XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

ON MEASUREMENTS VIA SHAPE DESIGNED SIGNALS

R. Zielonko
Chair of Electronic Measurement
Faculty of Electronics, Telecommunications and Informatics
Technical University of Gdansk, 80-952 Gdansk, Poland

Abstract: Paper presents two classes of unconventional measurement signals with special, designed shapes, in context of their application for the identification measurements. Firstly, measurement idea, choosing proper observables and signal shape design principle are explained. In the next part of the paper mirror signals (power and Poisson) and polynomial signals (Chebyshev, Legendre) are presented. An example illustrating practical applications of polynomial signals is included.

Keywords: Identification measurements, shape designed signals

1 INTRODUCTION

Recently, became commercially available arbitrary waveform generators, which can generate signals of different shapes. Thus, emerges the possibility of application of non-conventional-shape signals to measurement. In TU Gdañsk metrological properties of several classes of special-shape signals (power, Walsh, polynomial, complementary) have been investigated. Some results, with particular consideration of shape design principles, are published in [1], [2]. There was shown, that shape-designed signal enables to transfer the main effort of measurement from processing of output signal of the object under measure to the synthesis of an input stimulating signal of proper shape.

In this paper, two classes of non-conventional signals are presented in application to linear object identification via input-output measurements. The identification of technical objects is important problem in investigation of object dynamics [3], system modelling and fault diagnosis of analog circuits.

In the first part, the measurement idea and signal shape design principle are explained. In the next part, there are presented mirror signals, designed with the aid of mirror reflection principle and relative polynomial signals (Chebyshev, Legendre) with application example. The phenomena of compensation of high amplitude power signals inside low amplitude polynomial signal is emphasized.

2 MEASUREMENT IDEA

The idea of object identification via input-output measurements with application of shape designed signals is explained in Fig. 1. The object under measure (OUM) is stimulated by the sequence of special-shape input signals $u_i(t)$. Some features of the output signals, so-called observables y_i, are measured. We denote the vector of observables by $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$ and the vector of identified parameters of the measured object model by $\mathbf{p} = (p_1, p_2, \ldots, p_n)^T$. The parameters p_i can describe the identified model (as transfer-function coefficients) or can be values of internal elements of diagnosed circuit or system.

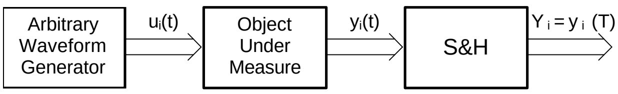

Figure 1. Illustration of measurement idea.

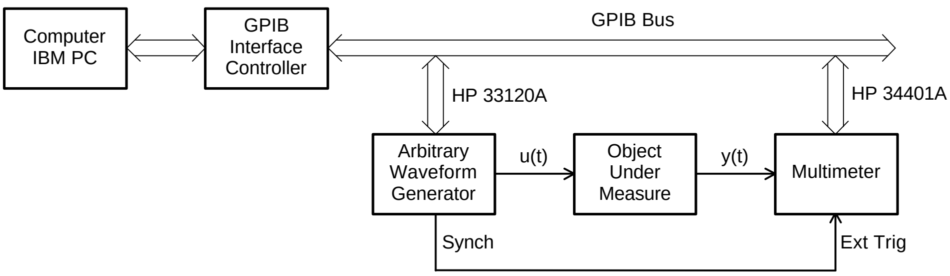

Figure 2. Architecture of the measuring system with using of shape designed signals.

The attraction of this idea is fact, that it is possible choosing such input signals and observables that:

- measuring procedure is simple; in the most cases it needs only one sample from an output signal,
- relations between observables and identified parameters are simple and have unique solution.

The architecture of the measuring system used for practical realisation above idea and for investigation all classes of unconventional signals presented below, is shown in Fig. 2.

The system has been organized from commercially available instrumentation: the arbitrary waveform generator HP 33120A and the multimeter HP 34401A. It is controlled by PC with the aid of GPIB bus. Power programmable unit for supply of an active object under measure is also provided. Arbitrary waveform signal consists 12 bit D/A converter and can perform up to 16000 samples. The system is not expensive and can be realized in typical equipped laboratory.

