

# THE FAULT LOCATION ALGORITHM BASED ON TWO CIRCUIT FUNCTIONS

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*The paper presents an algorithm for detection and location single parametric faults in analogue electronic circuits based on making use of input-output measurements. It utilises two circuit functions (for instance: voltage transmittance  $K_U$  and input opened admittance  $Y_{in}$ ) measured on the same frequency for the fault location. Each of circuit functions is regarded as bilinear transformation. When we put together these transformations into a three-dimensional space (for instance:  $Re(K_U)$ ,  $Im(K_U)$ ,  $|Y_{in}|$ ) it is wrested a family of curves representing the changes of respective elements' values. This composition causes increasing selectivity of the fault location, because the curves do not cut themselves and furthermore they are more separated, than it has place in classical applications of bilinear transformation. The algorithm consists of two parts. First part is aimed at determination of the optimum measuring frequency for a simultaneous measurement of two circuit functions. Second part achieves the fault location. It was chosen 3-order low-pass Butterworth Filter to verify the algorithm.*

*Keywords: Bilinear transformation, Detection single parametric faults*

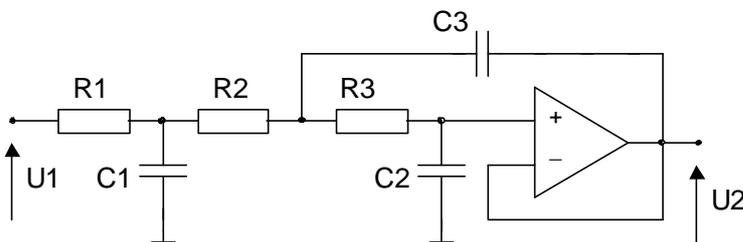
## 1 INTRODUCTION

For many electronic circuits as well as electric models representing technical, physicochemical and biological objects, there is no possibility of access to the interior of circuit. Therefore diagnostic based on input-output measurements has growing importance. One of the methods for this diagnostic is bilinear transformation [1][2]. It is used for location of the parametric faults in not very complex two-, and four terminal networks RC.

Geometric interpretation of bilinear transformation  $F(p)$ , where  $p$ - element's value, is complex plane, on which we draw up family of arcs representing changes of individual elements' values (Fig. 2). The location of the single fault consists in measuring of function  $F(p)$  and plotting the measurement point on plane [1]. If the measurement point lies of one of these arcs, than the fault element is the one, for which curve has been made [1][3].

From Fig. 2 showing the family of arcs drawn up for 3-order low-pass Butterworth Filter (Fig.1) we see, that the curves are near each other, and in some places they are criss-crossed, making

impossible in this way unambiguous location of the single fault. Additionally, location of the fault element will be more difficult, because measurements are burdened with errors, which make, that the measurement result does not necessary need to lie on the curve representing the fault element (Fig. 2).



**Figure 1.** 3-Order Low-Pass Butterworth Filter under investigation (DUT)  
, where:  $R1=R2=R3=10k\Omega$ ,  $C1=4.3nF$ ,  $C2=3nF$ ,  $C3=10nF$ .

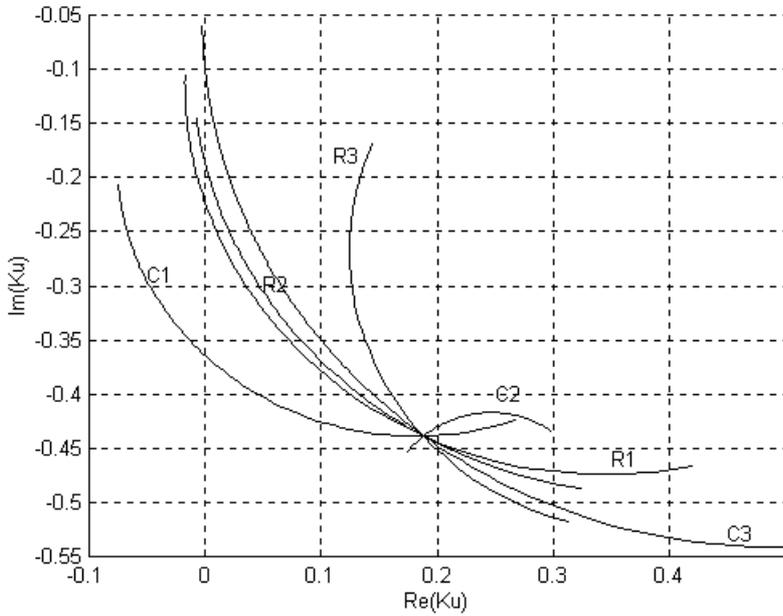
In this article we propose an algorithm that solves discussed problem. In our considerations we assume following presumptions:

- Diagnosed circuit is linear.
- There can be only one parametric fault in circuit.
- We know circuit typology and nominal values of all elements.

