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APPROXIMATIONS IN MEASUREMENT-TODAY STILL NECESSARY?

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Abstract: Because of missing a-priori information, tolerances of the elements used in microelectronics and other circuits, nonlinearities and to check the results of models and calculations or programs especially in measurement estimation and approximation methods using the physical background are advantageous. Also in education these methods to get a first impression of the solutions of problems should be trained. In the paper a lot of typical examples will be treated as signal analysis and aliasing errors, linearisation, dynamic errors and optimal filtering including consequences for the selection of measuring systems.
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1 INTRODUCTION

Solution of problems of system sciences and especially of measurement is not possible without the knowledge of a-priori information of the input signal. On the other hand this signal is not known otherwise it would not to be measured. Therefore typical in measurement approximations and estimations to get for instance model-signals are very important.

In general these methods still today are very significant to get models or to check the results of computations e.g. to find errors in programming of computers by means of an estimation of the magnitude of the solution. In the period of the slide rule the engineer usually has to do this and students today are not sufficient educated in this direction.

In the paper typical and important examples will demonstrate the method as well as the way of thinking. It will be shown how to get results with significance for practice as for instance the selection of measuring systems or the estimation of measuring errors and possibilities to avoid or minimise these.

2 SIGNAL ANALYSIS AND ALIASING ERRORS

Since the error definition used in most cases because of the closed solutions is the mean-square error ε^2 the signal is to be described by the power density function $S(\omega)$. The most favourable approximation is

$$S(\omega) = 1 / [\, 1 + (\omega / \omega_o)^2] \qquad (1)$$

that means the signal is band-limited to ω_o. The advantage of especially this model-signal is that one gets closed solutions [1].

As an important example the aliasing errors using this approach may be calculated: Using the frequency domain the aliasing error ε_{al}^2 may be estimated to be the same as the cut-off error $\varepsilon_{cut\text{-}off}^2$ with the sampling frequency ω_s

$$\varepsilon_{al}^2 = \varepsilon_{cut\text{-}off}^2 = 2 \int S(\omega)\, d\omega = 2\, [\, \pi/2 - \arctan (\omega_s / 2\omega_o) \,] \qquad (2)$$

Finally it may be referred to the influence of the errors from the processing after sampling as given in details in [2].

3 LINEARISATION

If the input-output function $y = f(x)$ of a system is weak non-linear the first approximation is the tangent while the second approximation is the well-known method of describing functions [3]. It may be emphasised on the other hand that the linearisation of a transfer-function (step-answer function) $h(t)$ by a rampfunction - as done in the next chapter - may lead to difficulties because the system cant be realised and therefore the superposition law is not valid [4].

4 DYNAMIC ERRORS

To estimate dynamic errors instead of the real step answer a ramp function with the transient time T_T and as an input a pulse-shaped function with the pulse width ΔT is used as shown in figure 1. Now three typical cases may be distinguished:

a) $T_T = \Delta T$. Here the pulse is distorted but the pulse height is still o.k. (dashed curve).
b) $T_T > \Delta T$. This leads to an error of the pulse height (short dashes).
c) $T_T < \Delta T$. The pulse shape can be represented (chain curve).

The investigations show that a heavily prolonged trailing edge means a dynamic error - as especially practicients with a lot of experience know.
On the other hand it is possible to estimate the error using the relation of the flank sides of the output signal: The real height of the input signal H is - using the symbols of figure 1 - thus correcting the measured wrong height H*

$$H \cong H^* \, T_T / \Delta T \qquad (3)$$

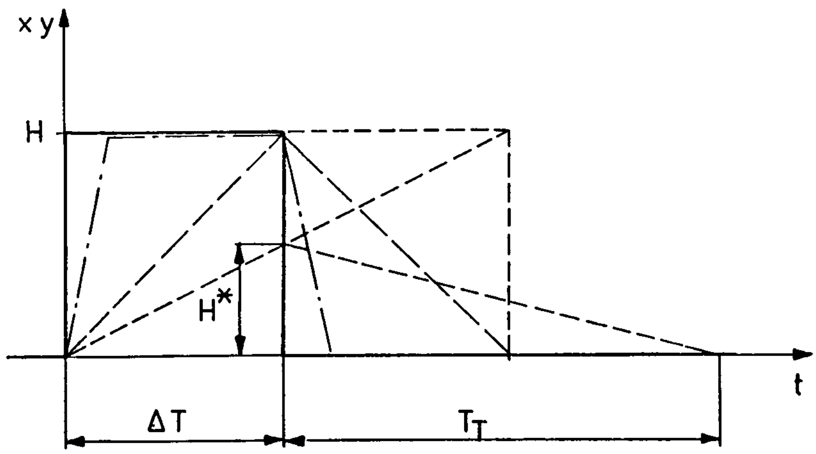

Figure 1. Estimation of dynamic errors

It may be emphasised that here the superposition law is not valid as mentioned before in chapter 3.
The next better approximation is a system of first order , that means the transient response to be an e-function. In this case the superposition law is fulfilled because the system can be realised.

