verified measuring instruments: consideration of long-term drift and external influences by two different error limits [2] (MPEV -maximum permissible error on verification, MPES -maximum permissible error in service)

In many economies with developed legal metrology systems, two kinds of error limits have been defined, i.e. the maximum permissible error on verification and the maximum permissible error in service, which normally is twice the maximum permissible error on verification. Here the maximum permissible error on verification equals a "maximum permissible error on testing" which is valid only at the time of verification. For the user of the measuring instrument, the maximum permissible error in service is the error which is legally relevant.

Fig. 2 explains this approach. During the period in which the verification is valid, the indicated value given by an instrument, may drift. In addition, the measurement uncertainty will usually increase due to the actual operating conditions, and other influences. In par-

ticular, the following influences must be considered:

- measurement uncertainty from the metrological test during verification,
- operation conditions and external interference during normal operation,
- long-term behaviour, drifting, ageing, and durability.

The maximum permissible error on verification may, therefore, be exceeded here. However, the requirements regarding the maximum permissible error will in general be met. As a result, verification implies a high probability that under normal conditions of use the instrument will give results within the tolerances of the maximum permissible error in service during the entire period that the verification is valid.

#### 3.3 Assessment of compliance

In practice, measuring instruments are considered to be in compliance with the legal regulations

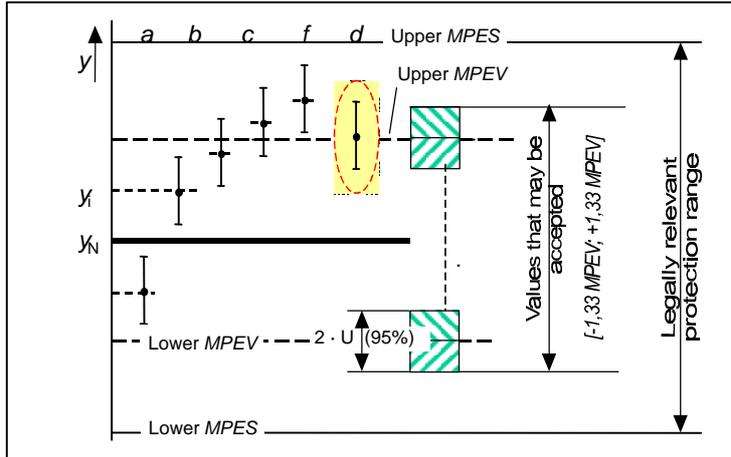
- if the indicated value is smaller than or equal to the maximum permissible error on verification when the test is performed by a verification body under unified test conditions and
- if the uncertainty of measurement of the previous calibration at the 95% probability level is small compared with the prescribed error limit.

In today's legal metrology, the measurement uncertainty is usually considered to be small enough if the so-called "one-third uncertainty budget" is not exceeded:

$$U(95\%) \leq 1/3 \cdot MPEV \tag{2}$$

where *MPEV* is the maximum permissible error on verification.

Consequently, this measurement uncertainty becomes an uncertainty of the conformity decision [3]. The criteria for the assessment of compliance are illustrated in Fig. 3: compliance with the requirements of the verification regulations is given in cases *a*, *b*, *c* and *d*. Cases *e* and *f* will result in rejection, although all the values, including the uncertainty of measurement, lie within the tolerances fixed by the maximum permissible errors in service.



Consequently, the maximum permissible error on verification of a newly verified measuring instrument will in the worst case be exceeded by 33%. However, as the legally prescribed maximum permissible errors in service are valid for the instrument users, there is therefore no risk in the sense that no measured value - even if the measurement uncertainty is taken into account - will be outside this tolerance band.

**Fig. 3:** Illustration of the criteria for the assessment of compliance and of the range of possibly accepted intrinsic measurement errors. (*MPEV* - maximum permissible error on verification, *PES* - maximum permissible error in service)

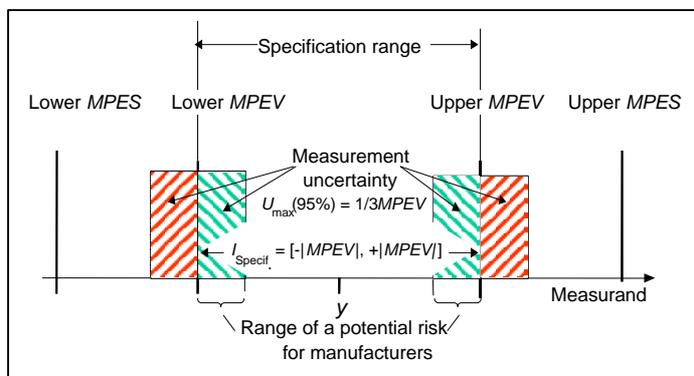
### 3.4 Manufacturer's risk

When performing conformity assessments of measurement results to meet legal requirements, the uncertainty of measurement becomes an uncertainty of decision.

