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OBJECT-PREDICATE RECIPROCITY IN THE NOMINAL MEASUREMENT

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Abstract: Within a formal framework, where each object is characterized by a set of predicates as a binary sequence of 0 and 1, a nominal level of measurement is discussed. The nominal scale is reinterpreted as a set of predicates, and nominal measurement as an assignment of binary sequences to the objects. One remarkable character of this formalization is a reciprocal relationship between the objects and the predicates. Taking this reciprocity as a heuristic guiding principle, a new notion of generalized indiscernibility between two sets of objects is introduced, which enable us to examine various complicated relationship among subsets of objects. As an application to nominal, non-quantitative measurement, algorithm of clustering is proposed, and its merit is discussed in comparison with a method of pattern recognition based on the principle of minimum entropy. The result suggests the importance of structural inter-group relationship that cannot be reduced easily to simple numerical variables. The notion of object-predicate reciprocity will helps us to analyse complicated object-predicate relations reflected in the result of nominal measurement, and to enlarge the scope of the newly emerging measurement science.

Keywords: nominal scale, classification, measurement science

1 INTRODUCTION

At the early stage of 1940s Stevens introduced four scales of measurement, nominal, ordinal, interval and ratio scale, aiming at releasing the definition of measurement from a restricted class of empirical operations, and establishing a more general theory of measurement [1][2]. Ever since Stevens theory of measurement received an axiomatic formalization, and the ordinal, the interval and the ratio scales were much discussed and studied in this course of development [3], while the nominal scale rarely had a theoretical consideration.

In terms of the empirical operations the nominal scale is supposed to correspond to determination of the equality. In nominal scaling, numbers are used only to represent identity or difference with respect to some attribute, and a measurement in nominal scale, in essence, amounts to a classification of objects, while there have been arguments that classification procedure should not be regarded as measurement [4]. The question as to whether classification is really nominal measurement is not completely resolved.

As for Stevens, in his 1956 paper [2], he emphasized the importance of the nominal scale, and referred the emerging information theory as an example which upholds his argument: a transmission of a sequence of symbols is nothing but ‘the treatment of data at the nominal level of measurement’ [2]. As early as 1976, Finkelstein analysed the concept of pattern recognition from the point of view of measurement theory, and pointed out that the procedure of pattern recognition is closely analogous to that of nominal measurement [5]. He also proposed regarding a description of objects by using non-numeral symbols as an extension of the concept of measurement [6].

Recently, in an effort to establish a new framework of measurement science, the nominal scale came to be reconsidered, and be received a new treatment as basis of testing, diagnosis, identification and pattern recognition [7]. Muravyov proposed an integrated formal framework for measurement in the nominal, the ordinal and the ratio scales, reducing a general measurement problem to a kind of combinatorial optimization procedure [8][9]. Reviewing recent trends and advances of measurement science [7], Van Biesen pointed out that a structural complexity of relations in a set describing the objects under investigation is increasing from the ratio to the nominal scale, and argued that in a sense ‘measurement in nominal scale yields more information than measurement in ratio scale’ [7, p. 17].

We do not intend here to give a comprehensive discussion on the nature of the nominal scale, nor to answer all problems connected with nominal measurement. The aim of the present paper is rather to propose a new view of a formal aspect of nominal measurement, and to establish such a view as a one of the basic principles to analyse the complicated relationship between the objects and the predicates, or attributes, to describe the objects. Assuming that each object is characterized by a set of predicates as a binary sequence of 0 and 1, we formulate nominal measurement as an assignment of binary sequences to the objects, and reduce it to a classification of objects by predicates [10]. The formalization is completely symmetrical between the role of objects and that of predicates, and allows us to consider not only a classification of objects by predicates but also a classification of predicates by objects. Combining these two, we introduce the notion of object-predicate reciprocity. This heuristic principle of reciprocity leads us to a definition of indiscernibility between two sets of objects, which enable us to examine various complicated relationship among subsets of objects. We apply this notion to pattern recognition, propose a heuristic approach to clustering, and discuss its merit in comparison with a method of pattern recognition based on the principle of minimum entropy [11]. Our result also clarifies so far unexplored aspect of the formal entropy function, and points out its limitation as a measure of similarity and cohesion. The following consideration will also includes some insights into this line of interest.