5 OPTIMAL FILTERING

In reality noise appears. To get the best signal-to-noise ratio it is evident to reduce the output signal in those regions where the noise preponderates and vice versa. This is the idea of optimal filtering founded by *Wiener et al.* [1].
Here also an approximation may be treated leading to new results with respect to systems with optimal information flow [1;3]:

Let us assume the input signal to be band-limited due to Eq. (1) and to correct the total system a series-connected system - today realised by a computer - may be used. By means of this correction system the bandwidth is extended from f_o (original system) to f_c (corrected system) e.g. the degree of correction is

$$a = f_c / f_o \qquad (4)$$

As Figure 2 shows the dynamic error ρ^2 is decreasing and the noise error P_z is increasing with increasing degree of correction a. The sum of both - the total error ε^2 - is running through a minimum, corresponding to the optimal filter. It may be emphasised that one has to pay for the advantage of error reduction by increasing parameter sensitivity [1].

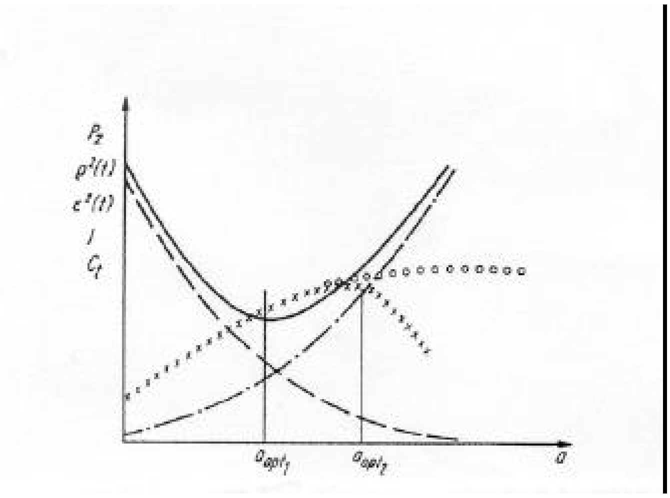

Figure 2. Errors and information flow as a function of the degree of correction
- - - dynamic error; -.-.- noise error; -------- total error
xxxxxxx information flow; oooo channel capacity

Last no least let us deal with the problem of optimal information flow: *Shannon* [5] introduced the channel capacity C_t with the noise power P_z and the signal power P_x.

$$C_t = f_c \ lb (P_x / P_z + 1) \qquad (5)$$

Using instead of the signal-to-noise ratio P_x / P_z the signal-to-error ratio or the inverse relative error we get the information flow I as presented also in Figure 2. Because of multiplying with f_c the maximum of the information flow is situated right hand side of the minimum error!

6 CONCLUSIONS

To select a suitable measuring system a priori information of the input signal is necessary. Especially in measurement this information is not available. Therefore approximations are used leading for instance in the very important case of dynamic measurement to the recommendation to analyse the output signal as mentioned before. Often in practice small or medium errors have more detrimental effects than large measuring errors: The latter case normally becomes evident during the testing period while with smaller errors due to the safety margins the products constructed due so theses measurements will withstand the test period but after a certain time according to the fatigue curve the elements will break down.

REFERENCES

[1] E.-G. Woschni, *Informationstechnik, 4th Ed.*, Verlag Technik, Berlin, 1990.
[2] E.-G. Woschni, Signal Processing after Sampling - Today's Problems and Solutions, *Proceedings of the International Symposium „Measurement Technology“,* Hayama, Japan, 1996.
[3] P.H. Sydenham (ed.), *Handbook of Measurement Science,* Vol.1, J. Wiley, Chicester, New York, 1982, p.135.
[4] E.-G. Woschni, Zur Gültigkeit des Überlagerungsgesetzes bei Näherungsbetrachtungen, *Wiss. Zeitschr. d. TH Karl-Marx-Stadt,* 1975, p. 627-631.
[5] C.E. Shannon, A Mathematical Theory of communication, *BSTJ* 27, 1948, p. 379-423, 623-656.

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