3 CHOOSING OBSERWABLES AND SIGNAL SHAPE DESIGH PRINCIPLE

The most popular input-output description of object models is the transfer function $K(s)$. For lumped, time-invariant, linear, dynamic system $K(s)$ can be written in form of the rational function:

$$K(s) = \frac{a_o + a_1 s + a_2 s^2 + ... + a_l s^l}{1 + b_1 s + b_2 s^2 + ... b_m s^m} \text{, which can be expanded into a Taylor series } \quad K(s) = \sum_{i=0}^{\infty} k_i s^i \ . \qquad (1)$$

Coefficients of transfer function $K(s)$ are related to Taylor coefficients k_i by a set of linear equations:

$$k_i = a_i - \sum_{j=1}^{i} k_{i-j} b_j \ , \quad i = 0,1,...,l; \qquad k_{l+i} = -\sum_{j=1}^{m} k_{i-j+1} b_j \ , \quad i = 1,2,...,m \ . \qquad (2)$$

Coefficients a_i, b_j; can be obtained from k_i by solving the above set of equations or, more efficiently, by using the Pade approximation recurrent algorithm of Baker [4].

If we choose as observables so-called moments of impulse response $k(t)$ of the measured object

$$m_i = \int_0^{\infty} t^i k(t) dt, \quad i = 0,1,2,\dots \text{, which are strictly related to Taylor coefficients } m_i = (-1)^i i! k_i \ , \qquad (3)$$

we can achieve the both mentioned above advantages of considered approach: simple measuring procedure and simple relations between observables and identified parameters.

3.1 Mirror kernel reflection method of signal shape design

For designing sequence of input signals we can use so called *mirror kernel reflection design principle* formulated by author [5]. This principle is useful, when the OUM can be represented by a set of linear functionals:

$$P_i = \int_0^{\infty} f_i(t) k(t) dt, \qquad i = 0,1,2,...,n \qquad (4)$$

where $f_i(t)$ is a kernel function, $k(t)$ - object impulse response. The approximants $P_i(T)$ of functionals (4)

$$P_i(T) = \int_0^{T} f_i(t) k(t) dt, \quad i = 0,1,2,...,n \ , \qquad (5)$$

may easily be measured by the one sample method shown in Fig.1, if the input signals $u_i(t)$ are *mirror reflections of the kernel functions*

$$u_i(t) = f_i(T - t) 1(t) \ , \quad i=0,1,2,...,n, \qquad (6)$$

where $1(t)$ is an unit step function. The above can be proved by using convolution integral

$$y_i(t) = \int_0^{t} u_i(t - t) k(t) dt = \int_0^{t} f_i(T - (t - t)) k(t) dt \ . \qquad (7)$$

A sample of the output signal at a time instant $t = T$

$$y_i(T) = \int_0^{T} f_i(T - T + t) k(t) dt = \int_0^{T} f_i(t) k(t) dt = P_i(T) \ , \qquad (8)$$

is equal the desired approximant of the functional (4).

It is easily seen from Fig. 1 and (8), that the measuring procedure is simple. It can be reduced to taking out only of a single sample from the output signal at the time instant $t = T$.

4 MIRROR SIGNALS

Signals designed with the aid of the mirror kernel reflection principle we shall name the mirror signals. Below two classes of mirror signals are presented.

4.1 Mirror power signals

From comparison of (3) and (4), it is evident, that moments m_i are some particular cases of functionals (4) with the kernel functions $f_i(t) = t^i$, $i = 0,1,2,...$, (9)

Therefore, according to the mirror kernel reflection principle, aproximants of the moments

$$m_i(T) = \int_0^T t^i k(t) dt \ , \quad i = 0,1,2,... \ , \tag{10}$$

can be measured with using mirror power signals $u_i(t) = (T - t)^i 1(t)$, $i = 0,1,2,...$ (11)

The signal (11) is mirror reflection of function $f_i(t)=t^i$ against the symmetrical axis running through the middle of interval [0,T]. For limiting dynamic range of input signals, more convenient to measure are normalized moments $m_i = \int_0^T (t/T)^i k(t) dt$, by normalized mirror power signals $u_i(t) = \left(1 - \frac{t}{T}\right)^i$. (12)