2 REINTERPRETATION OF THE NOMINAL SCALE

In this section we formulate nominal measurement as a kind of classification procedure [10]. We first review briefly the formal theory of measurement [12][13].

2.1 Formalization of nominal measurement

Let Q be a set of quality manifestations: $Q=\{q_1, q_2,...\}$, R be a set of empirical relations: $R=\{r_1, r_2,...\}$, N be a class of numbers, and P be a set of numerical relations defined on N. Then measurement M is defined as a homomorphism from an empirical relational system < Q, R > to a numerical relational system < N, P >:

$$M: < Q, R > \to < N, P >. \tag{1}$$

That is, M maps Q to N in such a way that each empirical relation is related to a corresponding numerical relation one-to-one, through which we regard < N, P > as a numerical representation of < Q, R >.

Now let us take an empirical equivalence relation ~ as R, and a set Z of symbols, instead of N, with a symbol equality =. Nominal measurement is defined as a homomorphism from a relational system < Q, ~ > to a symbol system < Z, = >. The equivalence relation ~ induces a partition of Q into equivalence classes, and all members of an equivalence class are mapped into the same element of Z. Thus each element of Z can be considered a class name, and whether it is numeral or not is immaterial [5].

In the nature of the formal theory this formalization states nothing about a quality under consideration, nor about a definition of the equivalence, while a clear concept of quality precedes the establishment of a scale of measurement [5]. Besides, formulation of measurement procedure in nominal scale is also a subject of interest [7]. Therefore, if a definition of the equivalence could be made clear, and criteria of the determination be given, it may help us to put forward the formal approach to nominal measurement. Bearing this respect in mind we formulate characterization of objects by means of predicates as follows:

Let X be a set of objects under consideration, $X = \{ x_1, x_2, ... \}$, and Y be a certain set of predicates, or attributes, $Y = \{ y_1, y_2, ... \}$, which are relevant and meaningful to describe the members of X. That is, we assume that for all pairs of x_i and y_j the proposition `the object x_i satisfies, or affirms, the predicate y_j' is meaningful whether it is true or false. For simplicity, we further assume that each predicate is applied to each object either affirmatively or negatively. Then we have an object-predicate table whose (i, j)-component is equal to 1 or 0 according to whether object x_i affirms or negates predicate y_j.

Let us take a subset A from Y. When two members of X have the same row of the table with respect to A we say these two are indiscernible with respect to A, and denote $x_i \sim^A x_j$. This equivalence relation induces a partition on X. We call this partition a classification of X with respect to A, denoting C(A).

Such a formalization allows us to consider nominal measurement as a kind of classification procedure, and a set of predicates as a nominal, non-numerical scale: an empirical equivalence relation with respect to a certain set of predicates is related to a coincidence of corresponding binary sequences, or rows of the table, and all members of an equivalence class are mapped to the same binary sequence.

As a reinterpretation of the nominal scale, our formalization looks somehow queer from the ordinary view of measurement theory, yet we presume our approach has some merits appealing as follows: First, through such a formalization, and by emphasizing the role of predicates, we can make clear that a measurement calls for inevitably some characterization or description of objects, and that a certain choice of predicates as classifier reflects the intention of measurement and directs its aspect. Second, Stevens introduced the nominal scale as a scale whose structure remains unchanged under a permutation of the numbers assigned to the objects, This character is inherited in our formalization: characterization of objects remains essentially the same under a permutation of predicates. Third, Finkelstein pointed out that measurement is not naming nor labeling, and that it should involve the symbolization of empirical relations of objects [6]. In our formalization these relations are assured by empirical meanings of predicates, and conversely, the implication relation of predicates follows from the inclusion relation of equivalent classes of objects. Fourth, numerical structure is only one of the possible structures to be brought into a set of predicates. Starting from the predicate, not from the scale value, will help us to enlarge the scope of measurement theory beyond mere numericalization [10]. Therefore our formalization will be allowed to claim to be a candidate for a possible reinterpretation of the nominal scale.