This leads to two effects which each have a definable probability,

- A small percentage of instruments which are actually acceptable, fail the verification process.
- A small number of instruments which despite having (true) intrinsic measurement errors which exceed the maximum permissible errors on verification, (*MPEV*), pass verification.

Fig. 4 illustrates the ranges of values concerned.



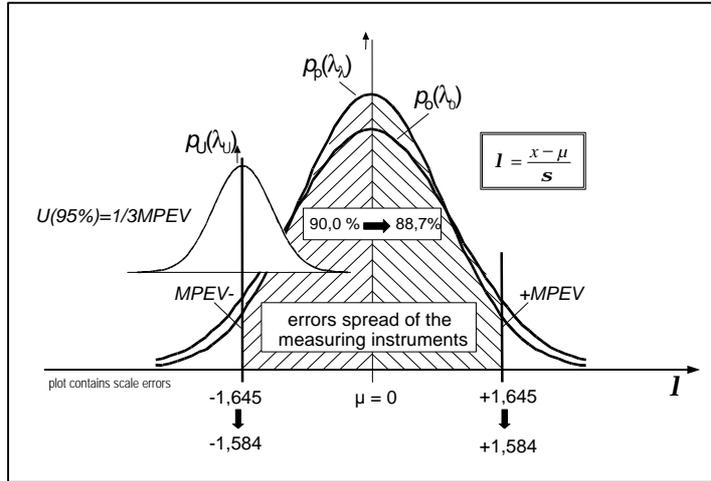
**Fig. 4** Illustration of the ranges of the uncertainty of conformity decision.

Depending on the error spread of the instrument population to be tested, the first effect can be expected to happen more often than the second. In view of the large number of instruments to be verified, this involves an economic risk for manufacturers, since it might appear that too many instruments fail the legal requirements. It is the combination of these two effects which leads to the risk of a financial loss for the manufacturer. To obtain a reasonable estimate of the first effect, the (unknown) real error spread of the respective instrument population can be compared

with the observed output scatter of the verification process. The probability distribution to be expected for verified instruments,  $p_o(x)$ , can be calculated by convolution of the actual distribution of the instrument population to be tested,  $p_p(x)$ , and the measuring process,  $p_u(x)$ :

$$p_o(x) = p_p(x) * p_u(x) = \int p_p(z) \circ p_u(x - z) dz \tag{3}$$

Assume that both input probability plots have a Gaussian distribution and the above equation applies. Calculation shows that for a population of instruments for which the theoretical "true" acceptance rate should be 95%, the actual acceptance rate will be 93.7%. This means that less than 1,5% of instruments have been "unfairly" rejected due to the influence of the measurement uncertainty. Fig 5 illustrates this change in the probability distribution. Consequently, the legal verification system involves only a low risk that good quality products may suffer unwarranted rejection at verification.



**Fig. 5** Illustration of the change in the probability distribution due to the influence of the measurement uncertainty.

### 3.5 Uncertainty corresponding to the specified error limits

In practice, it is often necessary to determine the uncertainty of measurements which are carried out by means of verified instruments. If only the positive statement of compliance with the legal provisions is known, e.g. in the case of verified instruments without certificate, the uncertainty of these measurements can only be derived from the available information about the prescribed error limits (*on verification* and *in service* respectively) as well as about the related uncertainty budgets according to equation (2). On the assumption that the systematic intrinsic errors of verified measuring instruments are predominant compared with the uncertainties obtained in the metrological test, the range of values between the acceptance limits on verification may be assumed to be a range of equiprobable values. The probability of indications of verified instruments which, due to the impact of the uncertainty, are actually beyond the acceptance limits declines in proportion to the increase in distance from there limits. Therefore, the following treatment, based on the German legal metrology system, seems to be justified:

- Immediately after verification, a trapezoidal probability distribution of the occurring errors according to plot (a) of Fig. 6 is taken as a basis for the determination of the uncertainty contribution of the instrument. The following may therefore, be assumed for the standard uncertainty contribution.