4.2 Mirror Poisson signals

Alternative representation of a linear object can be set of modified moments of $k(t)$

$$M_i(a) = \int_a^\infty t^i e^{-at} k(t) dt \ , \quad i = 0,1,2,... \ , \tag{13}$$

whose are uniquely related with the coefficients $k_i(a)$ of the Taylor series expansion of the transfer function $K(s)$ at a general point $s = a$, by the relation

$$k_i(a) = \frac{(-1)^i}{i!} M_i(a) \ , \quad i = 0,1,2,... \tag{14}$$

It is evident, that modified moments $M_i(a)$ are particular cases of the functionals (4) with the kernel

$$f_i(t) = t^i e^{-at} \ , \quad i = 0,1,2,... \tag{15}$$

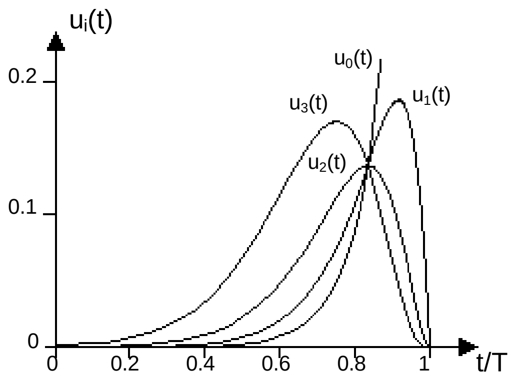

Figure 3. Examples of the mirror Poisson signals for a=2.

named Poisson functions. Therefore, according to the mirror kernel reflection principle approximants of the modified moments

$$M_i(a,T) = \int_0^T t^i e^{-at} k(t) dt \ , \tag{16}$$

can be measured by the signals

$$u_i(t) = (T - t)^i e^{-a(T-t)} 1(t) \ , \ i=0,1,2,..., \tag{17}$$

being mirror reflections of the Poisson functions (15).

In Fig. 3, there are shown shapes of some mirror Poisson signals for a=2. Although these signals have complicated shapes, actually there are no technical problems with their generation, with the help of arbitrary waveform generators.

5 POLYNOMIAL SIGNALS

In measuring high order moments via mirror power signals come some difficulties as consequence of high level of input signal in case of ordinary moments $m_i(T)$ or existence in Eq. (12) normalizing coefficient $1/T^i$ in case of normalized moments. It results in decrease of measured signal $y_i(T)$ for high i and in consequence increase of measuring errors due to the noise. In order to increase the level of output signals one can increase the level of input signals but only within bounds of linearity of the object under measurement.

We can overcome above difficulties replacing power or mirror power signal of i-th degree by a polynomial signal of the same degree, which is in fact a weighed sum of power signals. If the weights have different signs, then respective power signals will partly compensate each other in period [0,T].

Thus a total oscillation of polynomial signal in above period can be reasonable even if individual power signals have high amplitudes. We can observe the phenomena of compression of high-amplitude power signals in low-amplitude polynomial signal.

It can be shown that moment m_n is a linear combination of polynomial observables (samples of y_i responses to a given sequence of polynomial signals). Very convenient in this respect appear Chebyshev polynomials, because they have the greatest leading coefficient (coefficient of highest power) from among all polynomials having equal amplitudes in a fixed period.

5.1 Chebyshev polynomial signals

Normalized Chebyshev polynomials of the first kind are defined on the interval [0,1] by the formulas: $C_0\left(\frac{t}{T}\right)=1,\quad C_n\left(\frac{t}{T}\right)=\sum_{i=0}^{n} w_{ni}\left(\frac{t}{T}\right)^i,\quad w_{ni}=(-1)^{n-i}4^i\frac{n}{n+1}\binom{n+1}{n-1}$ (18)

The explicit formulas of first few Chebyshev polynomials (shapes in Fig. 4) are given below:

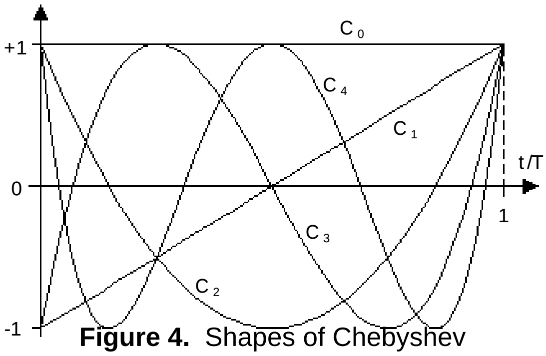

Figure 4. Shapes of Chebyshev polynomials of the first kind.

$$C_0=\left(\frac{t}{T}\right)=1,\quad C_1\left(\frac{t}{T}\right)=2\left(\frac{t}{T}\right)-1,$$
$$C_2\left(\frac{t}{T}\right)=8\left(\frac{t}{T}\right)^2-8\left(\frac{t}{T}\right)+1, \tag{19}$$
$$C_3\left(\frac{t}{T}\right)=32\left(\frac{t}{T}\right)^3-48\left(\frac{t}{T}\right)^2+18\left(\frac{t}{T}\right)-1.$$