2.2 Formal theory of classification: classification of objects by predicates

The formalization sketched above leads us to a formal theory of classification [10]: Let us take two sets A and B of predicates, and consider two ways of classification C(A) and C(B). When C(B) is a refinement of C(A) we say C(A) is less fine than C(B), and denote $C(A) \prec C(B)$. When two classification are the same we denote C(A)~C(B). Then we can examine basic natures of classification procedure, and derive many interesting properties of operations of classifiers. For example, if $C(A_1)$ is less fine than $C(A_2)$ then this relation is preserved for the addition of any set B of predicates:

$$C(A_1) \prec C(A_2) \Rightarrow C(A_1 \cup B) \prec C(A_2 \cup B). \qquad (2)$$

However, as for the subtraction, such a relation does not hold in general:

$$C(A_1) \prec C(A_2) \Rightarrow C(A_1 \setminus B) \prec C(A_2 \setminus B). \qquad (3)$$

A close examination shows that if we obey the logical rules faithfully the operation of classifiers is not so simple as it might appear. It needs careful treatment.

3 PRINCIPLE OF OBJECT-PREDICATE RECIPROCITY

In this section, noticing the formal symmetry between the objects and the predicates, we introduce a heuristic guiding principle of reciprocity, and discuss its potential significance in nominal measurement.

3.1 Classification of predicates by objects

The basic assumption of our consideration is that we are given a rectangular matrix whose entries are either 0 or 1. Hence, from a formal point of view, we can consider not only a classification of objects but also a classification of predicates by objects. Combining these two views of classification, we have

Proposition 3.1 If two predicates a and b are indiscernible with respect to a set X of objects then these two predicates give the same classification of X:

$$a \sim^{A} b \Rightarrow C(a) \prec C(b). \qquad (4)$$

The converse, however, is false in general. Even if two predicates give the same classification, they are not necessarily indiscernible with respect to X [10].

Considering the formal symmetry between the objects and the predicates this `oneway-ness' seems rather peculiar. One may conceive that two predicates that give the same classification should be identified as classifier, and that if it is not so then there is something wrong in our definition of indiscernibility, however it seems natural.

3.2 Principle of object-predicate reciprocity

Such an observation leads us to the notion of object-predicate reciprocity: The indiscernibility of classifiers and the coincidence of corresponding classifications should be equivalent. In other words, the characterization of objects by means of predicates should be balanced with that of predicates by means of objects. This choice is partly because of our aesthetic taste for the formal symmetry, but partly because of consideration as follows:

Measurement, in general, is an acquisition of knowledge about the external world. Therefore, first of all, it should be objective. However, in the field of modern philosophy of science, some philosophers argue that there exists no such thing as a completely objective knowledge, and that every experimental data is `infected' with some theoretical construction inevitably [14][15]. Such a view may be intuitively acceptable because measurement data is always subject to various factors ranging from physical limitation of instrumentation to our intention of measurement. One possible way to meet the situation, and to maintain the objectivity, is to remove empirical, extra-logical factors as far as possible, and to concentrate on a purely formal relationship reflected in the measurement data.

Besides, secondary, when we extract information from measurement data we want to have as much information as possible. Combining two aspects of the measurement data, that is, a classification of objects by predicates and that of predicates by objects, we may obtain more information, which will help us to examine `structural complexity of relations' [7, p.17] between the object and the predicates.

In the following consideration, we take this notion of reciprocity for our heuristic guiding principle, and examine where such a notion leads us to.