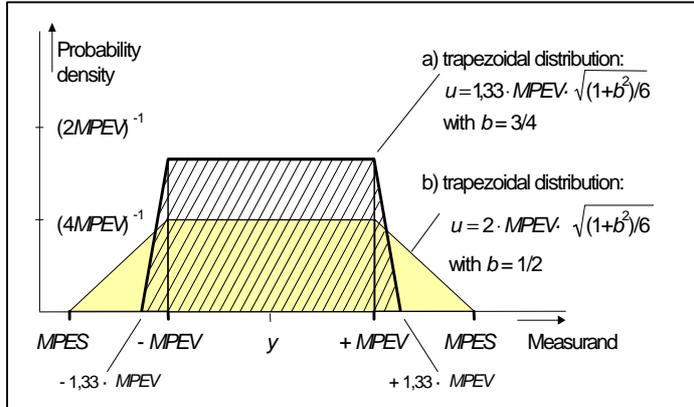
$$u_{Instr} = a \cdot \sqrt{(1 + b^2)} / 6 \approx 0,70 \cdot |MPEV| \tag{4}$$

where  $a = 1,33 \cdot |MPEV|$   
 $b = 3/4$

- After prolonged use and under varying environmental conditions, it must be supposed that, in the worst case, the measurement errors extended by the measurement uncertainty reach the values of the maximum permissible errors in service. As a result the trapezoidal distribution is represented by plot (b) of Fig. 6. In this case, the following may be assumed for the standard uncertainty contribution:

$$u_{\text{Instr}} = a \cdot \sqrt{(1 + b^2)/6} \approx 0,90 \cdot |MPEV|. \quad (5)$$

where  $a = 2 \cdot |MPEV|$   
 $b = 1/2$



**Fig. 6** Estimate of an uncertainty of verified instruments corresponding to the specified error limits

#### 4 SYSTEM COMPARISON

Table 1 shows a comparison between verification and calibration, which is partially based on *Volk-mann* [5]. In conclusion, it can be said that if the measuring instrument is used as intended, verification offers complete, basic assurance of correct measurement. The user need have no expert knowledge. It allows a reliable prediction to be made of the validity period of the next verification. The verification system is a strong tool in both legal metrology and quality assurance, especially when large numbers of measuring instruments are concerned.

In comparison to this, calibration forms an open and variable system, that is practically unlimited as far as the measurement task is concerned. A calibration certificate gives a statement of the instrument's quality at the moment of calibration. It requires sound expert knowledge on the part of the user to carry out and evaluate the measurements. In particular calibration is considered to be particularly suitable in scientific and industrial metrology.

Table 1: System comparison of verification and calibration

Characteristics	Verification	Calibration
<b>Bases</b>	<ul style="list-style-type: none"> <li>legal provisions</li> </ul>	<ul style="list-style-type: none"> <li>techn. rules, normatives, demands of customers</li> </ul>
<b>Objective</b>	<ul style="list-style-type: none"> <li>guarantee of indications within MPES during the validity period</li> </ul>	<ul style="list-style-type: none"> <li>relation between indication and conventional true value (at nominated accuracy level)</li> </ul>
<b>Prerequisite</b>	<ul style="list-style-type: none"> <li>admissibility for verification, above all stability (type approval)</li> </ul>	<ul style="list-style-type: none"> <li>instrument should be calibrateable</li> </ul>
<b>Validity of the results</b>	<ul style="list-style-type: none"> <li>within the period fixed for subsequent verification (as regards MPES)</li> </ul>	<ul style="list-style-type: none"> <li>at the moment of calibration and under specific calibration conditions</li> </ul>
<b>Evaluation of the results</b>	<ul style="list-style-type: none"> <li>by the verifying body</li> </ul>	<ul style="list-style-type: none"> <li>by the user of the measuring instruments</li> </ul>
<b>Traceability</b>	<ul style="list-style-type: none"> <li>regulated by the procedure</li> </ul>	<ul style="list-style-type: none"> <li>calibration laboratory to provide proof</li> </ul>

(MPES: maximum permissible error in service)

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