Chebyshev polynomials are symmetrical or antisymmetrical on interval [0,1], so they are self-mirror $C_n\left(1-\frac{t}{T}\right)=(-1)^n C_n\left(\frac{t}{T}\right),$ (20)

It is possible to express the power function $(t/T)^n$ by combination of Chebyshev polynomials

$$\left(\frac{t}{T}\right)^n=\sum_{i=0}^{n} h_{ni}C_i\left(\frac{t}{T}\right), \tag{21}$$

where $\sum_{i=0}^{n} h_{ni}=1,$ and $\sum_{i=0}^{n}(-1)^n h_{ni}=0$. (22)

The first few expressions in explicit form are given below

$$1=C_0,\quad \frac{t}{T}=(C_1+C_0)/8,\quad \left(\frac{t}{T}\right)^2=(C_2+4C_1-3C_0)/8,\quad \left(\frac{t}{T}\right)^3=(C_3+6C_2+15C_1+10C_0)/32. \tag{23}$$

Sample of the response to Chebyshev stimulus $C_n(t/T)$ we denote

$$y_n\overset{\text{df}}{=}y_n(T)=\int_0^T C_n\left(1-\frac{t}{T}\right)k(t)\mathrm{d}t \tag{24}$$

and name Chebyshev observable. From (22) it follows

$$y_n=\int_0^T(-1)^n C_n\left(\frac{t}{T}\right)k(t)dt. \tag{25}$$

By substituting (20) to (25) and comparing with (12) we obtain

$$y_n=(-1)^n\sum_{i=0}^{n} w_{ni}\int_0^T\left(\frac{t}{T}\right)^i k(t)dt=(-1)^n\sum_{i=0}^{n} w_{ni}m_i \tag{26}$$

Therefore, between Chebyshev observables and normalized moments m_n are simple relations. We can also express moment m_n in terms of Chebyshev observables. From (12) and (23)

$$m_n=\sum_{i=0}^{n} h_{ni}\int_0^T C_i\left(\frac{t}{T}\right)k(t)dt=\sum_{i=0}^{n}(-1)^i h_{ni}y_i,\qquad n=0,1,\ldots \tag{27}$$

and in an explicit form:

$$m_0=y_0, m_1=(y_0-y_1)/2,\qquad m_2=(3y_0-4y_1+y_2)/8,$$
$$m_3=(10y_0-15y_1+6y_2-y_3)/32,\qquad m_4=(35y_0-56y_1+28y_2-8y_3+y_4)/128. \tag{28}$$

So, it is easy to calculate moments from Chebyshev observables. Due to the different signs of the weights in relations (27) between moments and Chebyshev observables and properties (22) compensation of systematic additive errors and reduction of random ones take place. The random error propagation from the set of observables to the set of moments can be described by the variance propagation index $r_n\overset{\text{df}}{=}\frac{s^2_{m_n}}{s^2_{y_n}}$. (29)

As all measurements are performed in the same measuring scheme (Fig. 1) and observables are taken as response signal samples $y_i(T)$ in quick succession, we may assume stationary random errors with equal variance σ_y^2 for each $y_i(T)$. From (27), (29) and (22) we have

$$r_n=\sum_{i=0}^{n} h^2_{ni}\le 1. \tag{30}$$

In the Table 1 are shown the error propagation indexes for different n of Chebyshev, Legendre and optimal polynomials. It is seen, that choosing Chebyshev signals leads to reduction of random component of error and partial compensation of systematic errors due to different signs of the weights.

Table 1. Error propagation indexes for Chebyshev, Legendre and optimal polynomials

n	2	4	6	8	10	Polynomials
r_n	0,41	0,32	0,27	0,24	0,22	Chebyshev
	0,39	0,29	0,24	0,21	0,19	Legendre
	0,33	0,20	0,14	0,11	0,09	Optimal

5.2 Application example

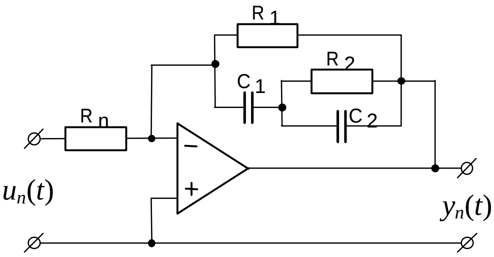

Figure 5. Circuit under test.