3.3 Weak indiscernibility and its extension

Let us modify the definition of indiscernibility in order for the reciprocity to be maintained [10]: We say two predicates a and b are weakly indiscernible with respect to X, denoting $[a] \sim^X [b]$, if either of two cases $a \sim^X b$ and $\bar{a} \sim^X b$ holds, where $\bar{a}$ is the negation of a: a column corresponding to $\bar{a}$ is obtained by inverting 0 and 1 in a column corresponding to a. Then we have the desired equivalence

$$[a]\sim^X[b] \Leftrightarrow C(a)\sim C(b), \quad (4')$$

and it can be extended to a general case as follows [10]: Let A and B be two sets of predicates. By augmenting the members of A by the use of $\wedge$, $\vee$ and $\neg$ until finally nothing new can be produced, we can obtain a collection $\mathcal{A}$ of all possible predicates engendered by the members of A. Such a collection is called the completed Boolean lattice of predicates. With this terminology, we define the generalized indiscernibility between two sets of predicates by the equality of corresponding completed Boolean lattices. Then we have

$$[A] \sim^X [B] \Leftrightarrow C(A)\sim C(B). \quad (5)$$

This result suggests us that when we compare two sets of predicates we should see not the set of predicates itself but all possible logical combinations of the predicates. If we consider a set of predicates as a nominal scale then the result implies that two scales that give the same completed Boolean lattice should be identified. In other words, our nominal scale is determined uniquely up to the lattice isomorphism.

By virtue of the formal symmetry between the objects and the predicates, our result also suggests us a definition of the generalized indiscernibility between two sets of objects. Then we have the equivalence

$$[X_I] \sim^X [X_J] \Leftrightarrow C(X_I) \sim C(X_J). \quad (5')$$

4 APPLICATION TO PATTERN RECOGNITION

From a point of view of measurement theory, procedure of pattern recognition is closely analogous to nominal measurement [5]. In this section we apply the notion of generalized indiscernibility to pattern recognition, and propose a heuristic approach to clustering. We start with a brief review of the method of interdependence analysis (IDA) [16] [17].

4.1 Interdependence analysis (IDA)

In the field of pattern recognition various kinds of algorithms to carry out clustering tasks have been developed, a large number of which are based upon the notion of distance between two objects. Satosi Watanabe, the physicist, emphasized the importance of taking `more-than-two-elements correlation' into account, and proposed the method of interdependence analysis (IDA) by using a formal entropy function as a measure of similarity among groups of objects [16][17]. The gist of the algorithm can be summarized in our terminology as follows:

Let X be a set of objects, A be a set of predicates, and T be a corresponding object-predicate table.

We take a subset of X, say X_I, and form a subtable of T by picking up the rows corresponding to the members of X_I. Among its columns some column types appear repetitively. Denoting a relative frequency of appearance of the ith column type pi, we define the formal entropy of X_I by

$$S(X_I) \equiv -\sum_i p_i \log p_i. \qquad (6)$$

Let us take another subset of X, say X_J. The formal interdependence between X_I and X_J with respect to A is given by

$$J(X_I, X_J) \equiv S(X_I) + S(X_J) - S(X_I \cup X_J). \qquad (7)$$

The function J thus defined is nothing but a counterpart of the redundancy in the information theory à la Shannon. Regarding this J as a measure of cohesion, or similarity, the method of IDA claims that a set X of objects should be divided so that the corresponding interdependence J is minimized. By showing example Satosi Watanabe proved that his method is indeed capable of exposing very subtle multilateral properties among groups of objects. His aim was to formulate pattern recognition procedure as a process of entropy minimization in a broad sense [17]. In this respect we have the following proposition [11].

Proposition 4.1 Let X_I and X_J be two sets of objects. If a classification of predicates by X_I is less fine than a classification by X_J then the formal entropy of X_I is never larger than that of X_J:

$$C(X_I) \prec C(X_J) \Rightarrow S(X_I) \le S(X_J). \qquad (8)$$

4.2 Heuristic approach to clustering: structural decomposition (SD)

Based on this relationship we can formulate a heuristic procedure to find out a suitable partition of X in accordance with the principle of minimum entropy as follows: Let n be a cardinality of X, k be an integer ≥ 1,

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
x_1	1	1	1	1	0	0	0	0
x_2	1	1	0	0	1	1	0	0
x_3	0	0	1	1	0	0	1	1
x_4	0	1	0	1	0	1	0	1

Table 1. Four girl students in a dormitory [16] [17]. Each row has the same number of 1's and 0's. Besides, each pair has same Hamming distance.