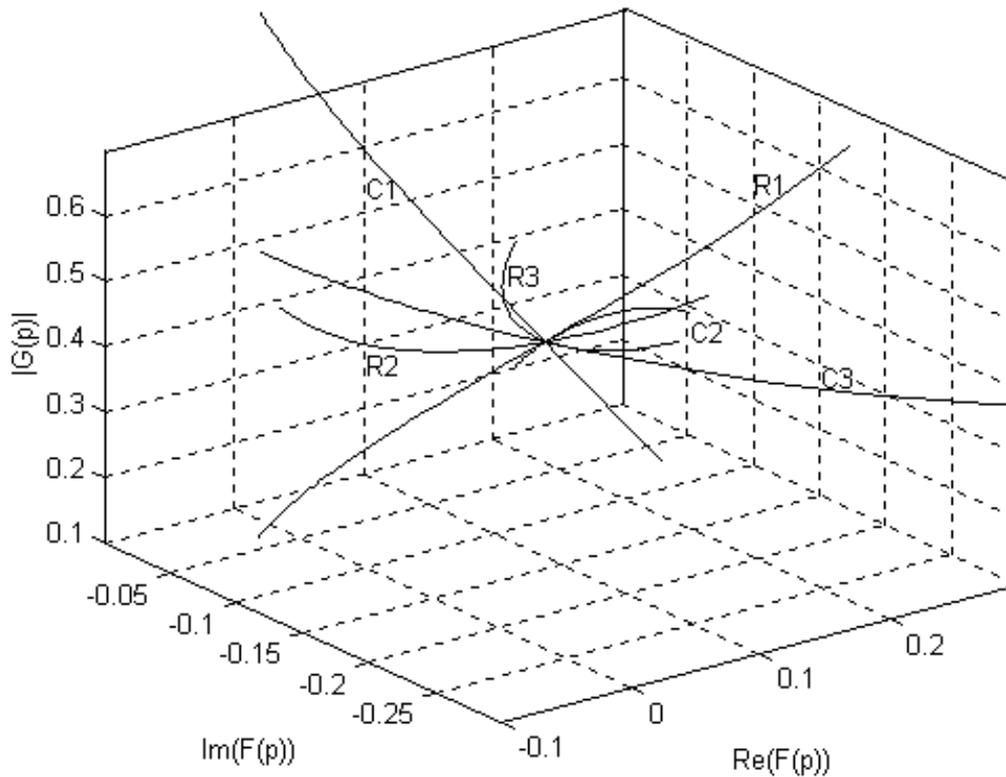
- We make test at only one measurement frequency.
- There is a measurement error of circuit functions parameters.

The works that have been made so far, analysed problem on complex plane, using for it measurement of one circuit function [1][2][3][4]. In this article we propose simultaneous measurement

of two circuit functions:  $F(p)$  - voltage transmittance and  $G(p)$  - input admittance. Using both circuit functions, we take modelling of changes of elements' values from plane to three-dimensional space. Picture 3 shows in 3D space the family of three-dimensional curves representing changes of elements' values of filters presented on picture 1. As we can see on picture, we result with lack of criss-crossing of curves representing changes of elements' values and increase of distance between them. It has made the fault location more unambiguous (selective) what will be presented in chapter 3. Analyse was made with program Matlab.



**Figure 2.** The graphic representation of the bilinear transformation  $F(p)$  - the arc families.



**Figure 3.** The graphic representation of the composition of two circuit functions  $F(p)$  and  $G(p)$  - the curve families.

## 2. AN ALGORITHM FOR DETECTION OF THE SINGLE FAULTS

Location of the single parametric fault is based on the following thesis: The fault element is the one for which three-dimensional curve is the nearest to the measurement point.