Chebyshev signals with amplitude 10V have been used for element identification of the circuit shown in Fig. 5. The transfer function of the circuit is

$$K(s) = \frac{a_0 + a_1 s}{1 + b_1 s + b_2 s^2} \text{, where:} \tag{31}$$

$$a_0 = R_1 / R_n,\ a_1 = (C_1 + C_2) R_1 R_2 / R_n,$$
$$b_1 = (C_1 + C_2) R_2 + C_1 R_1,\ b_2 = C_1 C_2 R_1 R_2.$$

By solving Eq. (2) and taking into account (3) we obtain explicit formulas for normalized moments in terms of transfer function coefficients and then comparing with the above relations we have

$$m_0 = UR_1 / R_n, \quad m_1 = UR_1^2 C_1 / R_n T, \quad m_2 = 2UC_1^2 (R_1 + R_2) R_1^2 / R_n T^2,$$
$$m_3 = 6U[C_1 (R_1 + R_2)^2 + C_2 R_2^2] C_1^2 R_1^2 / R_n T^3. \tag{32}$$

Equation (32) can be used for element values identification on the base Chebyshev observables. The identification procedure had the following steps: measuring Chebyshev observables, calculation moments, and then calculation components from the moments solving Eq. (32). Solution (32) can be obtained also in the explicit form.

For $R_1 = R_2 = R_n$ = 10 kΩ, $C_1 = C_2$ = 10 nF, T = 1 ms and Chebyshev signal amplitude U = 10 V in the system Fig. 2 we obtained measurement results (as the means of 3 measurements):
y_0=10,06 V; y_1=8,03 V; y_2=5,21 V; y_3=5,44 V
from which moments were calculated
m_0=10,06 V; m_1=1,015 V; m_2=0,409 V; m_3=0,181 V.
Component values were calculated from the moments, using (32)
R_1=10,06 kΩ; R_2=10,029 kΩ; C_1=10,03 nF; C_2=10,12 nF.
Error of identification is less then 1,2 %.

5.3 Legendre polynomial signals

It is possible using the other polynomials as measurement signals. Attractive in some aspects are Legendre polynomials defined on the interval [0,1] by formula

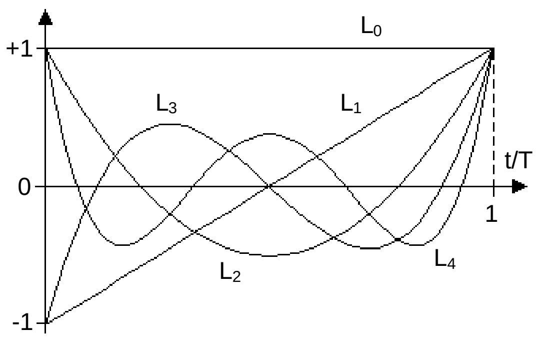

Figure 6. Shapes of Legendre polynomials.

$$L_n \overset{\text{df}}{=} \frac{1}{n!} \frac{d^n}{dx^n} [x(x-1)],\ n = 0,1,2,\ldots,\ x = \frac{t}{T} \tag{33}$$

The explicit forms of the first few normalized polynomials (shapes in Fig. 6) are:

$$L_0\left(\frac{t}{T}\right) = 1, \qquad L_1\left(\frac{t}{T}\right) = 2\left(\frac{t}{T}\right) - 1,$$
$$L_2\left(\frac{t}{T}\right) = 6\left(\frac{t}{T}\right)^2 - 6\left(\frac{t}{T}\right) + 1, \tag{34}$$
$$L_3\left(\frac{t}{T}\right) = 20\left(\frac{t}{T}\right)^3 - 30\left(\frac{t}{T}\right)^2 + 12\left(\frac{t}{T}\right) - 1,$$

They are self-mirror, then as previously:

$$\left(\frac{t}{T}\right)^n = \sum_{i=0}^{n} n_{ni} L_i\left(\frac{t}{T}\right), \tag{35}$$

where:

$$n_{ni} = \frac{(2i+1)(n!)^2}{(n-i)!(n+i+1)!}, \qquad \sum_{i=0}^{n} n_{ni} = 1, \qquad \sum_{i=0}^{n} (-1)^i n_{ni} = 0. \tag{36}$$