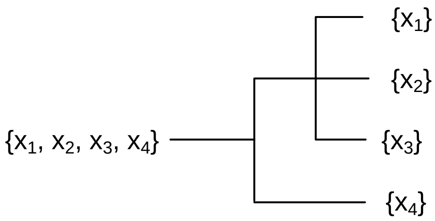

Figure 1. Clustering of the four girl students in a dormitory. After removing $\{x_4\}$, we have no ground for choosing any particular dichotomy.

X_k be a collection of subsets of X whose member contains k elements, and $n(X_I)$ be the number of equivalent classes of a classification $C(X_I)$ ($X_I \subset X$). Consider all possible dichotomies of X into k elements and n-k elements, and find out a dichotomy $\{ X_*^{(k)}, X \setminus X_*^{(k)} \}$ such that

$$n(X \setminus X_*^{(k)}) \le n(X \setminus X_i^{(k)}) \quad (\forall \ X_i^{(k)} \in X_k). \qquad (9)$$

We denote a collection of such dichotomies D_k^*. If $n(X \setminus X_i^{(k)})$'s are the same for all members of X_k then we consider D_k^* to be empty. We apply the following prescription first to the entire set X, then to each of the resulting subsets of X. We continue this procedure until each subset becomes a subset with a single element.

1° Starting from k=1, we form D_k^* for k=1, 2, ..., [n/2]. If all D_k^*'s are empty then we have no ground for choosing any particular dichotomy, and the prescribed partition is the polychotomy into $\{x_i\}$'s.

2° Otherwise, let k^* be the smallest integer such that $D_{k^*}^* \neq f$. If $D_{k^*}^*$ has a single dichotomy $\{ X_{k^*}, X \setminus X_{k^*} \}$ then it is the prescribed partition.

3° Suppose that $D_{k^*}^*$ contains more than one dichotomy. Compare these dichotomies by the IDA method. We denote the dichotomies obtained by IDA d_1^*, d_2^*, ..., $d_{m_k}^*$. Note that we can define a predicate a' which corresponds to a dichotomy into X' and X \ X' by C(a')={X', X \ X'}. Let a_i^* be a predicate which corresponds to a dichotomy d_i^*, and A^* be the entire set of such predicates: $A^* = \{ a_1^*, a_2^*, ..., a_{m^*}^* \}$. The prescribed partition is the polychotomy into the equivalent classes of $C(A^*)$.

Roughly speaking, the gist of this procedure is to find out a large subset of objects that gives less fine classification of predicates. We may call this heuristic approach a method of structural decomposition (SD) since this partitioning is based on the structural intergroup relationship among subsets of objects. This procedure is mere heuristics, and should not be regarded as an established algorithm. But it seems to be fairly reasonable. Let us give two examples.

Example 4.1 This example is known as a problem of `four girl students in a dormitory' [16][17]. In Table 1 each row has four 1's and four 0's, and the members are equivalent in this respect. Besides, each pair of rows has same four column types (1, 1), (1, 0), (0, 1) and (0, 0) two by two. That is, each pair has same Hamming distance, and we cannot find any special relationship in so far as we observe the members pairwise. The method of IDA deals with this example successfully, and the result is intuitively acceptable: $J(\{x_1, x_2, x_3\}, \{x_4\})=0$, otherwise $J=1$, and we have a taxnomic tree as shown in Figure 1. But our method also give the same result by comparing the fineness of classification without calculating any entropy function: we have both of $C(x_4) \prec C(x_i)$ and $C(X \setminus \{x_4\}) \prec C(X \setminus \{x_i\})$ for i=1, 2, 3.