The algorithm is composed with two parts. The task for first pre-testing component is to determine optimal frequency with which there are the best circumstances for simultaneous measurement of two circuit functions and detection of the single faults. After choosing proper frequency, there is database generated containing value of optimal measurement frequency and co-ordinates of the nominal point. Second component of the algorithm localises the fault element by choosing curve in space, which is the nearest (according to determined criteria) to the measurement point.

In three-dimensional space each curve can be described by parametric equation. For analysed case form of dimensional curve corresponding with changes of value of  $i$ -th element is given analytically in the following way:

$$\begin{cases} x_i = \text{Re}(F_i(p_i)) \\ y_i = \text{Im}(F_i(p_i)) \\ z_i = |G_i(p_i)| \end{cases} \quad (1)$$

where:  $i=1, \dots, N$ ,  $N$  - number of elements in circuit (For DUT  $N=6$ ),  $F$  - the transmittance voltage,  $G$  - the input admittance of circuit

Both algorithms pre-testing and testing use proposed description of curve.

### 2.1 Determining of optimal measurement frequency

The main pre-tested problem is determining optimal measurement frequency with which take place sufficient sensitivity of both circuit functions in dependence on all elements of tested circuit. For determining of optimal measurement frequency we assume criteria described in [4]. Because it has concerned only case on plane, we have introduced changes connected with transition to three-dimensional space. The algorithm is following. On the beginning we set starting frequency, frequency step  $f_{step}$  and number of steps  $S$ . Next, for each frequency  $f=f_{step} \cdot s$ , where  $s=1, \dots, S$  we make the following actions:

- Calculation of co-ordinates of the nominal point  $(x_{nom}, y_{nom}, z_{nom})$ , where all curves (described by equation (1)) representing changes of each elements' values criss-cross.
- Calculation of co-ordinates of the finals of these curves  $(x_{1i}, y_{1i}, z_{1i}) (x_{2i}, y_{2i}, z_{2i})$ , where  $i=1, \dots, N$ ,
- Determining of a distance between the nominal point and each of the finals of curves:  $d_i$  - distance from the first,  $d_{i+N}$  - distance of the second final of three-dimensional curve from the nominal point

$$d_i = \sqrt{(x_{1i} - x_{nom})^2 + (y_{1i} - y_{nom})^2 + (z_{1i} - z_{nom})^2} \quad (2)$$

$$d_{i+N} = \sqrt{(x_{2i} - x_{nom})^2 + (y_{2i} - y_{nom})^2 + (z_{2i} - z_{nom})^2} \quad (3)$$

- Determining of deviation coefficient length between curves  $\acute{a}(s)$  [4].

Next, we determine  $\acute{a}_{min}$  as minimal value among calculated set of  $\acute{a}$ :  $\mathbf{a}_{min} = \min_s \{\mathbf{a}(s)\}$ .

On this bases we can define optimal measurement frequency for which will be made measurements of circuit functions, while testing of the circuit. For tested circuit it amounts to  $f_{opt}=1700\text{Hz}$

### 2.2 Location of the single fault in the circuit

The aim of algorithm is location of the single faults in the circuit. An algorithm is looking for curve, which is the nearest to the measurement point plotted in three-dimensional space.

An algorithm is composed of the following steps:

1. From the pre-testing algorithm we take the optimal measurement frequency  $f_{opt}$  (for the tested circuit  $f_{opt}=1700\text{Hz}$ ) and co-ordinates of the nominal point  $(x_{nom}, y_{nom}, z_{nom})$
2. We determine set of points, represented by three vectors of co-ordinates  $x_i, y_i, z_i$  for  $i$ -th curve.

3. We calculate minimal distance of the measurement point  $P_m$  with co-ordinates  $(x_m, y_m, z_m)$  from  $i$ -th curve called a coefficient of nearness  $\hat{a}(i)$ .
4. Steps 2 and 4 are made for  $N$  curves.
5. From the set of calculated coefficients of nearness  $\{\hat{a}(i)\}$ ,  $i=1, \dots, N$  we determine minimal value  $\hat{a}_{min}$ . On the bases of it we determine which curve is the nearest to the measurement point, and in this way, which element is fault.