The first expresions in explicit form are

$$1 = L_0, \quad \frac{t}{T} = (L_1 + L_0)/2, \quad \left(\frac{t}{T}\right)^2 = (L_2 + 3L_1 + 2L_0)/6, \quad \left(\frac{t}{T}\right)^3 = (L_3 + 5L_2 + 9L_1 + 5L_0)/20\,. \tag{37}$$

The relations between Legendre observables y_i (samples of the response to Legendre stimulus) and moments m_i are similarly like (27) and in explicit form

$$\begin{aligned} &m_0 = y_0, \qquad m_1 = (y_0 - y_1)/2, \qquad m_2 = (2y_0 - 3y_1 + y_2)/6, \\ &m_3 = (5y_0 - 9y_1 + 5y_2 - y_3)/20, \qquad m_4 = (14y_0 - 28y_1 + 20y_2 - 7y_3 + y_4)/70. \end{aligned} \tag{38}$$

Similarly as in the Chebyshev polynomials in this case the weights in (38) have different signs and satisfy (36), then the compensation the systematic additive errors and reduction of random errors is achieved.

The random error propagation indexes calculated from (36) for different n are shown in Table 1 for comparison with corresponding values r_n for Chebyshev polynomials. Advantage is seen, which is noticeable for high n.

5.4 Optimal polynomial signals

It is possible using the other polynomials optimal designed as measurement signals. Parameter optimization design method [5] enables to design a family of optimal polynomial signals (Fig. 7)

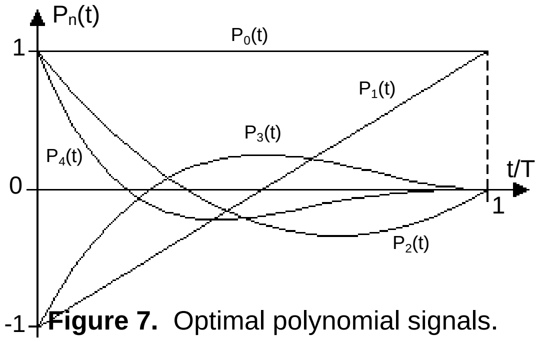

Figure 7. Optimal polynomial signals.

$$P_n(t) = \sum_{i=0}^{n} (-1)^{n-i} (i+1) \binom{n}{i} \left(\frac{t}{T}\right)^i, \; k = 0,1,2\ldots, \tag{39}$$

which minimize error propagation index.

The error propagation indexes of optimal polynomials for different n are included in the Table 1. Comparison the indexes for Chebyshev and Legendre polynomials and for optimal polynomials shows that these last assure significantly greater error reduction. Polynomial signals enable measuring higher order moments in comparison with mirror signals.

6 SUMMARY

Presented in this paper measurement idea and families of non-conventional measuring signals with designed shapes, as well as many other non-conventional signals investigated in TU Gdañsk (Walsh, complementary) represent the new approach to measurement and testing, when the main effort of a measurement procedure is transferred from processing of an output signal to the synthesis of an input signal of a proper shape. This approach is update because arbitrary waveform generators became commercially available and are standard equipment of each laboratory. Presented approach can lead sometimes to faster measuring and testing methods, one shot type. It is also alternative and complementary to the digital signal processing based measuring technique.

The interesting phenomena of compensations of high amplitude power signals inside low amplitude polynomial signal leads to reduction random errors and compensation of systematic additive errors.

REFERENCES

[1] Królikowski A., Zielonko R., Tlaga W., New diagnostic methods for analogue electronic circuits and systems based on input-output measurements, *Measurement* 4, 1984, pp. 175-179.
[2] Bartosiñski B., Zielonko R., New classes of complementary signals, *Electronics Letters* 9, 1987, pp. 433-434.
[3] ¯uchowski A., Symulacyjna metoda wyznaczania momentów charakterystyki impulsowej liniowych przetworników pomiarowych. *Sympozjum: Modelowanie i Symulacja Systemów Pomiarowych*, Krynica-Kraków 1992, pp. 9-15.
[4] Baker G. H., Essentials of Pade Approximants, *Academic Press*, New York 1975.
[5] Zielonko R., Some theoretical foundations of analog signal shape design for measurement and testing, , *Measurement* 17, 1996, pp. 29-37.

AUTHOR: Prof. Dr.Ing. Romuald ZIELONKO, Chair of Electronic Measurement, Faculty of Electronics, Telecommunications and Informatics, TU of Gdansk, Narutowicza 11/12, 80-952 Gdansk, Poland, Phone Int +0048 583472255, Fax Int +0048 583472255, E -mail: zielonko@pg.gda.pl