Example 4.2 As for Table 2 results do not coincide with. Since all entries of a row corresponding to y_5 are 1 the first step of IDA may be rather trivial, while the second step is seemingly unbalanced. In our heuristic approach, after removing two members y_1 and y_2, remaining nine members are classified into three groups, and the object-predicate table is rearranged as shown in Table 3. This result shows that each of three groups is characterized by the right-hand side of the table.

The method of IDA has a number of theoretical merits, but is sometimes affected by a minute difference of the entropy value. Our heuristic approach avoids this difficulty by taking structural inter-group relationship reflected in the fineness of classification of predicates into account.

By combining the method of IDA and that of SD we have a new method of clustering, which we may call a structural interdependence analysis (SIDA). This new method also works reasonably well [11].

		a_1	a_2	a_3	a_4	a_5	a_6
X	x_1	1	0	0	0	0	0
	x_2	1	0	1	0	0	0
	x_3	1	1	1	0	0	0
	x_4	1	0	0	0	1	1
	x_5	1	0	1	0	1	1
	x_6	1	0	1	1	1	1
Y	y_1	1	1	1	0	0	1
	y_2	1	0	1	1	1	0
	y_3	1	1	1	0	1	1
	y_4	1	0	0	1	1	1
	y_5	1	1	1	1	1	1

Table 2. The object-predicate table for Example 4.2. In this case a clustering by IDA does not coincide with the result of SD. This table was obtained from actual data on the efficacy of interferon treatment of chronic hepatitis C [10].

		a_1	a_2	a_3	a_4	a_5	a_6
Z_1	x_1	1	0	0	0	0	0
	x_2	1	0	1	0	0	0
	x_3	1	1	1	0	0	0
Z_2	x_4	1	0	0	0	1	1
	x_5	1	0	1	0	1	1
	y_3	1	1	1	0	1	1
Z_3	y_4	1	0	0	1	1	1
	y_6	1	0	1	1	1	1
	y_5	1	1	1	1	1	1

Table 3. The second stage of the clustering by SD. After removing $\{y_1, y_2\}$, remaining nine objects are classified into three groups Z_1, Z_2 and Z_3. Note that same pattern appears repeatedly in the first half of each group.

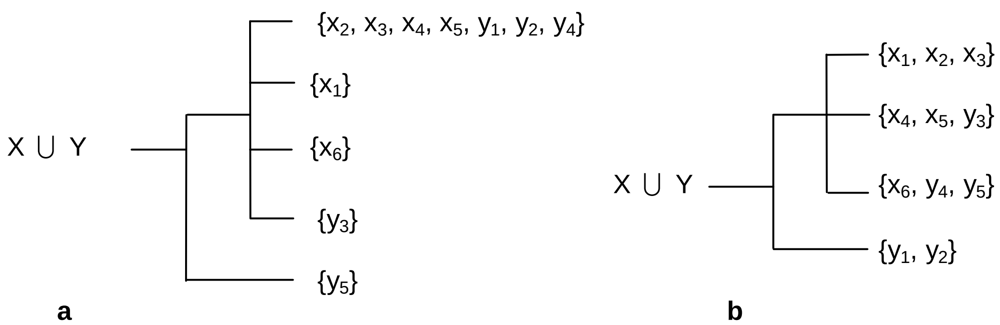

Figure 2. Clustering of the eleven members of Table 2. **a** shows a taxnomic tree obtained by IDA, **b** shows the one by SD. a seems not to fully reflect the object-predicate relation of the data, while b does.

5 CONCLUSION

The nominal scale, least studied of all the four traditional scales, is free from the numerical structure such as transitivity, and may be given a prominent place as a basis of a wide class of non-quantitative measurement. We have formulated nominal measurement as a kind of classification procedure, introduced a heuristic principle of reciprocity, and shown its importance by applying it to a clustering problem in pattern recognition. We have pointed out also so far unexplored aspect of the formal entropy function, and clarified its limitation as a measure of interdependence. The notion of object-predicate reciprocity provide us with a new view of measurement in the nominal scale. It will helps us to analyse complicated object-predicate relationship, and to enlarge the scope of the newly emerging measurement science.

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