The key element in this algorithm is step 3. From the earlier calculations each curve is represented by three vectors of co-ordinates  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{z}$ . We can also describe each of them by equation (1). It is theoretically possible to determine each equation describing dependence on change of each co-ordinate given curve, by bilinear transformation. But the forms of result functions are very complex (the level of complexity rise with amount of elements), and besides this, each function we should determine analytically for each element separately. There are 18 equations for tested circuit. Second, very important disadvantage of this approach is lack of universality. The determined set of function fits only to one circuit typology, therefore proposed algorithm would be not universal.

We decided to use algorithm searching for minimal distance of the measurement point from given curve - described below. The points representing the curve are placed with uneven density. Therefore for increasing precision of determining coefficient of nearness  $\hat{a}$ , we decided to use parabolic interpolation, made for three of curve's points which are the nearest to the measurement point, and next we determine minimal distance between curve (4) and the measurement point:

1. We search for a point  $P_j(x_j, y_j, z_j)$  from the set of co-ordinates  $\mathbf{x}$ ,  $\mathbf{y}$ ,  $\mathbf{z}$  representing given curve, the point, which is the nearest to the measurement point  $P_p$  and two points  $P_k(x_k, y_k, z_k)$  i  $P_l(x_l, y_l, z_l)$  surrounding point  $P_j$ , where point  $P_k$  is nearer to the measurement point  $P_p$  than point  $P_l$ , where  $k, l = j \pm 1$ .
2. We make parabolic interpolation for range  $(P_j, P_k)$  on the bases of points  $P_j$ ,  $P_k$ ,  $P_l$  and we result with the following description of curve [5]:

$$\begin{cases} x = x \\ y = Ax^2 + Bx + C \\ z = Dx^2 + Ex + F \end{cases} \quad (4)$$

This approximation is sufficient, because from the property of testing algorithm follows, that for location of faults it is enough, that the measurement point is nearer to given curve than other curves, and moreover, the shape of curves is similar to parabolic (Fig. 3).

3. Next, we determine absolute error  $\Delta d$  that would be made by impedance meter HP 4192A at measurement of the value corresponding with point  $P_j$  (in verification of this method use of the meter HP 4192A is planned).
4. On the bases of  $\Delta d$  we determine amount of points approximate curve (4), lying between  $P_j$  a  $P_k$ , according to formula:

$$m = \frac{\sqrt{(x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2}}{\Delta d} \cdot M \quad (5)$$

where:  $M=10$  – coefficient of decreasing approximation error in relation to the measurement error.

5. At the last stage on the bases of (4) we determine co-ordinates of points lying between  $P_j$  a  $P_k$  and distances between these points and the measurement point. Next, from the set of calculated distances we find minimal value, which is coefficient of nearness  $\hat{a}$  for given curve.

### 3 AN INFLUENCE OF ADDITIONAL DIMENSION ON QUALITY OF THE SINGLE FAULTS LOCATION

By connection of two bilinear transformation  $F(p)$  and  $G(p)$  in one, represented by three-dimensional space (Fig. 3) we get increased distance between curves representing changes of individual elements' values (curves are more separated each from other). In this way we achieve fact, that the location of single faults in circuit is more unambiguous. Additionally, we eliminate possible criss-crossing of curves (Fig. 2), what make unambiguous determining of the fault element impossible.

Here we introduce the proof that by described method we achieve fact, that location of the single faults is more unambiguous than in methods based on bilinear transformation [1][3] analysing only on plane.

1. Fact, that location of the single faults is more unambiguous is connected with increased distance between curves (between any points on any two curves).
2. Let be given two bilinear functions:

$$F_i(p_i) = \frac{A_i p_i + B_i}{C_i p_i + D_i} \quad (6a)$$

$$G_i(p_i) = \frac{E_i p_i + F_i}{H_i p_i + K_i} \quad (6b)$$

where:  $A_i, B_i, C_i, D_i, E_i, F_i, H_i, K_i$  – complex coefficients for functions made for  $i$ -th element, where  $i=1, \dots, N, N$  – number of elements in circuit.

In the same way we can describe function for  $j$ -th element, where  $j \neq i, j=1, \dots, N$ .

3. Because each from complex coefficients for functions drawn up for  $i$ -th and  $j$ -th element has the following form:

$$\begin{aligned} A_i &= A_i(p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_N) \\ A_j &= A_j(p_1, \dots, p_{j-1}, p_{j+1}, \dots, p_N) \end{aligned} \quad \text{where: } j \neq i \quad (7)$$

that  $A_i \neq A_j, B_i \neq B_j$  etc.

From that, we get function in form:

$$F_i(p_i) \neq F_j(p_j) \quad (8)$$

and

$$G_i(p_i) \neq G_j(p_j) \quad (9)$$

4. We can write parametric equations for curves on plane, representing changes of value of  $i$ -th element and  $j$ -th element in the following way: [4]

$$\begin{cases} x_i = \text{Re}(F_i(p_i)) \\ y_i = \text{Im}(F_i(p_i)) \end{cases} \quad (10a)$$

$$\begin{cases} x_j = \text{Re}(F_j(p_j)) \\ y_j = \text{Im}(F_j(p_j)) \end{cases} \quad (10b)$$

From this, we can define square of distance between any point lying on curve drawn up for  $i$ -th element, and any point lying on curve drawn up for  $j$ -th element:

$$l^2 = (x_i - x_j)^2 + (y_i - y_j)^2 \quad (11)$$

5. In 3D space we get following parametric equation of curves representing  $i$ -th and  $j$ -th element:

$$\begin{cases} x_i = \text{Re}(F_i(p_i)) \\ y_i = \text{Im}(F_i(p_i)) \\ z_i = |G_i(p_i)| \end{cases} \quad (12a)$$

$$\begin{cases} x_j = \text{Re}(F_j(p_j)) \\ y_j = \text{Im}(F_j(p_j)) \\ z_j = |G_j(p_j)| \end{cases} \quad (12b)$$

From this we can write square of distance between two any points lying on individual curves representing  $i$ -th and  $j$ -th element in this way:

$$d^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \quad (13)$$

6. Using formula: (11) and (13) we can write in form:

$$d^2 = l^2 + (z_i - z_j)^2 \quad (14)$$

Because from the formula (9) and (12) we can write, that  $z_i \neq z_j$  so from this:

$$(z_i - z_j)^2 \geq 0 \quad (15)$$

An in this way from (14) and (15) follows, that:

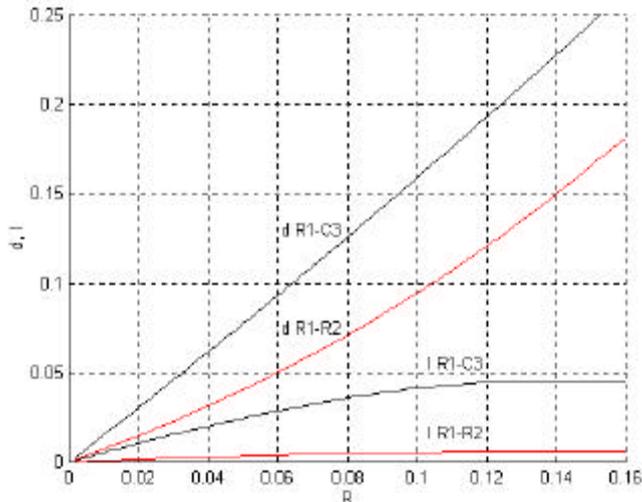
$$d^2 \geq l^2 \quad (16)$$

So distance between the same points in three-dimensional space is always larger than their distance in two-dimensional space. These distances are equal only for the nominal point, what means lack of faults in circuit, and in this case amounts to 0.

#### 4 CONCLUSION

Presented algorithm for location of the single faults in tested circuit DUT, based on simultaneous measurement of two circuit functions solves the problem of location of faults in case, when the measurement point does not lie on any of curves. It is characterised by simplicity of work and in this way speed of work and is easily implemented in programming environment. Moreover, introduction of 3D dimension makes location of single faults in circuit more unambiguous (even a few dozen times (Fig. 4)). It is the next step [4] for implementing presented method in practice, because all

measurements are burden with errors and the elements in tested circuits have tolerance. Of course described algorithm will be developed further, because it does not still concern all possible cases that can occur while testing circuits, for example: when all elements are in limits of tolerance. Introduced algorithm will be base for methods and algorithms for detection of the single faults with measurements at one frequency. The aim of these methods is to be implemented in algorithms for multifrequency measurements making possible detection any amount of faults in tested circuits.



**Figure 4.** Distance between points lying on curves R1 and R2 as well as R1 and C3, and equally distant from the nominal point, in function of distance from the nominal point R. Where: d - distance in three-dimensional space, l - distance in plane

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